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Modeling and non-linear responses of MEMS capacitive accelerometer

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Abstract. A theoretical investigation of an electrically actuated beam has been illustrated when the electrostatic-ally actuated micro-cantilever beam is separated from the electrode by a moderately large gap for two distinct types of geometric configurations of MEMS accelerometer. Higher order nonlinear terms have been taken into account for studying the pull in voltage analysis. A nonlinear model of gas film squeezing damping, another source of nonlinearity in MEMS devices is included in obtaining the dynamic responses. Moreover, in the present work, the possible source of nonlinearities while formulating the mathematical model of a MEMS accelerometer and their influences on the dynamic responses have been investigated. The theoretical results obtained by using MATLAB has been verified with the results obtained in FE software and has been found in good agreement. Criterion towards stable micro size accelerometer for each configuration has been investigated. This investigation clearly provides an understanding of nonlinear static and dynamics characteristics of electrostatically micro cantilever based device in MEMS.

1 Introduction

Technology has been improved in such a way that we can build products so small that it cannot be seen by our Human eye. In our 21st century, MEMS has been identified as one of the emerging and promising technology and has the ability to revolutionize both Industrial and consumer products by combining Silicon based microelectronics with micromachining technology [1]. Recently, MEMS devices are used almost in all fields with some well-known examples of MEMS devices applications in our daily life include micro actuators, transducers, Pressure and flow sensors [2]. Today, this fast developing Technology has emerged in such a way that a new set of applications have been opened up. For example, if we consider Medical field, we are now able to make medical and biomedical devices so small that they can be directly injected into our blood stream. These intelligently monitor blood substances and release drugs wherever necessary, thus selectively killing sick cells or germs [3, 5]. Most common MEMS device now days are MEMS accelerometer. .Normally accelerometer consists of a simple inertial mass which is suspended by springs, which is acted upon by acceleration forces which causes the mass to deflect from its initial position. Now this deflection is converted into an electrical signal, which is appeared at sensor output. MEMS technology has been applied to accelerometer, which is relatively new development. These type of transducers are typically constructed with a deformable diaphragm, beams with both ends fixed (commonly known as fixed-fixed beams) and cantilever beams (single side fixed beams).Their geometries are separated from a fixed ground plane by an air-gap of suitable thickness. To drive these devices we need drive mechanism, which includes constant voltage source or constant current source [10, 11]. In most of the cases, most electrostatic based MEMS devices are of constant voltage drive [12]. There are many types of accelerometers developed and reported in the literature. The vast majority is based on piezoelectric crystals, but they are too big and clumsy. People tried to develop something smaller that could increase capability and started searching in the field of microelectronics. They developed MEMS (Micro electromechanical systems) accelerometers. The electrostatic force developed with the constant voltage drive mode is non-linear and this gives rise to a phenomenon known as Pull-in voltage. This is a phenomenon which causes an electrostatically actuated beam to collapse on ground plane if the drive voltage exceeds permissible limit depending upon the geometry of our device. But we observe that accurate determination of pull in voltage is critical in our design process to determine sensitivity, instability [11]. We also have methods to determine pull in voltage of electrostatically fixed-fixed actuators. Another important which accounts for consideration is squeezed film effects in MEMS devices to perform squeeze-film damping measurements for different film thicknesses. In spite of a rigorous research has carried out in this area, very limited authors have considered combine effects of nonlinear electrostatic actuation and squeeze-film damping in their dynamic model. Hence, nonlinear dynamics of electrostatic systems still remains unexplored.

Therefore, an attempt has been made to study the nonlinear characteristics of electrically actuated micro beams for two different set of geometric configurations.

While the static analysis mainly focuses pull in voltage analysis for studying the structural instability, dynamics analysis has been carried out for investigating the transient behaviors at the dynamic pull in voltage. We investigate the linear and nonlinear characteristics of micro beam which is actuated under applied voltages and solving the boundary value problem numerically is discussed. Specifically, the effect of nonlinearities on both static and dynamics analysis has been investigated thoroughly. Results obtained numerically using MATLAB has been compared with results obtained in FE software.

