IEEE *Access*

Multidisciplinary ⋮ Rapid Review ⋮ Open Access Journal

Robust Stabilization of a Class of Nonlinear Systems via Aperiodic Sensing and Actuation

NILADRI SEKHAR TRIPATHY[1]**, (Member, IEEE), INDRA NARAYAN KAR**[2]**, (Senior Member, IEEE), MOHAMMADREZA CHAMANBAZ**[3]**, (Member, IEEE), and ROLAND BOUFFANAIS** [4]**, (Member, IEEE)**

[1]Dept. of Electrical Engineering, Indian Institute of Technology Jodhpur, Rajasthan, India-342037, e-mail: (niladri@iitj.ac.in)
[2]Dept. of Electrical Engineering, Indian Institute of Technology Delhi, New Delhi, India-110016, e-mail: (ink@ee.iitd.ac.in)
[3]Singapore University of Technology and Design, Singapore 487372, (e-mail: chamanbaz@sutd.edu.sg)
[4]Singapore University of Technology and Design, Singapore 487372, (e-mail: bouffanais@sutd.edu.sg)

Corresponding author: Mohammadreza Chamanbaz (e-mail: chamanbaz@sutd.edu.sg).

ABSTRACT This paper proposes a framework to design a robust controller for a class of nonlinear networked control systems using aperiodic feedback information. Here, the nonlinearity and parameter variations of system model are considered as sources of uncertainty. To tackle the uncertainty in system dynamics, a linear robust control law is derived by applying the optimal control theory. Two different architectures of closed-loop systems are considered. In the first one, system and controller are not collocated; instead they are interconnected by means of a shared communication network. In the second architecture, system, controller and actuator are all collocated with their respective outputs available at all time—instead, sensors and controller are connected through a shared communication channel. In both architectures, the feedback loop is closed through the network. Owing to its shared nature, the network may suffer from bandwidth limitations. To save the network bandwidth, state and input information are transmitted aperiodically within the feedback loop. With this aim, the paper adopts an event-triggered control technique so as to reduce the transmission overhead. Applying Input-to-State Stability theory, we derive two different event-triggered robust control laws that stabilize the uncertain nonlinear system. Finally, we show that the designed event-triggered controllers satisfy the trade-off between control performance and saving in network bandwidth in the presence of uncertainty. The developed control algorithm is implemented and validated through numerical simulations.

INDEX TERMS Bandwidth limitations, Event-triggered control, Input-to-state stability, Nonlinear systems, Robust-optimal control, Optimal event-triggering control.

I. INTRODUCTION

GENERALLY , in Cyber-Physical Systems (CPSs) or Networked Control Systems (NCSs) each physical component shares its own local information with other subsystems through a communication network. As a result of the shared nature of the communicating channel, controlling such systems with continuous or periodic control laws require large bandwidth resources [4], [15], [21]. In recent past, an event-triggered control technique has been introduced in [11]–[13], [37], [40], [41] to reduce the information requirements in order to achieve a stable control strategy.

Specifically, in the event-based control framework, when a prespecified event condition is violated, it determines the sensing and actuation instants at both sensor and actuator ends. This event-triggering law mainly depends on the system's present state or outputs. In the event-triggered control framework for continuous systems, the key issue is the stringent requirement in continuously monitoring the event condition occurrence. For instance, in [11], [12], the monitoring of the event-triggering condition is conducted periodically. To overcome the need for such a continuous/periodic monitoring, a self-triggered control approach has been developed

and reported in [3], [45]. In this self-triggered control approach, the subsequent time instant for event occurrence is determined using the system's state or output information at the previous sampling instant. For both classical event-triggering and self-triggering controls, a reduction in the overall network use can be achieved by increasing the time interval between triggering events. In the specific context of cyber-physical systems (CPSs) and networked control systems (NCSs), the primary role played by aperiodic sensing and actuating for continuous and periodic event-triggered control has been reported in [4], [15], [21].

