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Effect of Squeeze Film Damping and AC Actuation Voltage on Pull-in Phenomenon of Electrostatically Actuated Microswitch

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Abstract

The effect of squeeze air-film and AC actuation voltage on dynamic stability of microswitch actuated electrostatically has been investigated in squeeze film domain. The dynamics of the device has been developed considering the load arising from the squeezed air film between the microcantilever attached plate and the grounded substrate. Using trajectories in phase plane and time history, characteristics of the pull-in phenomena have been studied in the presence of DC voltage combined with AC component. Pull-in voltage is observed to be less as compared to the conventional micro-cantilever devices which are actuated only due to DC loading. Furthermore, the dynamic pull-in voltage tends to approach static pull-in voltage, when the squeeze film damping is considered. The study indicates that although electrostatic forces cause softening characteristics, geometric nonlinearity produces a stiffening effect on the microstructure and the nonlinearities play a significant role when pull-in occurs. The ability to resist the bending deformation has become higher as the overall damping has been enhanced due to the effect squeeze film damping. The consideration of higher order nonlinearities while modeling electrostatic forces, predicts more accurate response. This research gives a desirable insight of dynamic behavior of MEMS device under the influence of squeeze film effect.

Keywords: Microswitch; Squeeze film damping; Alternating Voltage; Dynamic Pull-in

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1. Introduction

MEMS devices which characterize by lightweight, small size, compactness and low cost form are an important integral part of many sensors and actuators. Micro-electro mechanical system is one of the emerging and promising

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technologies that have the ability to revolutionize both Industrial as well as consumer products. For designing Bio-MEMS devices, the study of their interaction with environmental fluid has always been a major consideration. Recently, it has been discovered that it is essential to consider the effects of various micro-fluidics phenomena, including squeezed film damping in the design of other MEMS devices like sensors and RF switches. Various techniques for predicting the dynamic performance of MEMS devices and estimating the effect of squeeze film damping due to the air-gap between the vibrating microstructure and a fixed substrate are presented in [1]. The squeeze-film damping can be modeled using the Reynolds equation derived from the Navier–Stokes equations and the continuity equation for simple structures. Keating [2] presented governing equations for quasi-steady and viscous-dominant compressible and incompressible fluids in devices, where the gap is much smaller than the plate width. Knudsen number (*Kn*), which is defined as the ratio of the mean free path of the gas molecule to the characteristic length of the flow is an important parameter in determining squeeze film damping. Modelling based on the modified Reynolds equation including inertia effects underestimates the damping due to end effects.However, it correctly predicts the trend for lower Knudsen numbers ($Kn<10$) as mentioned in [3]. For a complex structure, the squeeze-film damping is modeled on the assumption of continuum [4-6]. Li and Hu [7] presented an analytical molecular dynamics model for estimating the squeeze-film damping of a perforated microplate in case of high Knudsen numbers ($Kn > 10$) . Pandey and Pratap [8] presented the compact formula obtained for computing the squeeze film damping in MEMS cantilever resonators. This formula is valid for all flexural modes and beam geometries with a large range of width to length ratios. Steeneken et al. [9] proposed a model that predicts damping force by using speed and acceleration, determined by differentiating the measured motion.

Considering the operation reality, applied voltage often appears in varied fashions of DC voltage (step voltage), AC voltage (harmonic like voltage) and combined DC and AC voltage. Estimating only the static pull-in condition will be ill-suited for investigating suitably the structural stability and sensitiveness of the device due to pull-in phenomenon. Hence, it is extremely necessitated to evaluate and predict the dynamic pull-in rather than static pull-in when the device is actuated by the bias voltage source. Accurate determination of pull-in voltage phenomenon which causes MEMS device to collapse if the drive voltage exceeds the permissible limit is critical in design process as it determines sensitivity and instability of device. Accurate model prediction evidently needs to be considered for the effective design for applications that operate in and around pull-in zone. Considering the advantages of using electrostatic MEMS, accurate mathematical model representing the physical system of MEMS is important for designing safe and efficient MEMS devices. Here we propose one such model that considers all the effects with respect to electrostatic forces in MEMS and also focuses on challenges that arise due to influence of nonlinearities, while modeling the mechanical behavior of MEMS devices and mainly their dynamics behavior.