2. Model description

Two different configurations of microbeams are investigated. The floating configuration in which the other end of the cantilever beam is free as shown in Fig. 1. The fixed configuration where the other end of cantilever beam is fixed as shown in figure 2. The above figures shows undamped cantilever beam of length *l*, width *w*, Young's modulus *E* and thickness *t* separated from the ground plane by an initial gap of d_0 when

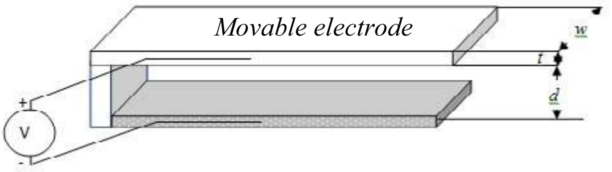

Fig.1*a*: Clamped-free configuration

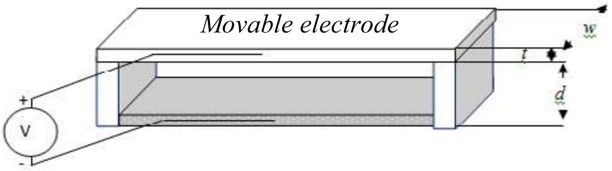

Fig. 1*b*: Double Clamped configuration

The final non-dimensional form of equation of motion is as follows.

$$\ddot{w} + 2\zeta\dot{w} + w'''' = f\big|_{electrostatic} - f\big|_{squeeze\ film\ effect}$$

Here,

$$\zeta = \left(C/2\right)\sqrt{l^4/EIA}\,,$$

$$f\big|_{electrostatic} = \left(\varepsilon b l^4 V^2 / 2EId_0^3\right)\left(1-w\right)^{-2},$$

$$f\big|_{squeez\ film} = \left(\frac{\mu b^3}{d_0^3\left(1-w\right)^2}\right)\sqrt{\frac{l^4}{EIA}}\left(\frac{\dot{w}}{1+(d_0-w)Kn-w}\right).$$

Now in order to analyze the static and dynamic behavior of the beam, reduced order model based on Galerkin's decomposition with undamped Linear Eigen modes as base functions is developed. The deflection of the beam is given as:

$$w(x,\tau) = \sum_{i=1}^{n} \varphi_i(x) q_i(\tau).$$

Here, $q_i(t)$ is time varying generalized coordinates and $\varphi_i(x)$ are eigenmodes of the associated linear undamped homogeneous equation. In our case, we consider first mode so the equation becomes: $w(x,\tau) = \varphi(x)q(\tau)$

3. Results and discussion

The above differential equation is solved using MATLAB to obtain numerical solution. The equation includes a nonlinear term so the system can have multiple solutions and the solution depends on the initial guess we took and the accurate solution is obtained for absolute tolerance of 10^{-6}.Commands such as **BVP4** and **BVP5** have been used to implement in MATLAB. The micro beam was modeled using following specifications:

Property	**Value**
Relative permittivity	4.5
Young's Modulus	153GPa
Poisson ratio	0.223
Density	2330 kg/m^3
Name of the material	Poly silicon

Air gap was modeling using relative permittivity value 1. The beam resides in an air filled chamber that is electrically insulated. The upper side of the chamber has a grounded electrode. Polysilicon is assumed to be heavily doped so that electric field penetration into the structure can be neglected. The micro beam with length 300 µm, width 20 µm, thickness 2 µm has been taken. Meshing was done using with maximum element size = 4 and minimum element size = 0.064 and using free Quad elements.

Case 1: Clamped-free configuration

Figure 2 show the deflection of the micro beam for the cantilever and double clamped beams actuated under applied voltage where red color shows the deflection with maximum amplitude obtained by using ES software. Figures 3 and 4 depict the variation of deflection of the clamped-free beam and subsequent end gaps under electrostatic actuation. It is observed that deflection slope is sharply increased for high value of applied voltage. The deflection of free end of the microcantilever under the influences of applied voltage has been illustrated in Fig. 4 and it has been observed that the pull in condition is anticipated when both stable and unstable solution coalesces at a certain value. Here, while upper branch solution represents the unstable solution, lower branch solution is indicating the stable solution. The pull-in results obtained in MATLAB has been compared with the same obtained in ES software and has been found very good agreement. The time history for the tip deflection obtained in Fig. 5 predicts the dynamic pull-in voltage. It is observed that for certain voltage above the critical value, the motion exposes to a divergent motion and the beam abruptly collapses to on the ground electrode. The qualitative phase plot for the corresponding time history is illustrated in Fig. 6.

