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Efficient Three Dimensional Nonlinear Thermo-Mechanical Analysis of Structures Subjected to Fire

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Abstract

In this paper, a computationally efficient integrated frame-work is developed for coupled thermo-mechanical analysis of 3D frames. It can account for physical phenomena like large deformations, temperature dependent material degradation, permanent plastic deformations and fire induced spalling which are prevalent at elevated temperatures. The developed frame-work utilizes three way sequential coupling between thermal, mass transport and structural analysis. A two level discretization scheme is incorporated where 1D beam column elements are utilized for structural analysis, the cross-section of these beam-column elements are further discretized into matrix of segments. Aforementioned strategy entails sequential coupling of effects of non-uniform temperature and pore pressure across the cross-section into structural analysis. Subsequently structural analysis is performed with an updated Lagrangean based formulation with force deformation relationships deduced form classical Euler-Bernoulli beam column theory. Critical physical phenomena like cracking, crushing, spalling and transient states of strain in case of concrete and yielding in case of steel are duly accounted. Cross-sectional reduction due to spalling are accounted for by replacing the spalled segments with void segments in the subsequent time steps. Numerical examples of steel and concrete structures subjected to various fire scenarios are presented to demonstrate the accuracy of developed framework. Furthermore, progressive collapse analysis is carried out for concrete and steel 3D subjected to fire.

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1. Introduction

In recent years, structural design for fire has received increasing attention and a number of design procedures have been developed over past few decades. These design procedures are classified into two categories, prescriptive and performance-based procedures. Prescriptive methods yield fire rating as a function of clear cover/insulation thickness and cross-sectional dimensions whereas performance based procedures yield fire ratings based on scenario based engineered fire analysis of structural member/system subjected to fire [1]. Critical physical phenomena that must be incorporated for scenario based analysis for accurate prediction of thermo-mechanical response include a) geometric and material nonlinearities, b) temperature-dependent material properties, c) natural fire scenarios, and d) fire induced spalling. Incorporating these phenomena, several structural fire analysis tools have been developed over the past few decades. They are broadly classified into two types: empirical methods and advanced calculation methods. Empirical methods, for instance [2,3], predict the response of structures under fire with the aid of simplified design equations. They are usually scenario specific and cannot be utilized in all cases. Advanced calculations are the numerical finite element (FE) models which work at micro and macro levels and perform time variant structural analysis under fire.

FE Micro models essentially utilize 3-D FE for structural fire analysis of steel/concrete composite structures. For instance, Khennane and Backer [4] developed 3-D FE model for structural fire analysis of concrete structures. Yu et al. [5] developed 3-D FE based micro model and successfully implemented in the VULCAN solutions [6]. Franssen developed a 3-D FE micro model and successfully implemented in the SAFIR program [7]. Commercial software like ANSYS and ABAQUS have also been utilized for structural fire analysis of steel/concrete composite structures. For instance, Kumar [8] demonstrated structural fire analysis of steel/concrete structures utilizing 3-D FE micro models in ANSYS. Even though, FE micro models account for complex geometries and boundary conditions they are computationally intensive and face lot of convergence issues even for moderate fire exposures and demand numerous parameters to be calibrated [9]. Also, very few FE micro models incorporate actual elevated temperature strain decomposition as observed in laboratory experiments. Due to higher computational efforts, these are better suited for isolated and small scale structural assemblies subjected to fire.

FE macro models are computationally efficient compared to FE micro models and are ideal for structural fire analysis of full structural systems. Structural fire analysis of isolated member vs. complete structural system is an interesting subject for fire engineering community and is gaining significant attention in recent years. For instance, full-scale fire test conducted at BRE fire research laboratory at Cardington [10] indicated augmented fire resistance of structural system as whole compared to isolated members. The main reason for this phenomenon was interaction between heated and non-heated structural members. For a performance based design, such interactions need to be incorporated and fire rating needs to be quantified considering entire structural system considering various fire scenarios. Currently, two of the most used FE macro models for structural fire analysis of steel/concrete composite structural systems are VULCAN and SAFIR. VULCAN solutions [6] has been developed at the University of Sheffield while SAFIR program [7], at the University of Liege. Apart from these, there are some other notable formulations. For instance, Rigobello et al. [11] developed solid like finite elements for thermo-mechanical analysis of steel frames. Bionidi and Nero [12] developed cellular finite element based FE macro model for coupled thermo-mechanical analysis of RC frames. Caldas et al. [13] developed a 3-D FE macro model for coupled thermo-mechanical analysis of steel/concrete composite structures. However, these macro models too require FE discretization for reasonable levels of accuracy and structural fire analysis will become computationally intensive if large scale structural systems are considered. Moreover, these macro models are one-way coupled i.e. thermal analysis is performed first followed by structural analysis. This will be inaccurate when cross-sections of structural may change during fire (e.g. due to fire induced spalling). Also, in situations like moving fires, both by natural movement and movement induced by structural collapse, mutual coupling of physical phenomena is required.