Nomenclature

v	The transverse deflection of beam
b_b	The width of the beam
c	The cross-sectional area
d_0	The initial gap between electrode and the beam
I	The moment of inertia of the beam
Y	The *Y*oung's modulus
ρ	The material density
h	Thickness of the plate
μ	The air viscosity
M	The mass of rigid plate
L_C	The distance from beam end to the center of rigid plate
C	The damping coefficient
V	Applied DC Voltage

2. Model Description

A schematic diagram of MEMS switch, characterized by a microcantilever attached plate based device, which is actuated by combined AC as well as DC loading, is illustrated in Fig. 1. A standard Euler-Bernoulli principle has been adopted in order to depict the dynamic characteristics of the present MEMS device as well as describing its workability under combined DC and AC Actuation.

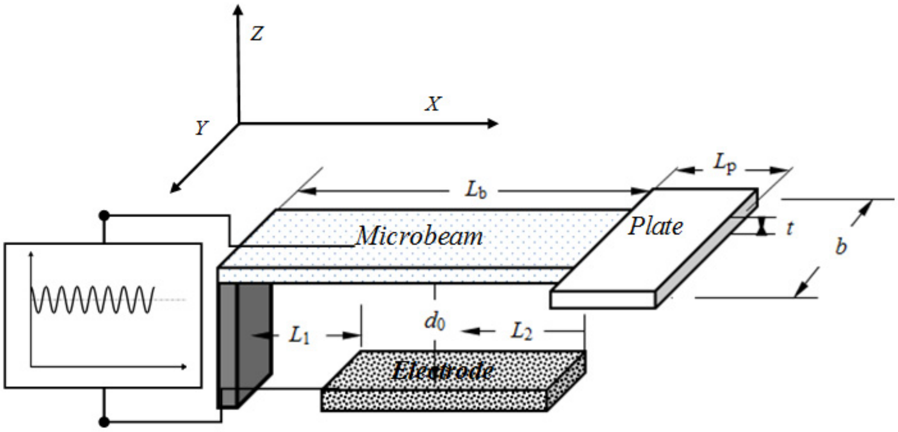

Fig.1: Schematic diagram of MEMS Switch

The nonlinear governing equation of motion has been expressed in terms of transverse direction.

$$\rho A\frac{\partial^2 v}{\partial t^2}+EI\left[v'''' + \left\{v'\left(v'\,v''\right)'\right\}'\right]+\rho A\left[v'\int_{L_b}^{\eta}\int_{0}^{\xi}\left(\ddot{v}v'+\dot{v}'^2\right)d\xi\,d\eta\right]'+\frac{\mu b_b{}^3}{(d_0-v)^3}\frac{\partial}{\partial t}(d_0-v)$$
$$=\frac{\varepsilon_0 b_b V^2}{2(d_0-v)^2}\left[1-\frac{\left(d_0/L_b\right)^2}{d_0{}^2}\left\{v'^2+2(d_0-v)v''\right\}\right]\left\{H(x-L_1)H(L_2-x)\right\} \quad 1(a)$$

Subjected to Boundary conditions:

$$\text{at } x=0,\quad v(0,t)=0,\quad v'(0,t)=0;$$
$$\text{at } x=L,\quad EIv'''(L,t)=M\ddot{v}(L,t)+Ml_c\ddot{v}'(L,t),$$
$$EIv''(L,t)=-l_cM\ddot{v}(L,t)+4Ml_c^2\ddot{v}'/3(L,t). \quad 1(b)$$

Using inextensible condition of the beam i.e., $(1+u')^2+v'^2=1$ where the longitudinal displacement and transverse displacement are represented by $u(x,t)$ and $v(x,t)$ along with Hamilton's principle, and substituting non-dimensional parameters:

$s=\left(x/L_b\right)$, $\tau=t\left(EI/\rho AL_b^4\right)^{1/2}$, $w=\left(v/d_0\right)$, $\beta=\left(d_0/L_b\right)^2$, $\alpha=\left(6\varepsilon_0 L_b^4/Eh^3d_0^3\right)^{1/2}$, $\bar{L}_2=\left(L_2/L_b\right)$, $\bar{c}=c\left(L_b^4/EI\rho A\right)^{1/2}$, $\bar{\mu}=\mu\left(b_b/d_0\right)\left(\rho AL_b^4/EI\right)^{1/2}$, $L=\left(L_C/L_b\right)$, $\bar{L}_1=\left(L_1/L_b\right)$, and $\bar{M}=\left(M/\rho AL_b\right)$ into the equation (1), one may obtain the following equation on motion.

$$\frac{\partial^2 w}{\partial\tau^2}+w''''+\beta\left\{w'\left(w'w''\right)'\right\}'+\beta\left[w'\int_1^{\bar{\eta}}\int_0^{\bar{\xi}}\left(\ddot{w}w'+\dot{w}'^2\right)d\bar{\xi}\,d\bar{\eta}\right]'+\frac{\mu b_b{}^3}{\left(1-w\right)^3}\frac{\partial}{\partial\tau}\left(1-w\right)$$

$$=\frac{\alpha}{\left(1-w\right)^2}\left[1-\frac{\beta}{3}\left\{w'^2+2\left(1-w\right)w''\right\}\right]\left\{H\left(s-\bar{L}_1\right)H\left(\bar{L}_2-s\right)\right\} \qquad 2(a)$$

Subjected to boundary conditions are as:

$$\text{at } s=0,\quad w=0,\quad \frac{\partial \mathrm{w}}{\partial \mathrm{s}}=0,$$

$$\text{at } s=1,\quad w'''=\bar{M}\frac{\partial^2 w}{\partial\tau^2}+\bar{M}L\frac{\partial^3 w}{\partial\tau^2\partial \mathrm{s}},\quad w''=-L\bar{M}\frac{\partial^2 w}{\partial\tau^2}-\frac{4}{3}\bar{M}L^2\frac{\partial^3 w}{\partial\tau^2\partial \mathrm{s}}. \qquad 2(b)$$

The dynamic pull-in instability relies exhaustively on the time dependent terms viz., inertial nonlinearity, air-film damping and AC component of a bias voltage. As the dimension of air-gap is smaller as compared to the other lateral dimensions of the substrate, the effect of air-film damping is inevitably significant. Hence, the influences of squeeze film damping in calculating the dynamic pull-in voltage, which often considers as the structural limitation is inevitable for smaller air-gap thickness. Here, the primary aim is to investigate the impact of squeeze film damping on the dynamic pull-in instability of the microswitch under alternating actuation. A precise higher order of squeeze film $\left(\bar{\mu}b_b^3/\left(1-w\right)^3\right)\frac{\partial}{\partial\tau}\left(1-w\right)$ has been considered in obtaining the pull-in voltage in dynamic condition. Neglecting the terms of higher order distribution of electrostatic pressure, higher order of squeeze film and nonlinear geometric and inertial, the present equation of motion can also identically represent with the equation obtained in [10] assuming small deflection. Nonetheless, the outcomes obtained here are distinct and included various facets of MEMS technology to develop an accurate mathematical model with pertinent quality assessment of pull-in instability in electrostatically actuated MEMS devices.

3. Generalize Decomposition

Dynamics of deflection of the micro beam actuated electrostatically actuation can be written as:

$$w\left(s,\tau\right)=\sum_{n=1}^{n}\gamma_j\left(s\right)e^{i\omega\tau},\qquad n=1,2,3\cdots\infty,\qquad 0\le s\le 1. \qquad (3)$$

Here, $e^{i\omega\tau}$ is the time modulation and $\gamma_j(s)$ is an admissible function. Following admissible function of ith mode has used similar to that of the eigen-function of the cantilever beam with extended tip mass expressed as

$$\gamma_n\left(s\right)=\left\{cosh\left(\beta_n s\right)\text{-}cos\left(\beta_n s\right)\right\}+\Gamma_n\left(sinh\left(\beta_n s\right)-sin\left(\beta_n s\right)\right). \qquad (4)$$