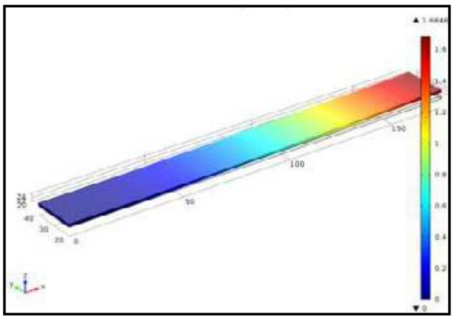

Fig. 2: Variation of deflection under electrostatic actuation.

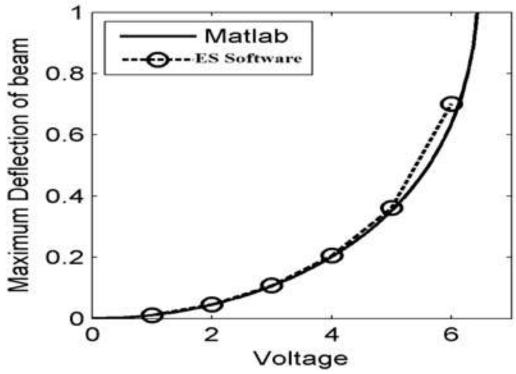

Fig. 3: Variation of tip deflection under electrostatic actuation.

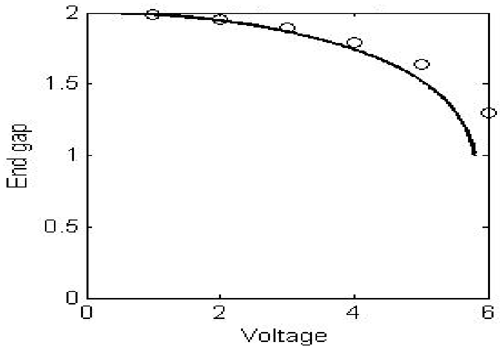

Fig. 4: End gap between electrode and microbeam

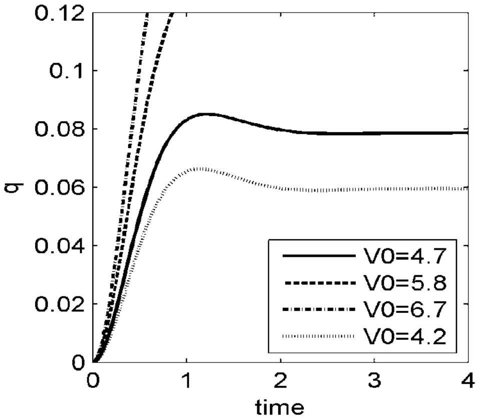

Fig. 5: Deflection time history under various applied DC voltages

Case II: Double clamped configuration

Similar investigation has been carried out for microbeam with double clamped. While Fig. 7 shows the deflection pattern of the double clamped micro beam subjected to under applied voltage obtained by using ES software. Variation of maximum deflection at mid-point of double clamped beam and corresponding end gaps under the influences of applied voltage has been illustrated in Fig. 8 and 9 and it has been observed that the pull in condition has occurred at a voltage in between 40 to 42 where both stable and unstable solution coalesces at a certain value. The critical value obtained in MATLAB numerically and in ES software is found in very good agreement. The mid-point deflection in times scale is illustrated in Fig. 10 and the periodic motion below the critical value represents the dynamic pull-in voltage. A slight increase in voltage at the critical value, the periodic motion exposes to a divergent motion and the beam abruptly collapses onto the electrode. A corresponding quantitative estimation of the dynamic pull in voltage has been obtained through the phase plot illustrated in Fig.11. It is observed that the dynamic pull-in of the micro beam took place at an applied voltage is well below than that of static counterpart.