There are two approaches which address above mentioned limitations. First, the moment curvature based macro model developed by Kodur et al.[14] and second, is the direct stiffness method (DSM) based formulation developed by Srivastava and Prakash [15]. The moment curvature based macro model developed by Kodur et al. [14]

incorporates physical coupling between heat transfer, moisture transport and mechanical analysis. However, its formulation restricts itself for structural fire analysis of isolated. Srivastava and Prakash [15] developed DSM based formulation for structural fire analysis of steel/concrete plane frames. The superiority of their formulation compared to existing frameworks is due to the consideration of three-way coupling between heat transfer, moisture transport and mechanical analysis in a DSM based formulation. DSM allows one to achieve greater computational efficiency with fewer sub-elements (or coarser spatial discretization). Also, it avoids the need of element-level numerical quadrature typically required by all FE macro models.

The present study presents development of a temperature-dependent plasticity model that has been incorporate in the existing DSM formulation, so that cooling scenarios can be addressed properly. Presently, there are two approaches for incorporating permanent plastic deformations: one is the lumped-plasticity approach and another is the distributed plasticity model. Even though, lumped plasticity models are computationally efficient, they may not yield accurate results where material possess significant strain hardening, which is the case for steel at elevated temperatures. To address these limitation, distributed plasticity models with full FE based formulation [6], [7] are most widely used. However, as discussed earlier they are computationally intensive. Hence, in a DSM based setting with distributed plasticity model, permanent plastic deformations are taken into account with sequential loading and unloading, as discussed subsequently.

2. Theoretical Formulation

Present study develops a large deformation 3-D Euler-Bernoulli beam-column and considers updated Lagrangian co-rotational formulation. A two level discretization scheme is incorporated where 1-D line elements are utilized for mechanical analysis whereas cross-section of each beam-column element is further discretized into a matrix of fibers for storing the cross-sectional details of temperature, pore-pressure and mechanical properties (Fig. 1). These cross-sectional details are facilitated by 2-D FE mesh in y-z plane utilized for heat transfer and mass transport analysis. Thus for each sub element heat transfer and mass transport analysis is performed utilizing 2-D FE mesh and these effects are coupled to the mechanical analysis through fibers linking 1-D and 2-D FE meshes. This means that uniform temperature and pore pressure profile are assumed for a line element. However, in cases where variations in exposure conditions are encountered, more sub elements are taken along length of a member.

2.1 Thermal analysis

For a given cross-section of beam-column element, thermal analysis is performed by solving the 2-D heat conduction equation

$$k\nabla^2 T + Q = \rho c \frac{\partial T}{\partial t} \qquad \text{* MERGEFORMAT (1)}$$

where, k is the thermal conductivity, c is the specific heat, and ρ is the mass density of material; Q is the heat source, t represents time, and T represents temperature. The combined effect of convective and radiative boundary conditions are considered as

$$k\nabla T \cdot \mathbf{n} = h\left(T - T_\infty\right), \qquad \text{* MERGEFORMAT (2)}$$

where, T_∞ is the ambient temperature, h is the temperature-dependent combined convective-radiative heat transfer coefficient, and $\mathbf{n}$ is the vector normal to the boundary. Above mentioned mathematical model is solved utilizing Galerkin FE approach and can be seen in detail in [15].