Here,

$$\Gamma_n = \frac{\sin \beta_n s\,(1-\overline{M}\beta_n{}^2 L) - \sinh \beta_n s\,(1-\overline{M}\beta_n{}^2 L) + \left(\overline{M}\beta_n\right)\{\cos \beta_n s - \cosh \beta_n s\}}{\cos \beta_n s\,(1-\overline{M}\beta_n{}^2 L) - \cosh \beta_n s\,(1+\overline{M}\beta_n{}^2 L) + \left(\overline{M}\beta_n\right)\{\sin \beta_n s - \sinh \beta_n s\}}$$

The value β_n can be obtained from the following frequency equation.

$$1+\frac{1}{3}L^2\overline{M}\beta_n^4 + \cos\beta_n \cosh\beta_n\left(1-\frac{1}{3}L^2\overline{M}\beta_n^4\right) + \cos\beta_n \sinh\beta_n\left(\overline{M}\beta_n - \frac{4}{3}L^2\overline{M}\beta_n^3\right) -$$
$$\cosh\beta_n \sin\beta_n\left(\overline{M}\beta_n + \frac{4}{3}L^2\overline{M}\beta_n^3\right) - 2\frac{4}{3}L\overline{M}\beta_n^2 \sinh\beta_n \sin\beta_n = 0. \qquad (5)$$

By adopting the Galerkin’s techniques, the weighting function has been selected similar to that of admissible function (4).

4. Results and discussion

The resulting differential equation (1) has been solved by numerically integrating using standard fourth order Runge-Kutta scheme. As the governing equation includes number of nonlinear terms, it is expected to have multiple solutions which are strictly controlled by the initial guess and convergence criteria. The two aspects i.e., effect of squeeze film and AC actuation on the estimation of dynamic stability of microswitch has been presented. The trajectories have been plotted in the phase plane in estimating the pull-in voltage by investigating the intersection of these trajectories with the horizontal axis passing through the origin. The obtained dynamic pull-in voltages have been verified by plotting the time response for various applied voltage. Here, the trajectories in the phase plane have been plotted for the both un-damped and system with squeeze film for a microswitch considering higher order deflection which takes care of geometric and inertial nonlinearities in addition to higher order of electrostatic force that were not explored earlier. Here, nonlinearity of squeeze film has also taken into consideration in calculating the pull-in voltage. The dynamic pull-in instability has been evident by the monoclinic orbits passing through the saddle-node or degenerate singularity when subjected to a step input voltage. Hence, the dynamic pull-in instability takes place due to the presence of monoclinic bifurcation as the periodic orbits collides with saddle-node point.

In subsection 4.1, the ascendency of squeeze air-film for on pull-in occurrence has been illustrated whereas in subsection 4.2, effect of alternating actuation in the development of pull-in instability has been described.

4.1. Effect of squeeze film damping

The rate of varying the applied voltage is negligible while applying a constant DC source. Hence, for estimating the static pull-in voltage, the effect of time variant terms has been eliminated. However, to accurately understand the benefits and limitation of the device, only the estimation of static pull-in voltage of the device is highly inappropriate when a bias voltage is applied. Hence, the pecdiction of critical voltage at which pull-in occures needs to re-evaluate under the consideration of micro-beam dynamics.

Since the static pull-in occurrence is irrespective of the presence of damping as well as the inertia terms, therefore applied voltage at which static pull-in occurs results a surplus of electrostatic force over the restoring force. It has been observed that the microbeam with higher flexural stiffness requires higher applied voltage to become statically unstable due to pull-in. Pull-in deflection due to static voltage has been shown in Fig. 2 and it is being observed that maximum deflection has occurred at applied voltage closest to 6.215. The pull-in instability occurs at voltage 5.375 when a bias voltage is applied as compared to the static counter-part and it has been observed that stability of the system loses at a voltage less than that of static pull-in voltage i.e., nearly 86% of the pull-in voltage in static condition. Hence, for smooth and safe operation, designer must consider the voltage causing dynamic pull-in voltage as a limitation while designing the microsystems that run under a bias voltage source. Furthermore, it is noticed that the dynamic pull-in voltage occurs at voltage within the range of 85- 95 % of static counter-part for undamped system under the influences of different design parameters.

