The fourth family: A simple explanation for the observed pattern of anomalies in B–CP asymmetries

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ABSTRACT

We show that a fourth family of quarks with $m_{t'}$ in the range of (400–600) GeV provides a rather simple explanation for the several indications of new physics that have been observed involving CP asymmetries of the b-quark. The built-in hierarchy of the 4×4 mixing matrix is such that the t' readily provides a needed *perturbation* ($\approx 15\%$) to $\sin 2\beta$ as measured in $B \to \psi K_s$ and simultaneously is the dominant source of CP asymmetry in $B_s \to \psi\phi$. The correlation between CP asymmetries in $B_s \to \psi\phi$ and $B_d \to \phi K_s$ suggests $m_{t'} \approx$ (400–600) GeV. Such heavy masses point to the tantalizing possibility that the 4th family plays an important role in the electroweak symmetry breaking.

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The spectacular performance of the two asymmetric B-factories allowed us to reach an important milestone in our understanding of CP-violation phenomena. For the first time it was established that the observed CP-violation in the B and K systems was indeed accountable by the single, CP-odd, Kobayashi–Maskawa phase in the CKM matrix [1,2]. In particular, the time dependent CP-asymmetry in the gold-plated $B^0 \to \psi K_s$ can be accounted for by the Standard Model (SM) CKM-paradigm to an accuracy of around 15% [3,4]. It has then become clear that the effects of a beyond the standard model (BSM) phase can only be a perturbation. Nevertheless, in the past few years as more data were accumulated and also as the accuracy in some theoretical calculations was improved it has become increasingly apparent that several of the experimental results are difficult to reconcile within the SM with three generations [SM3] [5,6]. It is clearly important to follow these indications and to try to identify the possible origin of these discrepancies especially since they may provide experimental signals for the LHC which is set to start quite soon. While at this stage many extensions of the SM could be responsible, in this Letter, we will make the case that an addition of a fourth family of quarks [7–11] provides a rather simple explanation for the pattern of deviations that have been observed [12]. In fact we will show that the data suggests that the charge 2/3 quark of this family needs to have a mass in the range of (400–600) GeV [13].

We now briefly mention the experimental observations involving B–CP asymmetries that are indicative of possible difficulties for the CKM picture of CP-violation.

1. The predicted value of $\sin 2\beta$ in the SM seems to be about 2–3 σ larger than the directly measured values. Using only ϵ_k and $\Delta M_s/\Delta M_d$ from experiment along with the necessary hadronic matrix elements, namely kaon "B-parameter" B_K and using $SU(3)$ breaking ratio $\xi_s \equiv \frac{f_{bs}\sqrt{B_{bs}}}{f_{bd}\sqrt{B_{bd}}}$, from the lattice, alongwith V_{cb} yields a prediction, $\sin 2\beta_{noV_{ub}}^{\text{prediction}} = 0.87 \pm 0.09$ [6] in the SM. If along with that $\frac{V_{ub}}{V_{cb}}$ is also included as an input then one gets a somewhat smaller central value but with also appreciably reduced error: $\sin 2\beta_{\text{full fit}}^{\text{prediction}} = 0.75 \pm 0.04$.

2. The celebrated measurement, via the "gold-plated" mode $B \to \psi K_s$, gives $\sin 2\beta_{\psi K_s} = 0.672 \pm 0.024$ which is smaller than either of the above predictions by ≈ 1.7 to 2.1 σ [6].

3. As is well-known penguin-dominated modes, such as $B \to (\phi, \eta', \pi^0, \omega, K_S K_S, \ldots) K_S$ also allow an experimental determination of $\sin 2\beta$ in the SM [14,15]. This method is less clean as it has some hadronic uncertainty, which was naively estimated to be at the level of 5% [15,16]. Unfortunately, this uncertainty cannot be reliably determined in a model-independent manner. However, several different estimates [17] find that

amongst these modes, $(\phi, \eta', K_s K_s) K_s$ are rather clean up to an error of only a few percent. In passing, we note also another intriguing feature of many such penguin-dominated modes is that the central value of $\sin 2\beta$ that they give seems to be below the two SM predicted values given above in #1 and in fact, in many cases, even below the value measured via $B \to \psi K_S$ (given in #2).