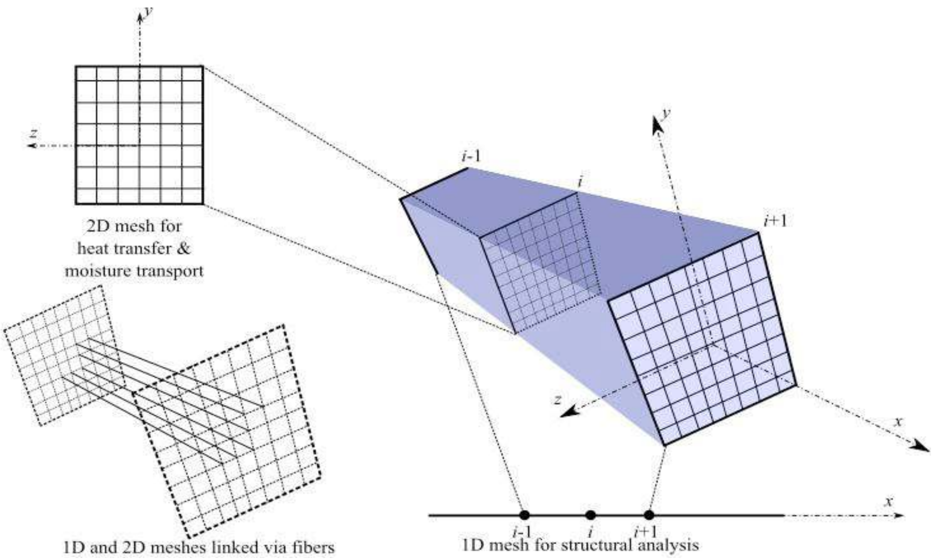


Fig. 1 Discretization of space frames [15]

2.2 Pore pressure analysis

Moisture co-exists in liquid and vapor phase in concrete structures at elevated temperatures and eventually cause spalling. In the present study, moisture transport in vapor phase is considered and modeled using Darcy's law. Assuming water vapor as ideal gas governing equations are written as

$$a_p \frac{dP_v}{dt} = \kappa_p \nabla^2 P_v + Q_p ,$$

* MERGEFORMAT (3)

where P_v is the pore pressure, Q_p is the mass flow rate, details about coefficient a_p and κ_p can be seen in [15]. Similar to heat transfer, Eq.(3) is solved utilizing Galerkin FE approach.

2.3Mechanical analysis

Mechanical analysis is performed utilizing a beam-column element developed from Euler-Bernoulli beam-column theory. It can account for large deformations, non-linear thermal gradients, spalling, temperature-dependent material degradation and permanent plastic deformations in an updated Lagrangian co-rotational formulation. As discussed earlier, structural frames are discretized with 1-D line elements and cross-section of these line elements are further discretized into matrix of fibers utilizing 2-D FE mesh. The details of the beam-column element in member and local co-ordinate system are shown in Fig. 2. For a given fiber, total strain in concrete and steel is assumed to follow an additive decomposition as

$${}^{c}\varepsilon_t = {}^{c}\varepsilon_m + {}^{c}\varepsilon_T ; \quad {}^{c}\varepsilon_T = {}^{c}\varepsilon_{th} + {}^{c}\varepsilon_{cr} + {}^{c}\varepsilon_{tr} ,$$

* MERGEFORMAT (4)

$${}^{s}\varepsilon_t = {}^{s}\varepsilon_m + {}^{s}\varepsilon_T ; \quad {}^{s}\varepsilon_T = {}^{s}\varepsilon_{th} + {}^{s}\varepsilon_{cr} .$$

* MERGEFORMAT (5)

Here, the left superscripts '*c*' and '*s*' denote concrete and steel respectively, ε_t is the total strain, ε_m is mechanical strain, ε_T is the total thermal strain, ε_{th} is thermal strain, $\varepsilon_{tr}$is transient strain, and ε_{cr} is creep strain. Thermal transient and creep strains are computed from empirical relationships and are explained in detail in [15]. From these strains mechanical strain is computed and moduli of each fiber is computed from stress-strain relationships given in [16,17]. The individual effect of all fibers are integrated to model the overall behavior of neutral axis. Subsequently mechanical behavior of beam-column element shown in Fig. 2 is mathematically modelled as

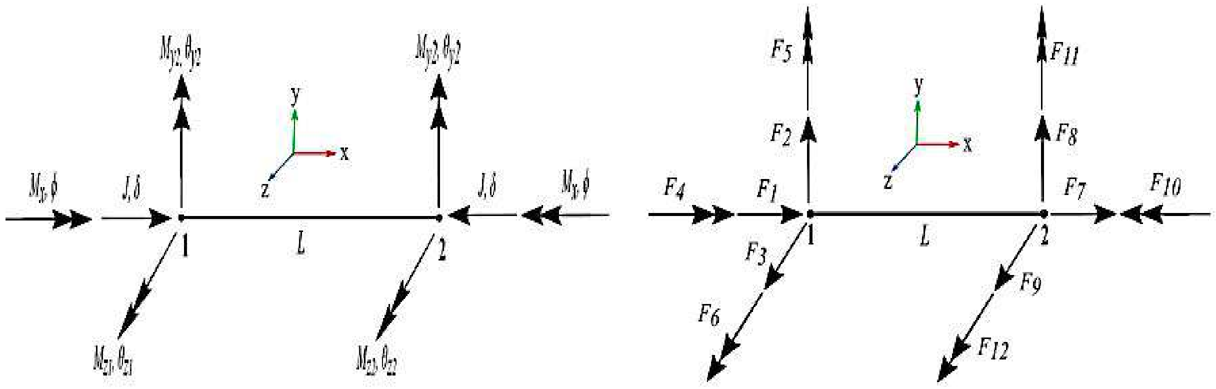


Fig. 2 Beam-column element in member co-ordinate system (left) and local co-ordinate system (right)

