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Nonlinear Frequencies and Unbalanced Response Analysis of High Speed Rotor-Bearing Systems

Phadatare H. P., Barun Pratiher*

Department of Mechanical Engineering, Indian Institute of Technology, Jodhpur, 342011, India

Abstract

This paper describes a step forward in calculating the nonlinear frequencies and resultant dynamic behavior of high speed rotor bearing system with mass unbalance. Nonlinear strongly coupled equations of motion has been formulated based on strain energy and kinetic energy equations for shaft, disk and unbalance mass with shaft undergoing large bending deformations. Here gyroscopic effects of disk as well as mass unbalance are also considered while vibration effect along the shaft axis is ignored. Time history and FFT analysis for finding the fundamental frequencies for the rotating are portrayed under variation of shaft diameter, frequency of the shaft speed, geometric nonlinearity and disk location. The present research shows an interesting development that the initial conditions are playing an important role in finding the nonlinear frequencies and this variation is strongly due to the presence of nonlinear geometric coupling. In addition, response analysis of the system has been developed due to mass unbalanced using time history. This paper enables an understanding and realization of operating zones of rotational speed of the shaft by which the excessive vibration can easily be avoided due to the resonant conditions occurred as the natural frequencies come closer to the frequency of the rotational speed.

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Keywords: Shaft-disk; Higher order deformation; Inextensible condition; Method of multiple scales; Free vibration analysis; Critical speeds;

1. Introduction

Rotor dynamics being appealing subset of vibration due to its varied applications as well as vast opportunities to explore the subject in multiple dimensions. Rotating machines exist in many high end engineering applications such as jet engines, helicopter rotors, turbines, compressors and the spindles of machine tools as well as routine applications such as pump, fan, blower, IC engine etc. which makes rotor dynamics an important field of study. Kinetic energy always exist in forms the internal energy of the system because of connection to the driving

* Corresponding author.
E-mail address: barun@iitj.ac.in

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system. Hence, the rotating elements possess enormous amount of operating energy which can be converted into vibrations due to which the prediction and analysis of the rotor system becomes essential.

Mass unbalance is very commonly observed in industrial application due to reasons such as material characteristic, machine design, non-uniform wear or corrosion, sticking of external particles etc. As the speed of rotation increases the impact of mass unbalance increases significantly and if not predicted and controlled may result in catastrophic failure occurs thus disrupting the smooth functioning of the same. Dynamics of rotating shafts is therefore necessary for an accurate design. Free vibration analysis is one of the vital steps for analysis in rotor dynamics. Over years, research on rotor dynamics is being carried on. For the instances, the effect of shear deformation and rotary inertia of a rotor on its critical speeds was studied by R. Grybos [1]. Choi et al [2] displayed the consistent derivation of a set of governing differential equations describing the flexural and the torsional vibrations of a rotating shaft where a constant compressive axial load was acted on it. Using the beam and shell theory, Singh and Gupta [3] investigated free damped flexural vibrations analysis of composite cylindrical tubes. Almasi [6] developed a model considering large deformations of a rotor based on virtual work theory. Chang and Cheng [7] studied the instability and nonlinear dynamics of a slender. Villa et al. [8] used invariant manifold approach to numerically investigate the responses of a nonlinear rotor-bearing system. Shabaneh and Zu [9] investigated the dynamic analysis of a single-rotor shaft system with nonlinear elastic bearings at the ends mounted on visco-elastic suspension.

In general linear analysis is done as it is easy. But there demand for more accurate and refined solution, consideration of nonlinearities has become important and that result into tedious process. Nonlinearities in the rotating system can be in the form of higher order large deformations in bending, geometric and inertial nonlinearities, gyroscopic effect, etc. A better result can be obtained considering the nonlinearities mentioned. Hosseni and Khadem [4] analyzed the free and forced vibration of nonlinear rotor-bearing systems by means of multiple scales method. Rizwan et al. [5] derived the mathematical modeling and investigated that nonlinear phenomenon of nonlinear rotor dynamics due to higher order deformations in bending considering geometric nonlinearity and gyroscopic effect.