4. Another apparent difficulty for the SM is understanding the rather large difference in the direct CP asymmetries $\Delta A_{CP} \equiv A_{CP}(B^- \to K^- \pi^0) - A_{CP}(\bar{B}^0 \to K^- \pi^+) = (14.4 \pm 2.9)\%$ [3]. Naively this difference is supposed to be zero. Using QCD factorization [18] in conjunction with any of the four scenarios for $1/m_b$ corrections that have been proposed [19] we were able to estimate $\Delta A_{CP} = (2.5 \pm 1.5)\%$ [5] which is several σ's away from the experimental observations. It is important to understand that by varying over those four scenarios one is actually spanning the space of a large class of final state interactions; therefore the discrepancy with experiment is serious [20]. However, given our limited understanding of hadronic decays makes it difficult to draw compelling conclusions from this difficulty for the SM3.

5. Finally, more recently the possibility of the need for a largish non-standard CP-phase has been raised [22,21] in the study of $B_s \to \psi\phi$ at Fermilab by CDF [23] and D0 [24] experiments. Since the above items suggest the presence of a beyond the SM CP-odd phase in $b \to s$ transitions as (for example) already emphasized in [5], such non-standard effects in B_s decays are quite unavoidable.

In the following we show that SM with a fourth generation [SM4] is readily able to address these difficulties and in particular the data seems to suggest the need for $m_{t'}$ within the range (400–600) GeV.

SM4 is a simple extension of SM3 with additional up-type (t') and down-type (b') quarks. It retains all the features of the SM. The t' quark like u, c, t quarks contributes in the $b \to s$ transition at the loop level [7]. The addition of fourth generation means that the quark mixing matrix will become a 4×4 matrix and the parametrization of this unitary matrix requires six real parameters and three phases. The two extra phases imply the possibility of extra sources of CP violation [9]. In order to find out the limits on these extra parameters along with the other observables we concentrate mainly on the constraints that will come from B_d–$\bar{B}_d$ and B_s–$\bar{B}_s$ mixing, $\mathcal{BR}(B \to X_s\gamma)$, $\mathcal{BR}(B \to X_s\ell^+\ell^-)$ [30], indirect CP violation in $K_L \to \pi\pi$ described by $|\epsilon_k|$ etc. Table 1 summarizes complete list of inputs that we have used to constrain the SM4 parameter space. With these input parameters we have made the scan over the entire parameter space by a flat random number generator and obtained the constraints on various parameters of the 4×4 mixing matrix. In Table 2 we present the one sigma allowed ranges of $|V_{t's}^* V_{t'b}|$ and ϕ_s' (the phase of $V_{t's}$), which follow from our analysis [28].

The SM3 expressions for ϵ_k and $Z \to b\bar{b}$ decay width have been taken from [31] and [32] respectively whereas the relevant expressions for $\frac{\Delta M_s}{\Delta M_d}$, along with the other observables can be found in [33]. The corresponding expressions in SM4 i.e., the additional contributions arising due to t' quark can be obtained by replacing the mass of t-quark by $m_{t'}$ in the respective Inami–Lim functions. For concreteness, we use the parametrization suggested in [34] for 4×4 CKM matrix [V_{CKM4}]. In ΔM_d and ΔM_s, apart from the other factors, we have the CKM elements $V_{tq}V_{tb}^*$ which can be replaced by (with $q = d$ or s),

$$V_{tq}V_{tb}^* = -\left(V_{uq}V_{ub}^* + V_{cq}V_{cb}^* + V_{t'q}V_{t'b}^*\right) \tag{1}$$

Table 1
Inputs used to constrain the SM4 parameter space; the error on V_{ub} is increased to reflect the disagreement between the inclusive and exclusive methods.