$$(EI)_z \frac{d^2 y}{dx^2} + Jy = -M_{z1} + x\left(\frac{M_{z1}+M_{z2}}{L}\right) + \sum_{i=1}^{n} A_i E_i \varepsilon_T^{(i)} y_i , \backslash\text{* MERGEFORMAT } (6)$$

$$(EI)_y \frac{d^2 y}{dx^2} + Jz = -M_{y1} + x\left(\frac{M_{y1}+M_{y2}}{L}\right) + \sum_{i=1}^{n} A_i E_i \varepsilon_T^{(i)} z_i , \ \backslash\text{* MERGEFORMAT } (7)$$

$$P = (AE)\left(\frac{u_3}{L} - c_{by} - c_{bz}\right) + \sum_{i=1}^{n} A_i E_i \varepsilon_T^{(i)}, \quad M_x = \frac{(GJ)}{L}\phi . \backslash\text{* MERGEFORMAT } (8)$$

where $(EI)_z$, $(EI)_y$ represent equivalent flexural rigidities with respect to member z and y axes, (GJ) represents torsional rigidity, (AE) represents axial rigidity , c_{by} and c_{bz} represent member bowing functions along y and z axes, J represents axial load and L is the length of member. Eq (6-8) are solved with given boundary conditions and equilibrium equations are written in global co-ordinate system as

$$\Delta\mathbf{F} = \left(\mathbf{K}_G\right)\Delta\mathbf{v} + \Delta\mathbf{F}_T , \qquad \backslash\text{* MERGEFORMAT } (9)$$

where $\Delta\mathbf{v}$ is the element displacement vector, $\mathbf{K}_G$ represents global stiffness matrix, $\Delta\mathbf{F}$ is the force vector due to externally applied loads, and $\Delta\mathbf{F}_T$ is the fixed-end force vector due to temperature in global coordinate system. Eq.(9) is solved iterative using Newton-Raphson method till predefined tolerance is achieved. Furthermore, thermal, moisture transport and mechanical solvers are sequentially coupled and solved in a staggered scheme as demonstrated in [15]. The computational efficacy of the developed framework is demonstrated with numerous verification and validation studies [15,18]

2.4 Sequential loading and unloading scheme

Preserving the computational merit of the developed framework, permanent plastic deformations are incorporated in to the developed framework by incorporating distributed plasticity in a sequential loading and unloading program. Fig. 3 show the stress strain state of steel at temperatures T_i and T_{i+1}. For an applied stress σ, $\varepsilon_{m,Ti}$ is the converged mechanical strain corresponding to temperature T_i. Then, plastic strain, $\varepsilon_{p,Ti}$ at temperature T_i is evaluated by elastic unloading of the structure. When temperature increases to T_{i+1}, new stress strain curve is constructed form temperature dependent yield stress and modulus of elasticity. Preserving the plastic strain at T_i ($\varepsilon_{p,Ti}$) reloading is done with elastic moduli E_{Ti+1} as shown in Fig. 3 and subsequently converged mechanical ($\varepsilon_{m,Ti+1}$)and plastic strains ($\varepsilon_{p,Ti+1}$) are evaluated at temperature T_{i+1}. Same procedure is repeated for all subsequent temperatures and coupled thermo-mechanical response is performed as discussed previously.