2. Mathematical Modeling And Approaches

The geometry of the rotor system considered for this work has been shown in Fig.1 where the shaft, considered being a beam of circular cross section of length L and radius R_s has been modeled by its kinetic and strain energies.

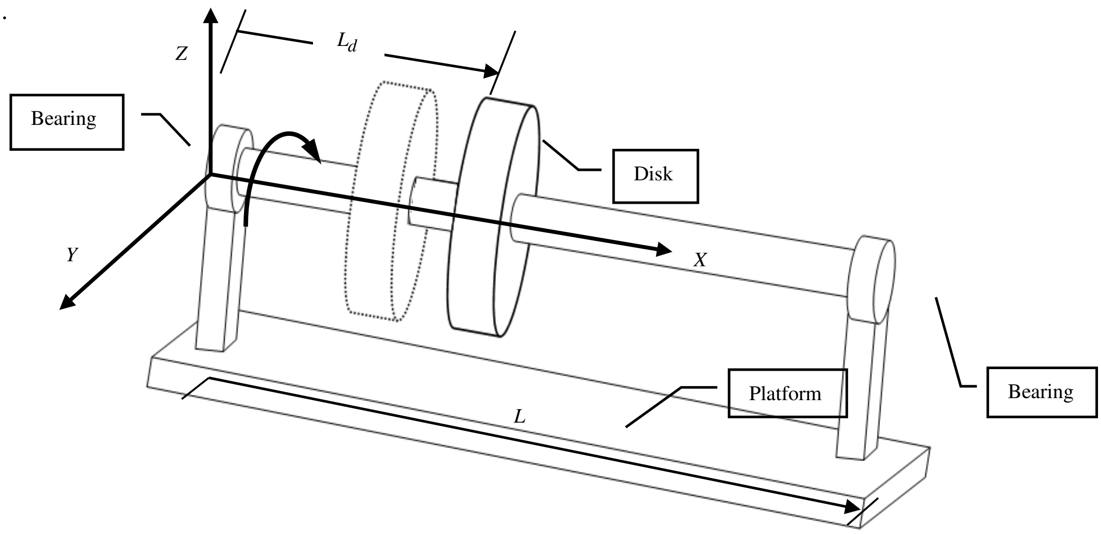

Figure 1: Rotor System with shaft and disk

The disk of external radius R_d positioned at a distance $L_d = L/3$, has been considered to be rigid and hence only requires kinetic energy for its characterization. The mass unbalance denoted by m_u has been located at a distance d from the geometric center of the shaft. The concept of Euler-Bernoulli beam theory has been used to derive the governing equations of motion which describes the dynamic characteristics of rotor bearing combinations. Following mathematical expressions have been developed to display the dynamic behaviors of the rotor bearing system in the directions of y and z as the deformation along x axis has been considered to be negligible

Similar to the procedures and concept adopted [4,5] and using generalized Galerkin's principle Following mathematical expressions have been developed to display the dynamic behaviors of the rotor bearing system in the directions of y and z as the deformation along x axis has been considered to be negligible.

$$\ddot{v}-\Omega\vartheta_1\dot{w}+d\vartheta_2 v+\left(\frac{\kappa_1}{2}+\kappa_2\right)\left(v^3+vw^2\right)+\zeta\dot{v}=m_u\Omega^2 d\, f\left(L_d\right)\sin\Omega t\,, \tag{1}$$

$$\ddot{w}+\Omega\vartheta_1\dot{v}+d\vartheta_2 w+\left(\frac{\kappa_1}{2}+\kappa_2\right)\left(w^3+wv^2\right)+\zeta\dot{w}=m_u\Omega^2 d\, f\left(L_d\right)\cos\Omega t\,. \tag{2}$$