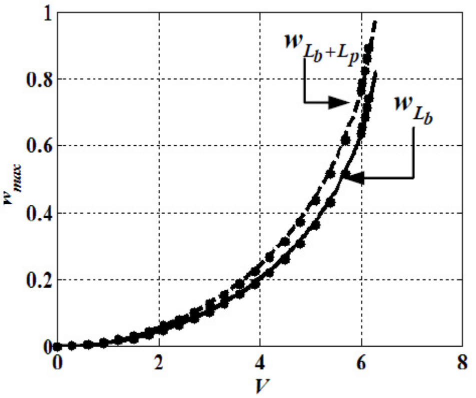

Fig. 2: Static pull-in at beam end (w_{L_b}) and plate at end ($w_{L_b+L_p}$) under actuation voltage for $(d_0/L_b)^2 = 4.4e^{-5}$.

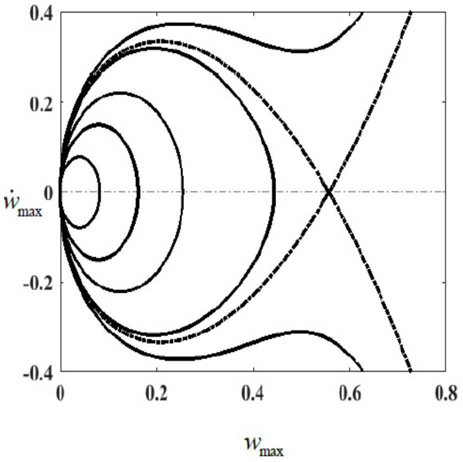

Fig. 3: Dynamic pull-in occurrence for system without considering the squeeze film damping for $(d_0/L_b)^2 = 4.4e^{-5}$. and electrode length 45 μm.

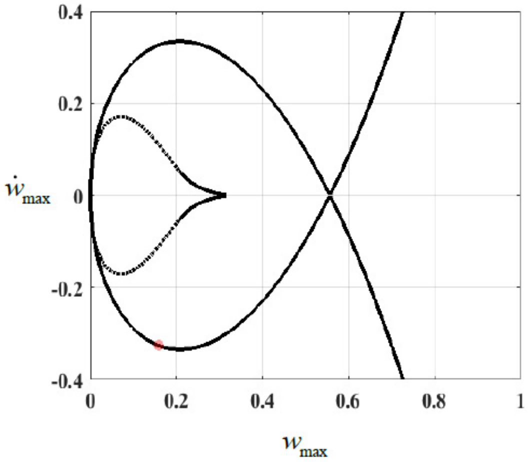

Fig. 4: Dynamic pull-in voltage of under-damped system with squeeze film for electrode length 45 μm, $\beta = 4.4e^{-5}$, and $\mu = 1.86\times10^{-5}$ N.s/m^2.

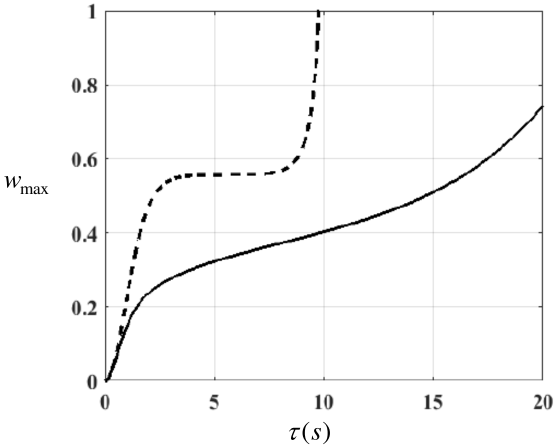

Fig. 5: Dynamic pull-in of (A) undamped system and (B) system with squeeze film effect