$B_K = 0.72 \pm 0.05$
$f_{bs}\sqrt{B_{bs}} = 0.281 \pm 0.021$ GeV
$\Delta M_s = (17.77 \pm 0.12)$ ps^{-1}
$\Delta M_d = (0.507 \pm 0.005)$ ps^{-1}
$\xi_s = 1.2 \pm 0.06$
$\gamma = (75.0 \pm 22.0)°$
$|\epsilon_k| \times 10^3 = 2.32 \pm 0.007$
$\sin 2\beta_{\psi K_s} = 0.672 \pm 0.024$
$\mathcal{BR}(K^+ \to \pi^+ \nu\nu) = (0.147^{+0.130}_{-0.089}) \times 10^{-9}$
$\mathcal{BR}(B \to X_c\ell\nu) = (10.61 \pm 0.17) \times 10^{-2}$
$\mathcal{BR}(B \to X_s\gamma) = (3.55 \pm 0.25) \times 10^{-4}$
$\mathcal{BR}(B \to X_s\ell^+\ell^-) = (0.44 \pm 0.12) \times 10^{-6}$
(High q^2 region)
$R_{bb} = 0.216 \pm 0.001$
$|V_{ub}| = (37.2 \pm 5.4) \times 10^{-4}$
$|V_{cb}| = (40.8 \pm 0.6) \times 10^{-3}$
$\eta_c = 1.51 \pm 0.24$ [25]
$\eta_t = 0.5765 \pm 0.0065$ [26]
$\eta_{ct} = 0.47 \pm 0.04$ [27]
$m_t = 172.5$ GeV

Table 2
Allowed ranges for the parameters, $\lambda_{t'}^s$ ($\times 10^{-2}$) and phase ϕ_s' (in degree) for different masses $m_{t'}$ (GeV), that has been obtained from the fitting with the inputs in Table 1.

$m_{t'}$	400	500	600
$\lambda_{t'}^s$	(0.08–1.4)	(0.06–0.9)	(0.05–0.7)
ϕ_s'	$-80 \to 80$	$-80 \to 80$	$-80 \to 80$

using the 4×4 CKM matrix unitarity relation, $\lambda_u + \lambda_c + \lambda_t + \lambda_{t'} = 0$ where $\lambda_q = V_{qb}V_{qs}^*$. The phase of V_{td} and V_{ts} will also be obtained by using this unitarity relation. In this way we can reduce the number of unknown parameters by using information from known parameters.

With a sequential fourth generation, the effective Hamiltonian describing the $b \to s$ transitions becomes

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}}\left[\lambda_u\left(C_1^u O_1 + C_2^u O_2 + \sum_{i=3}^{10} C_i^u O_i\right) \right.$$
$$\left. + \lambda_c \sum_{i=3}^{10} C_i^c O_i - \lambda_{t'} \sum_{i=3}^{10} \Delta C_i^{t'} O_i\right], \tag{2}$$

where C_i^q's are the Wilson coefficients, $\Delta C_i^{t'}$'s are the effective (t subtracted) t' contributions and O_i are the current–current operators. Using the above Hamiltonian, and following [5] we use the S4 scenario of QCD factorization approach [19] for the evaluation of hadronic matrix elements and the amplitudes for the decay modes $B \to \pi K$ and $B \to \phi K_s$ for $m_{t'} = 400$, 500 and 600 GeV respectively.

Using the ranges of $\lambda_{t'}^s \equiv |V_{t's}^* V_{t'b}|$ and ϕ_s', as obtained from the fit for different $m_{t'}$ (Table 2), we studied the allowed regions in the $\Delta A_{CP} - \lambda_{t'}^s$ plane for different values of $m_{t'}$. With the 4th family we see that there is some enhancement and ΔA_{CP} up to about 8% may be feasible which is till somewhat small compared to the observed value $(14.4 \pm 2.9)\%$. Again, as we mentioned this could be due to the inadequacy of the QCD factorization model we are using.