Here, v and w are represented the displacement in Y and Z directions. Normalized first mode shape of a Euler-Bernoulli with a constant cross section in bending and simple supported at both ends has been used and expressed as $f\left(x\right)=\sin\left(\pi x/L\right)$. It has been observed that equation of motion contains linear damping term and gyroscopic effect $\Omega\vartheta_1\dot{Q}+c\dot{P}$, nonlinear geometric coupled terms (i.e., P^3+PQ^2) due to mid-plane stretching effect of the shaft and forcing term (i.e., $m_u\Omega^2 d\, f\left(l_d\right)\sin\Omega t$) due to mass unbalance. The expressions for the coefficients can be obtained by using the generalized Galerkin's principle with the consideration of single of vibration technique similar to the procedures explained in [5]. In the next section, the effect of various design parameters in graphically investigating the linear and nonlinear natural frequencies of the rotor-bearing system along with studying of nonlinear responses due to mass unbalanced. These outcomes have been depicted via time history and FFT analysis by using Runge-Kutta Method of higher order.

3. Results And Discussion

In all numerical simulation a metallic shaft element with length L = 0.4 m, radius R_1 = 0.01 m, mass density (ρ) = 7800 kg/m^3, Young's modulus E = 2×10^{11} N/m^2, and the rigid disk with outer radius R_2 = 0.15 m and thickness h = 0.03 m, respectively. Here, distance of mass unbalance from the geometric centre of disk has been chosen as 0.15 m as that of the point laying on the outer surface of the disk element. Here damping constant c has been considered as 0.001 N-s/m^2 for all simulations. Time history and FFT analysis have been developed to investigate the linear and nonlinear natural frequencies and their behaviour under the variation of spinning speed, and geometric nonlinearity. Phase portrait has also been illustrated to understand the nature of response. The variation of mass unbalanced has also been addressed via time history.

3.1. Effect of Spinning Speed

In the present rotor-bearing system, shaft element is flexible in the directions except along the axis the shaft axis. Hence, vibration amplitude with definite deformation pattern is illustrated under various spinning speed. A periodic response in Figs. 2 and 3 has been observed when subjected to initial conditions i.e., $v(0)=0, \dot{v}(0)=0,$ $w(0)=0,$ and $\dot{w}(0)=0.$ The natural frequencies about both directions are same as 47.24 Hz as presented in Fig. 4. The natural frequencies have been obtained graphically for various values of spinning speeds. For each case, the initial conditions and location of disk have been kept constant and neglected the nonlinear parts of the equations while calculating the natural frequencies. It has been observed that with increase in spinning speed, not only increased amplitude of the response but also increases the difference between the forward and backward natural frequencies from the Figs. 5-8. Also, two-period, quasi-periodic motion has been observed for any spinning speed except zero. Simultaneously, it has been investigated that for low value of spinning speed, a beating phenomenon has been developed. The variation of natural frequencies with respect to the spinning speed has been developed in Fig. 9 and it has clearly been established that range between both frequencies becomes larger for higher value of rotational speed of the shaft.

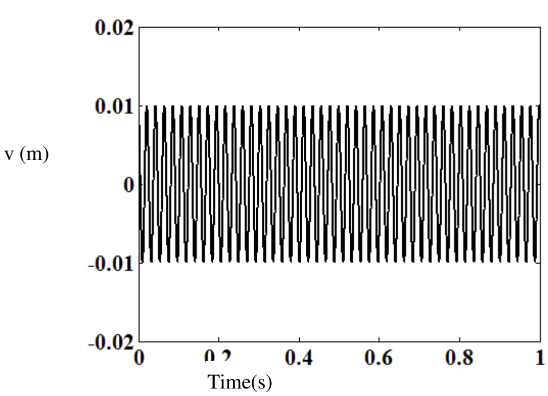

Fig. 2: Time history of shaft-disk system when subjected to initial excitation without spinning speed

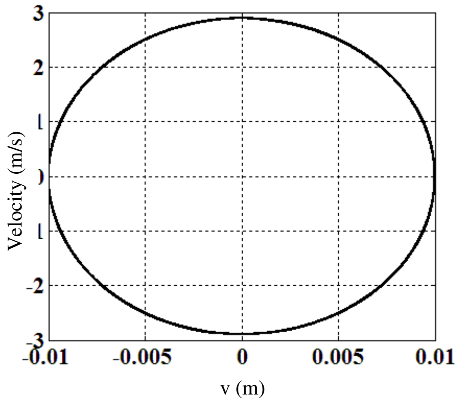

Fig. 3: Phase portrait of shaft-disk system when subjected to initial excitation without spinning speed