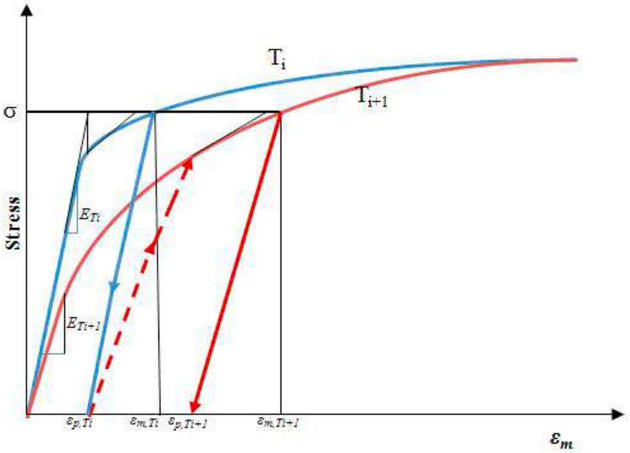
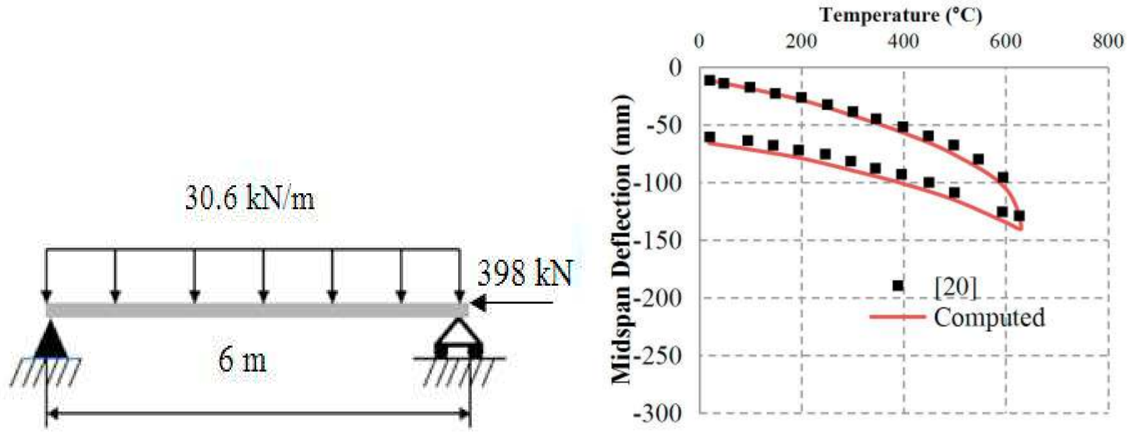


Fig. 3 Loading and unloading sequence

3. Simulation of a steel beam subjected to a heating-cooling regime

To highlight the temperature-dependent distributed plasticity model discussed earlier, a simply supported steel structure (356×171×51 UB) with axial load of 398 kN as shown in Fig. 4 is considered. To the given beam, a non-uniform temperature profile across the cross-section is considered with reference temperature *T* corresponding to the bottom flange, web and *T/2* corresponding to the top flange. For this beam, Rimawi et al. [19] simulated the thermo-mechanical response utilizing full FE based macro model, incorporating permanent plastic deformations. The reference temperature was increased during the heating phase upto 630°C followed by cooling phase where temperature was reduced to room temperature. The room temperature yield stress and modulus of elasticity were taken as 308 MPa and 210 GPa respectively and temperature dependent stress strain relationships were modelled using Ramberg-Osgood curves. The beam under consideration is modelled using proposed formulation with four sub elements and thermo-mechanical analysis is performed considering nonlinear stress strain relationships available in Eurocode-3 [16]. A comparison of thermo-mechanical response obtained from proposed formulation vs Rimawi et al. [19] for the considered fire scenario is shown in Fig. 4. Results indicated that proposed framework is in excellent agreement with the results obtained by Rimawi et al. [19]. It can be observed from the results that, mid-span deflection increase gradually followed by steep increase beyond 400°C due to accelerated material degradation. Also, it can be visible from the results that the path traced by the structure during cooling phase is completely different from the heating phase. This is due to the accumulation of permanent plastic deformations in the structure with rise in temperature.



(a) (b)

Fig. 4 (a)Steel structure under fire (b) Temperature vs. mid-span deflection

4. Conclusions

A computationally efficient framework in a DSM based setting is developed for coupled hydro-thermo-mechanical analysis of concrete/steel space frames. The developed framework incorporates three-way physical coupling between heat transfer, moisture transport and structural analysis. Proposed framework was based on an updated Lagrangian co-rotational formulation with force-deformation relationships developed from classical Euler-Bernoulli beam column theory. A two-level discretization scheme was incorporated where mechanical analysis utilizes 1-D line elements whose cross-sections were further discretized into matrix of fibers to store the details of temperatures, pore pressure and mechanical properties. Such a scheme enables three-way mutual coupling of physical phenomena in a computationally efficient manner. Permanent temperature dependent plastic deformations were incorporated into the framework in distributed plasticity based approach. To facilitate this, sequential mechanical loading and unloading of structure was performed at each time. Also, due consideration is given to nonlinear thermal gradients, fire induced spalling and transient states of strain and were integrated into member stability and bowing functions. This enables higher computational efficiency with fewer sub-elements required for mesh independent results. The capability of the proposed framework in a fire scenario with heating and cooling phases was demonstrated with a numerical example and compared with the predictions from existing methods available in the literature. It was observed that the developed formulation is reasonably accurate in predicting the thermo-mechanical response of steel structures.

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