In Fig. 1 (left-panel) we have shown the allowed regions in the $S_{\psi\phi}$–$\phi_{t'}^s$ plane for different values of $m_{t'}$ and in the right-panel
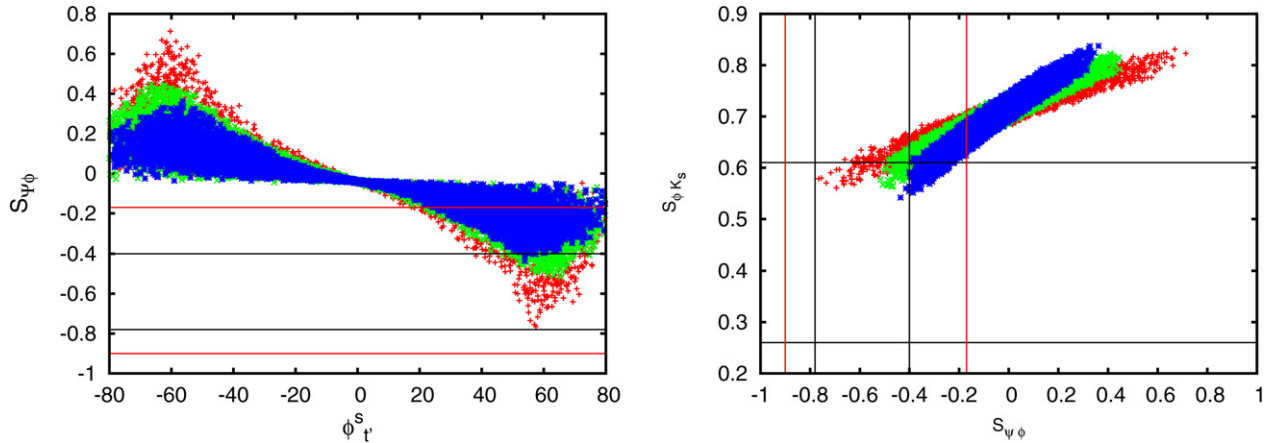
Fig. 1. The left-panel shows the allowed range for $S_{\psi\phi}$ in the $(S_{\psi\phi}-\phi^s_{t'})$ plane for $m_{t'} = 400$ (red), 500 (green) and 600 (blue) GeV respectively. Black and red horizontal lines in the figure indicate 1-σ and 2-σ experimental ranges for $S_{\psi\phi}$ respectively. The right-panel shows the correlation between $S_{\phi K_s}$ and $S_{\psi\phi}$ for $m_{t'} = 400$ (red), 500 (green) and 600 (blue) GeV respectively. The horizontal lines represent the experimental 1σ range for $S_{\phi K_s}$ whereas the vertical lines (black 1-σ and red 2-σ) represent that for $S_{\psi\phi}$. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this Letter.)

of Fig. 1 we have shown the correlation between *CP* asymmetries in $B \rightarrow \psi\phi$ and $B \rightarrow \phi K_s$. We follow the notation $S_{\psi\phi} = \sin(\phi^\Delta_s - 2\beta_s) = \sin 2\beta^{eff}_s$, where ϕ^Δ_s is the phase coming from mixing and $\beta_s = \arg(-\frac{V^*_{tb}V_{ts}}{V^*_{cb}V_{cs}}) = 1.1° \pm 0.3°$, is the phase of $b \rightarrow c\bar{c}s$ decay amplitude [21,35]. The range for new B_s mixing phase ϕ^Δ_s is given (@68% CL) by $\phi^\Delta_s \in (-18 \pm 7)°$ or, $\phi^\Delta_s \in (-70 \pm 7)°$. The corresponding 2-$\sigma$ and 1-σ ranges for $S_{\psi\phi}$ is given by [−0.90, −0.17] and [−0.78, −0.40] respectively. The large error on $S_{\phi K_s}$ and $S_{\psi\phi}$ does not allow at present to draw strong conclusions on $m_{t'}$, nevertheless the present experimental bounds disfavor $m_{t'} > 600$ GeV.