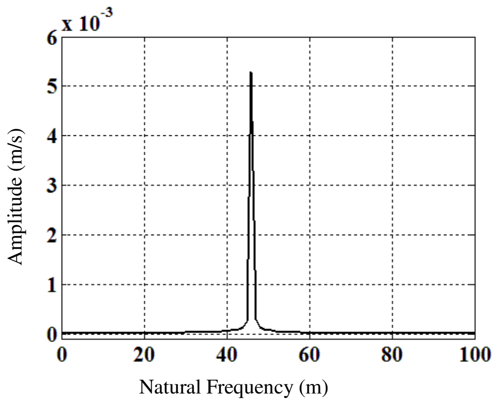

Fig. 4: FFT of shaft-disk system when subjected to initial excitation without spinning speed

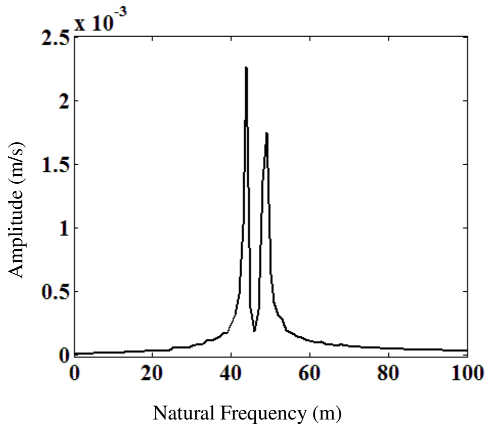

Fig. 5: FFT of shaft-disk system when subjected to spinning speed equal to 150 rad/sec

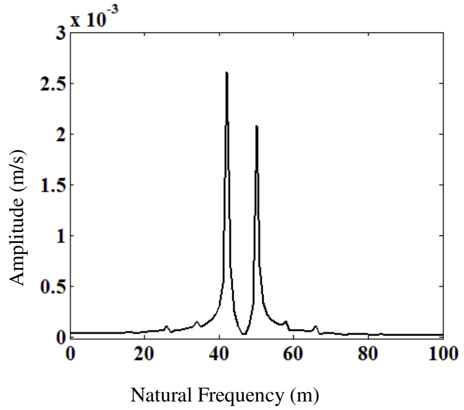

Fig. 6: FFT of shaft-disk system when subjected to spinning speed equal to 250 rad/sec

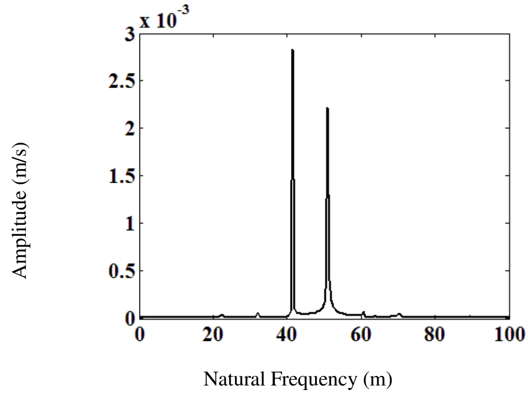

Fig. 7: FFT of shaft-disk system when subjected to spinning speed equal to 300 rad/sec

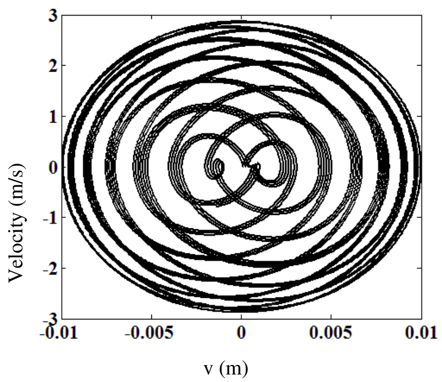

Fig. 8: Phase portrait of shaft-disk system when subjected to spinning speed equal to 300 rad/sec