It is often inevitable to forsake the effect of squeeze film in estimating the structural stability due to pull-in phenomenon where most of the MEMS devices are modeled as parallel plate or beams that confine a very thin air film between the microstructure and substrate. The building-up pressure in the air-gap for the viscous flow of air-film due to the dynamic motion of the microstructure opposes the movement of the microstructure. This development will help the system response amplitude getting down and hence, a high applied bias voltage is required to the system become collapsed onto the ground. This is why the effect of squeeze film or viscous damping is undesirable during the performance of microswitch/micro-gripper as it works moving into and out of the side due to pull-in occurrence to alternate ON and OFF condition of the electrostatically actuated microswitch/grippers. Howbeit, when the MEMS devices are designed to work as sensors, and actuators, it is expected to perform safely and smoothly within a reasonable range of applied voltage. In that sense, the range of applied voltage is moderately enhanced by the effect squeeze film damping and it has been observed that the microstructure with a relatively small air-gap works perfectly in higher range of applied bias voltage. The deflection of the microstructure has been shortened due to the presence of squeeze film thickness as shown in Fig. 4 and it has been observed that maximum pull-in deflection occurred at a applied voltage equal to 5.375 without considering the squeeze film effect i.e., undamped system, which comes around 86% of static pull-in voltage. The critical voltage has become increased to 6.1 for flattering dynamically unstable in the presence of squeeze film damping. The stability of the system has improved nearly 14 % for a low value of effective damping μ as compared to the case of without considering squeeze film damping. This percentage may increase for a higher value of damping ratio $\bar{\mu}$. Hence, it is truly evident that the inertia effect gets belittled by the presence of squeeze film in between the microstructure and substrate. It is interesting to note that the pull-in deflection ($w_{\max}$) at critical applied voltage has found to be around 0.32 as compared to 0.54 for the system without considering the squeeze air-film. Hence, the amplitude of dynamic motion has significantly foreshortened in the shadow of air damping and effective air-gap is reduced as highly viscous boundary layer has being introduced at the surface of substrate.

4.2. Effect of Alternating Voltage

Calculating or estimating the static pull-in voltage only while designing a MEMS device is appropriate if the rate of varying a bias voltage is insignificant i.e., $dV \approx 0$ at time $dt \neq 0$. Hence, it is highly demanded as an important design parameter to evaluate the dynamic pull-in voltage rather than determining the static pull-in voltage when the device is performed under a bias source of voltage. Interesting, it has been discovered that the point of dynamic instability does not significantly influenced by the scaling of fluctuation voltage about a steady DC voltage as shown in Figs. 6 and 7. Unlike step input, here pull-in instability has been developed due to dynamic bifurcation instead of static bifurcation and homoclinic orbits collide with hopf bifurcation rather saddle-node bifurcation. At this critical point, the period of periodic orbit becomes infinity and yielding an infinite duration to become the system as global unstable.

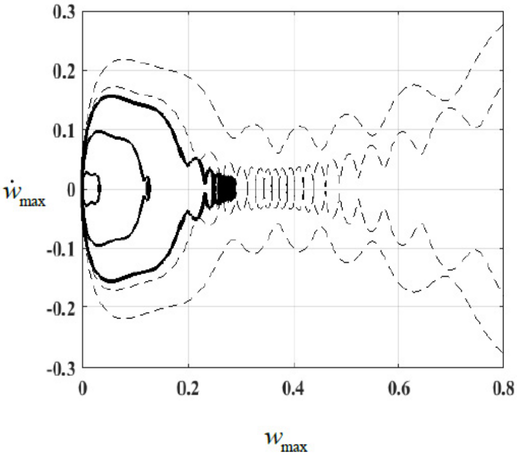

Fig. 6: Dynamic pull-in occurrence for system without considering the squeeze film damping for $(d_0 / L_b)^2 = 4.4e^{-5}$. and electrode length 45 μm.

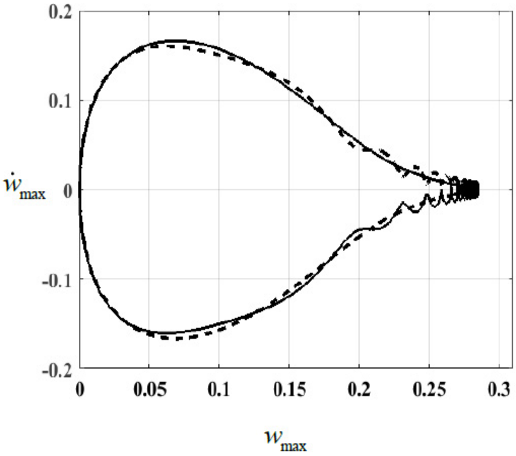

Fig. 7: Dynamic pull-in voltage of under-damped system with squeeze film for electrode length 45 μm, $\beta = 4.4e^{-5}$, and $\mu = 1.86 \times 10^{-5}$ N.s/m^2.