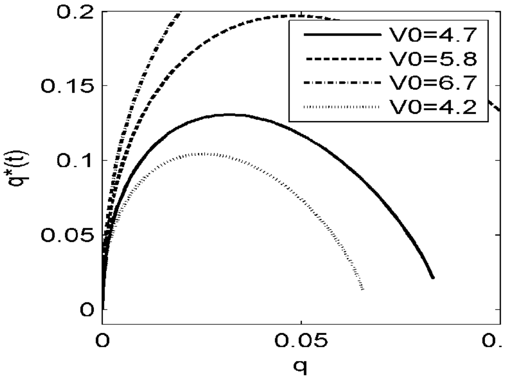

Fig 6: Phase plot of the model of the microcantilever excited by various applied DC voltages

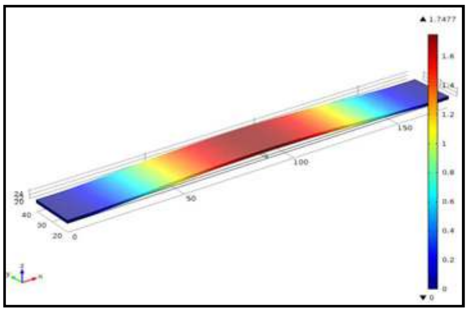

Fig. 7: Variation of deflection under electrostatic actuation for double clamped microbeam.

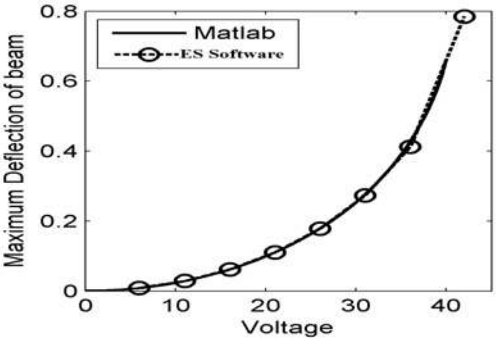

Fig. 8: Variation of tip deflection under electrostatic actuation.

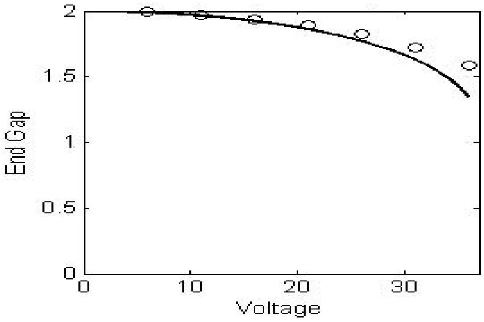

Fig. 9: End gap between electrode and microbeam

5. Conclusions

Electrically actuated micro beam has been analyzed theoretically when the electrostatic-ally actuated micro-cantilever beam is separated from the electrode by a moderately larger gap. Two distinct types of geometric configurations of MEMS accelerometer are analyzed. For each configuration, both static and dynamic pull in analysis have been carried out and a possible comparison between the configurations has been illustrated. Higher order nonlinear terms have been taken into account in the dynamic model in addition to nonlinear of gas film squeezing damping effect. The theoretical results obtained by using Engineering Simulation software has been verified with the results obtained in MATLAB and found to be in good agreement.

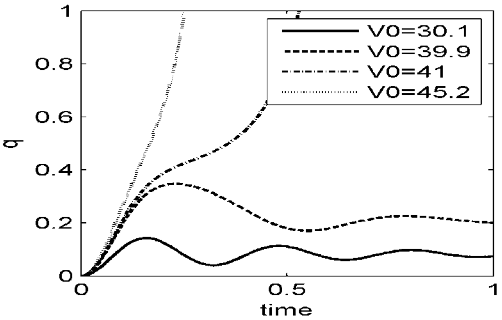

Fig. 10: Deflection time history under various applied DC voltages

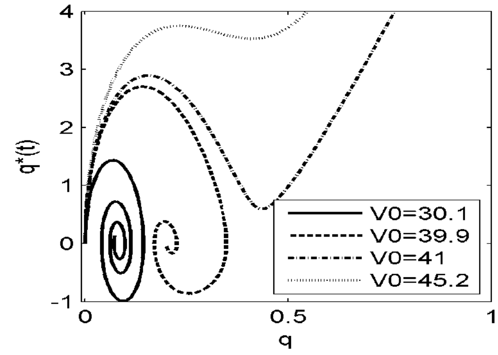

Fig 11: Phase plot of the model of the microcantilever excited by various applied DC voltages

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