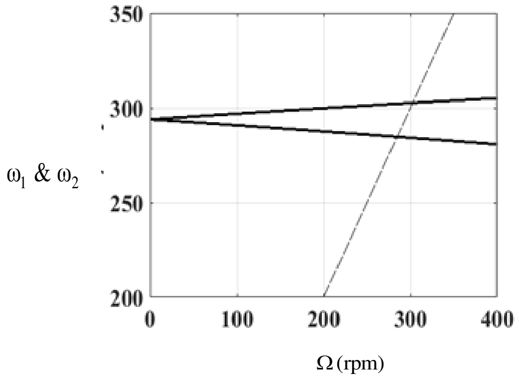

Fig. 9: Campbell diagram of shaft-disk system

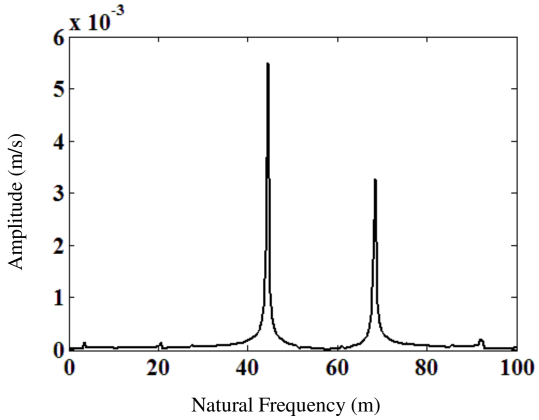

Fig. 10: Natural frequencies of shaft with disk located at *L*/5 when subjected to spinning speed equal to 200 rad/sec

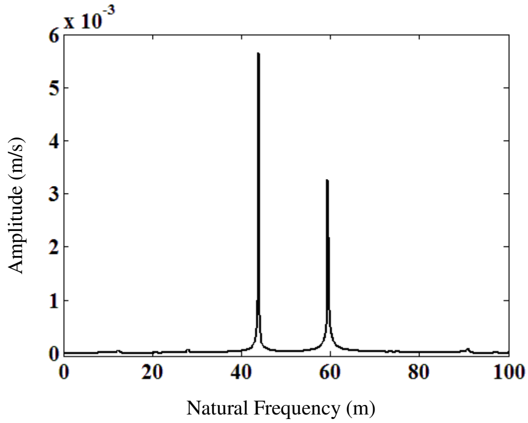

Fig. 11: Natural frequencies of shaft with disk located at *L*/4 when subjected to spinning speed equal to 200 rad/sec

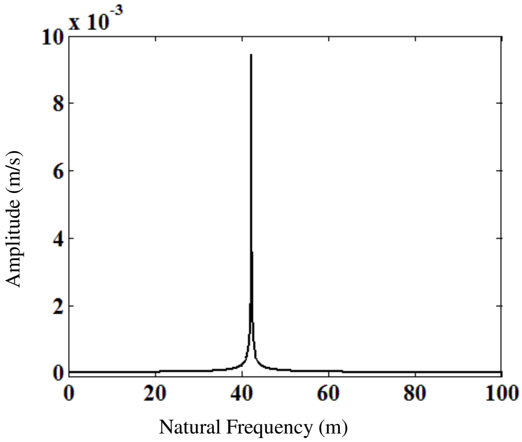

Fig. 12: Natural frequencies of shaft with disk located at *L*/2 when subjected to spinning speed equal to 200 rad/sec