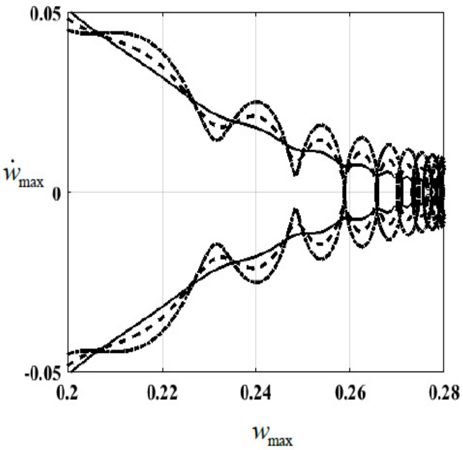

Fig. 8: Dynamic pull-in voltage of under-damped system with squeeze film for electrode length 45 μm, $\beta = 4.4e^{-5}$, and $\mu = 1.86\times10^{-5}$ N.s/m^2.

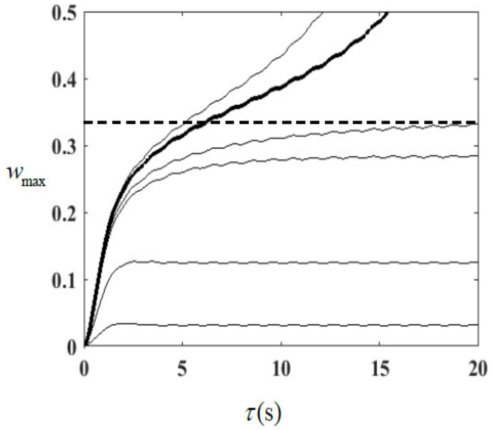

Fig. 9: Time response of under-damped system with squeeze film for electrode length 45 μm, $\beta = 4.4e^{-5}$, $\mu = 1.86\times10^{-5}$ N.s/m^2.

As observed in Figs. 3 and 5, at critical point, when a step voltage applied the movable structure is collapsed onto the ground electrode and for further increase in voltage, it remains on to the stationary electrode maintained a flat configuration. However, it is interesting to note that there is no upper limit of applied voltage for the flat configuration. When a combined DC and AC potential difference exists, at pull-in voltage, the movable structure will vibrate with an amplitude and digging periodically onto the ground electrode. For an alternating potential difference, the movable structure vibrate with amplitude of pull-in deflection occurred at the critical voltage. This amplitude increases with increase in value of AC component as shown in Fig. 7 and Fig. 8.

It is noteworthy and cleared from the Fig. 9 that influence of inertia is subservient in the presence of squeeze film effect and system has become a first order nonlinear vibratory system. As mentioned, the point of dynamic instability does not alter by the order or level of fluctuation of the applied voltage. However, the catastrophic failure may observe due to the degradation of fatigue resistance as the mechanical stresses incited into the movable structure are altered periodically during operation.

The initial displacement varies exponentially with time and reaches a steady state fixed solution when time goes infinity as shown in Fig. 9. The amplitude of exponentially varying response depends on the value of coefficient of squeeze film damping and spring constant and an aperiodic response may be observed for the microsystems under high influenced squeeze film effect.

5. Conclusions

The significance of squeeze film damping and alternating actuation voltage on dynamic pull-in voltage calculation has been investigated. The effect of squeeze film and alternating actuation does not influence the system static stability which is appropriate and significant for steady voltage inputs. The structural stability has been improved by squeeze film effect as the overall damping constant gets higher when the air-gap thickness is relatively small. The higher order air-damping cannot be avoided for the deformation of micro-cantilever based instrument in MEMS is larger under the external excitations. It is also observed that with some initial conditions, chaotic behavior may be observed in the transient part of the response due to the squeeze film damping for few initial periods.

The dynamic pull-in voltage has been observed to be the same for all the form of bias voltages i.e., step input, harmonic input and combined AC and DC input. Notwithstanding, unlike the case of step input, here for combined AC and DC voltage, the movable electrode vibrate with mean pull-in deflection which results fatigue failure of the movable structure. Hence, a catastrophic collapse would happen from the fatigue failure.

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