A very appealing feature of the 4th family hypothesis is that it rather naturally explains the pattern of the observed anomalies. First of all the heavy $m_{t'}$ generates a very important new source of electroweak penguin (EWP) contribution since, as is well known, these amplitudes are able to avoid the decoupling theorem and grow as $m^2_{t'}$ [7,36]. This helps to explain two of the anomalies in $b \rightarrow s$ transitions. The enhanced EWP contribution helps in explaining the difference in *CP*-asymmetries, ΔA_{CP} as it is really the $K^\pm\pi^0$ that is enhanced because of the color allowed coupling of the Z to the π^0. A second important consequence of t' is that $b \rightarrow s$ penguin has a new *CP*-odd phase carried by $V_{t'b}V^*_{t's}$. This is responsible for the fact that $\sin 2\beta$ measured in $B \rightarrow \psi K_s$ differs with that measured in penguin-dominated modes such as $B \rightarrow (\phi, \eta', K_sK_s, \ldots)K_s$.

Note also that $\Delta B = 2$ box graph gets important new contributions from the t' since these amplitudes as mentioned before are proportional to $m^2_{t'}$. Furthermore, they are accompanied by new *CP*-odd phase which is not present in SM3. This phase is responsible for the fact that the $\sin 2\beta$ measured in $B \rightarrow \psi K_s$ is lower than the value(s) "predicted" in SM3 [6] given in item #1 on page 1.

Finally, we note briefly in passing how SM4 gives a very simple explanation for the size of the new *CP*-phase effects in B_d versus B_s mesons. In B_d oscillations resulting in $B \rightarrow \psi K_s$, top quark plays the dominant role and we see that the measured value of $\sin 2\beta$ deviates by $\approx 15\%$ from predictions of SM3. It is then the usual hierarchical structure of the mixing matrix (now in SM4) that guarantees that on $\sin 2\beta$, t' will only have a subdominant effect. However, when we consider B_s oscillations then the role of t' and t get reversed. In B_s mixing the top quark in SM3 has negligible *CP*-odd phase. Therein then the t' has a pronounced effect. SM4 readily explains that just as t is dominant in $\sin 2\beta$ and sub-

dominant in $\sin 2\beta_s$, the t' is dominant in $\sin 2\beta_s$ and subdominant in $\sin 2\beta$.

We now briefly summarize some of the definitive signatures of the 4th family scenario in flavor observables [29]. The need for new *CP* phase(s) beyond the single KM phase [2] of course must continue to persist. This means that the three values of $\sin 2\beta$, the fitted one, the one measured via ψK_s and the one measured via penguin dominated modes (e.g. ϕK_s, $\eta' K_s$, etc.) should continue to differ from each other as more accurate analyses become available. Furthermore, B_s mixing should also continue to show the presence of a non-standard phase (e.g. in $B_s \rightarrow \psi\phi$) as higher statistics are accumulated. For sure SM4 will have many more interesting applications in flavor physics which need to be explored. For the LHC, one definitive prediction of this analysis is a t' with $m_{t'}$ in the range of ≈ 400–600 GeV and the detection of the t', b' and their leptonic counterparts deserves attention. EW precision constrains the mass-splitting between t' and b' to be small, around 50 GeV [37,38].

As far as the lepton sector is concerned, it is clear that the 4th family leptons have to be quite different from the previous three families in that the neutral leptons have to be rather massive, with masses $> m_Z/2$. This may also be a clue that the underlying nature of the 4th family may be quite different from the previous three families; for one thing it could be relevant to the dark matter issue [39]. It may also open up the possibility of unification with the SM gauge group [40]. KM [2] mechanism taught us the crucial role of the three families in endowing *CP* violation in SM3. It is conceivable that 4th family plays an important role [41–44] in yielding enough *CP* to generate baryogenesis which is difficult in SM3. Of course it also seems highly plausible that the heavy masses in the 4th family play a significant role in dynamical generation of electroweak symmetry breaking. In particular, the masses around 500 or 600 GeV that are being invoked in our study, point to a tantalizing possibility of dynamical electroweak symmetry breaking as the Pagels–Stokar relation in fact requires quarks of masses around 500 or 600 GeV for dynamical mass generation to take place [45–48]. Note also that for such heavy masses the values of Yukawa coupling will be large so that corrections to perturbation theory may not be negligible [49]. Finally, we want to emphasize that a fourth family of quarks does *not* violate electro-weak precision tests [50]. Clearly all this brief discussion is signaling is that there is a lot of physics involving the new family that needs to be explored and understood.

Acknowledgements

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