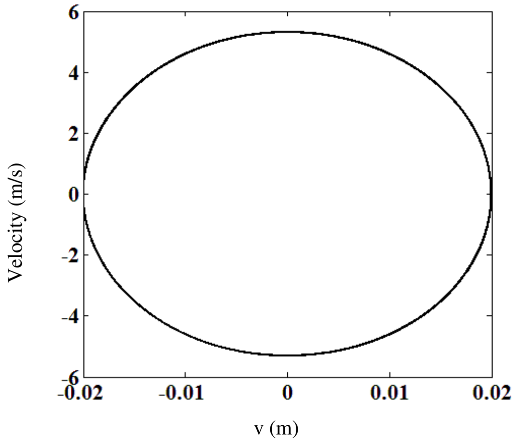

Fig. 13: Phase portrait of shaft with disk located at *L*/2 when subjected to spinning speed equal to 200 rad/sec

3.2. Effect of Location of Disk

The effect of disk location along the span of the shaft element in calculating the natural frequencies have been illustrated Figs. 10-13. It is observed that when the location of disk closed to the one of the bearing ends, a clear disparity of natural frequencies in magnitude has been portrayed. The value of forward natural frequency is nearly double the backward natural frequency when the location of disk is at either of these two bearing ends. It has also been noted that both frequencies is shared the same value when the disk is located exactly at the middle point of shaft. At this location, the shaft behaves a single element without rotational speed since the gyroscopic effect at this position due to disk is negligible and nullified. However, the maximum response amplitude occurs when the disk is at mid-point of the shaft element. It needs to be noted that simulations of this section has been made based on initial conditions i.e., $v(0)=0, \dot{v}(0)=0$, $w(0)=0$, and $\dot{w}(0)=0.$, A typical periodic response has been developed with the system of shaft having a disk located at mid-point of the shaft element as shown in Fig. 11.

3.3. Effect of Geometric Nonlinearity

The initial conditions are insignificant while calculating the natural frequencies with considering the small deformation theory which results linear coupled equation of motions. However, the initial conditions play an important role to evaluate the nonlinear natural frequencies and it has been observed that with increase in initial disturbance of $v(0)$, both frequencies are increased as shown in Figs. 14-16. However, the difference between these frequencies is unaffected by the variation of initial conditions. From the Fig. 17, it has clearly been depicted that the period of response is notable and varying with initial conditions. This variation in natural frequencies is greatly due to the strongly coupling of nonlinear terms. Hence, the effect of coupled terms P^3+PQ^2 and P^2Q+Q^3 is trivial for relatively small disturbance.

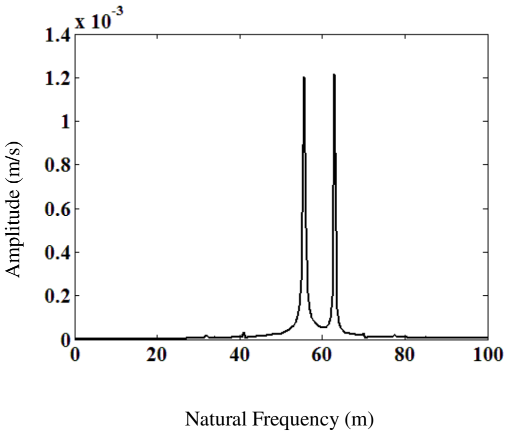

Fig. 14: Natural frequencies of shaft with disk for the initial conditions i.e., $v(0)=0.005$, $\dot{v}(0)=0$ $w(0)=0$, and $\dot{w}(0)=0$.

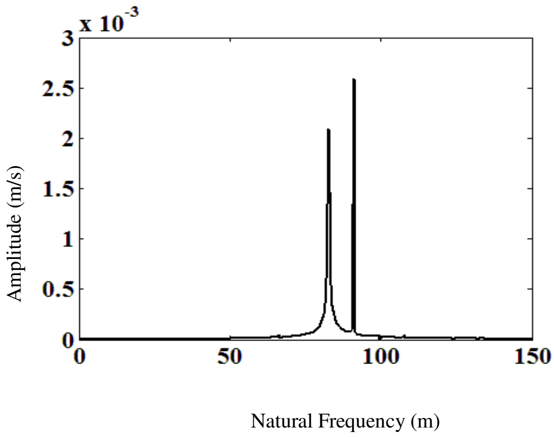

Fig. 15: Natural frequencies of shaft with disk for the initial conditions i.e., $v(0)=0.01$, $\dot{v}(0)=0$ $w(0)=0$, and $\dot{w}(0)=0$.

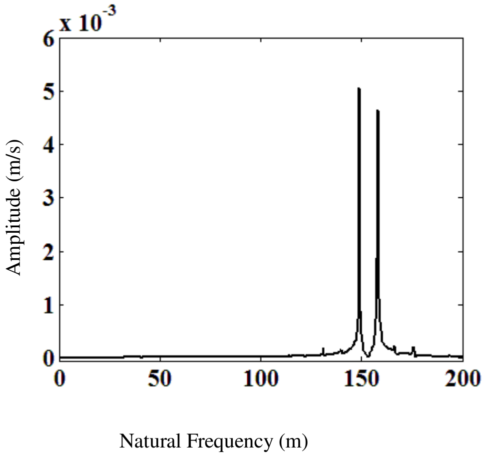

Fig. 16: Natural frequencies of shaft with disk for the initial conditions i.e., $v(0)=0.02$, $\dot{v}(0)=0$ $w(0)=0$, and $\dot{w}(0)=0$.

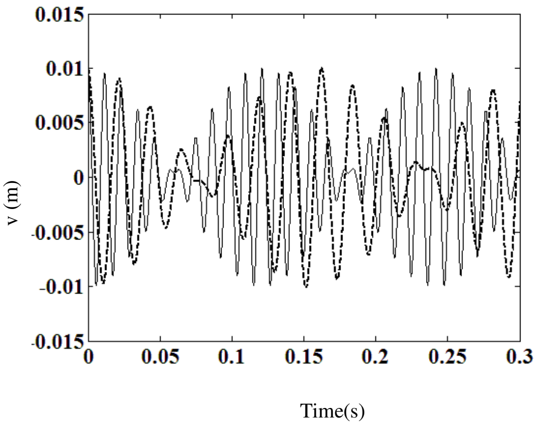

Fig. 17: Time history of shaft with disk for the initial conditions i.e., $v(0)=0.01$, $\dot{v}(0)=0$ $w(0)=0$, and $\dot{w}(0)=0$.

3.4. Effect of Unbalance Mass

The free vibration analysis has been investigated to calculate the natural frequencies under the influencing design parameters. The effect of mass unbalanced does not alter the fundamental frequencies of rotor-bearing system as this appears as forcing terms which results inhomogeneous equation of motions. Hence, steady state and its stability are greatly influenced by the forcing terms which are here due to the mass unbalanced located away from the geometric center of disk. The effect of unbalanced on the dynamic motion has been illustrated in Figs. 18-21 with keeping other parameters constant. There are three possible ways to vary the magnitude of the forcing parameter expressed as $m_u\Omega^2 d$ either varying the rotational speed or mass of the unbalanced or location of the mass unbalanced. The dotted line has been used to represent the effect of mass unbalanced whereas solid lines are used to depict the free vibration response. Here, the rotational speed of the shaft has been considered to study carefully about the effect of forcing parameter. It has been observed that with increase in speed of the shaft, the response amplitude becomes prominent. Simultaneously, from the Figs. 18-19, it has been noticed that the effect of mass unbalanced is insubstantial for the rotational speed less than 200 rad/s. Hence, it is noteworthy that upto rotational speed equal to 200 rad/s, transient part dominates over the steady state part and amplitude of steady state part becomes negligible.

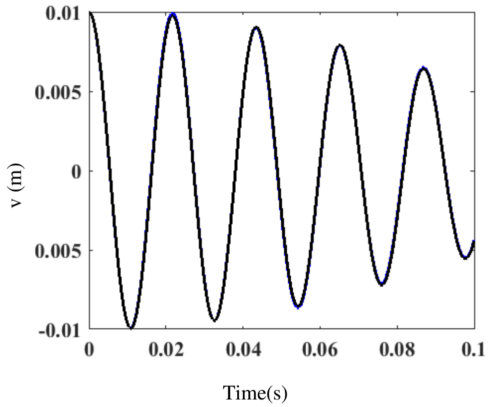

Fig. 18: Time history of shaft-disk system when subjected to spinning speed equal to 100 rad/s

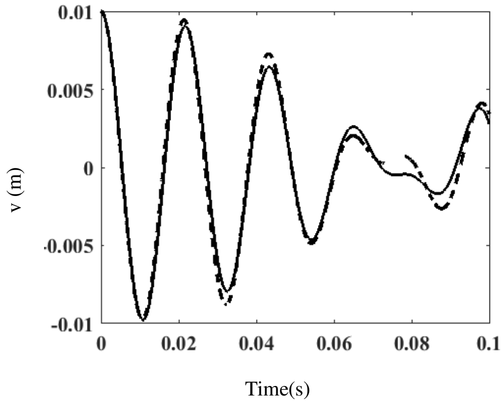

Fig. 19: Time history of shaft-disk system when subjected to spinning speed equal to 200 rad/s

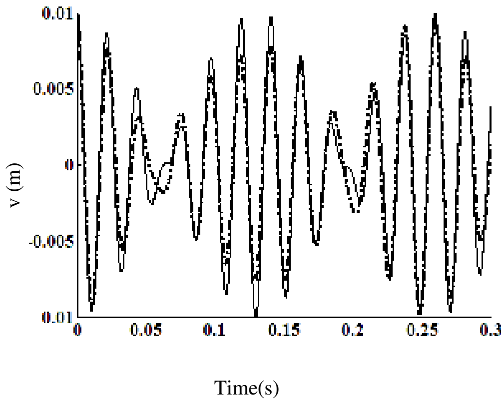

Fig. 20: Time history of shaft-disk system when subjected to spinning speed equal to 240 rad/s.

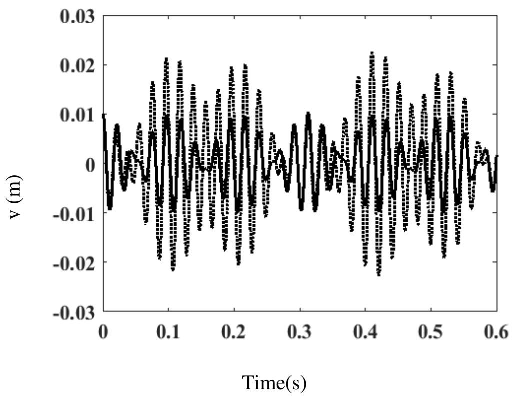

Fig. 21: Time history of shaft-disk system when subjected to spinning speed equal to 300 rad/s

4. Conclusion

A numerical investigation of depicting the linear and nonlinear frequencies of lightly damped rotor-bearing system and studying the nonlinear responses of the system subjected to mass unbalanced. Geometrically coupled nonlinear differential equations of motion have been derived considering gyroscopic effect of disk as well as mass unbalanced while vibration effect along the shaft axis is ignored. Time history and FFT analysis have been established for finding the fundamental frequencies for the rotating under variation of shaft diameter, effect of geometric nonlinearity and disk location while dynamic impact of mass unbalanced on the behavior of rotor bearing system has been investigated using time history. Following important observations have been depicted while studying the present system.

- It has been investigated that a beating phenomenon has been developed for low value of spinning speed and span between both frequencies becomes larger for higher value of rotational speed of the shaft.
- The value of forward natural frequency is nearly the double with respect to backward natural frequency when the location of disk is at either of these two bearing ends. The gyroscopic effect has been negligible when the disk is placed at a mid-point of the shaft element.
- The gap between the natural frequencies is unaffected by the changing the initial conditions and but the period of response is notable and varying with initial conditions due to the presence of geometric nonlinearity.
- With increase in speed of the shaft, the response amplitude becomes prominent when the rotor bearing system is subjected to mass unbalanced. It has been noticed that the effect of mass unbalanced is insubstantial for the rotational speed less than 200 rad/s.
- This article furnishes a proper understanding about the operating speed of the shaft element by which one can easily avoid the excessive vibration due to resonant conditions

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