The key deficiency with classical event-triggered control is the need to have access to an accurate model of the studied system in order to devise the event-triggering rule. In practice, system modeling inevitably simplifies the actual system operation and thereby introduces a certain level of inaccuracy, which have practical implications. It is worth highlighting that there is a vast breadth of problems related to addressing the issue of event-triggering control in the presence of uncertainty. Such uncertainty has several possible origins: nonlinearity, variation in the system's parameters, components unaccounted for in the dynamical model, and pervasive perturbations. These issues thereby necessitate the development of a specific controller. Recently, an attempt has been made to develop both state and output feedback resilient controllers under communication constraint and model uncertainty. Ghodrat & Marquez [9] have proposed an event-triggered control law for Lipschitz nonlinear systems. In their work, the design of the triggering rule and control law have been carried out concomitantly. Both state and output feedback event-triggered control laws have been developed. To develop output feedback law, they consider an observer dynamics with intermittent measurement. They have shown that the separation principle is satisfied under small sampling threshold between sensors-observer transmission channel. In [25], Liu & Huang have proposed an event-triggered output feedback robust control technique for a class of nonlinear systems. They have solved the global robust output regulation problem for nonlinear systems in the presence of uncertain parameters that belong to some arbitrarily large prescribed compact set. Liu & Jiang [24] discussed the concept of event-triggered robust stabilization of nonlinear systems using the small gain approach. To avoid infinitely fast sampling, they have proposed an Input-to-State Stability (ISS) gain condition and correspondingly an event and self-triggering mechanism subject to external disturbances. Recently, in [42], [43], an event-triggered robust control algorithm has been developed based on aperiodic feedback to deal with the presence of uncertainty, albeit limited to linear systems. Tripathy et al. have adopted an optimal control strategy to design such a robust control law [42], [43]. Originally, this control law has been developed by Lin et al. [22], [23] within the optimal control framework. The nominal dynamic (or a virtual dynamic) has been used to design the control law. To realize the robust control law in [43] and [42], a prior assumption is made in that the system model is considered

to be linear in nature. But in practice, most systems are nonlinear. Therefore, considering nonlinear systems is a far more realistic and pertinent control problem . Moreover, extending robust control results mentioned in [42], [43] for a class of nonlinear systems in the presence of bandwidth constraints in the communication channel is not straightforward. Indeed, the design of robust control input depends on results borrowed from the optimal control theory. In general, to design an optimal control law for a nonlinear system, it is essential to solve the Hamilton–Jacobi–Bellman (HJB) equation. Solving HJB is known to be computationally intensive and expensive since it essentially is a partial differential equation (PDE). Researchers have used different techniques to achieve this goal—e.g., neural networks and dynamic programming [1], [2], [6], [44], [48]. Recently, Yang & He [49] adopted an actor-critic based neural-network technique to address the robust stabilization problem of event-triggered nonlinear systems with input constraint. To design such a robust controller, they have solved an infinite-time nonlinear optimal control problem. However, these computation techniques remain computationally demanding. To overcome these challenges, a linear control law is proposed for a class of nonlinear systems, which can withstand uncertainties and limited availability of feedback information. This paper considers the input-to-state stability theory [10], [31], [36], [50] for analysis. Various researchers used the ISS theory for analyzing the robustness of event-triggered linear and nonlinear systems. The ISS theory results for linear system with external disturbance with observer-based output feedback control has been discussed in [50]. Ghodrat & Marquez [10] have applied the ISS theory to derive the event-triggering rule for a class of input-affine nonlinear systems under network constraints. They also showed that the proposed controller ensures stability in the presence of actuator errors and external disturbances.

In this paper, an event-triggered robust control algorithm is proposed to stabilize a class of nonlinear systems with aperiodic feedback information. Here, nonlinear systems with parametric uncertainty are considered. An attempt is made to rewrite the system dynamics as a linear model plus uncertainty. With this formulation, the system nonlinearity and parametric variation of the system's model are considered as a source of uncertainty. An event-based linear robust control algorithm is developed to stabilize this class of nonlinear systems with aperiodic feedback information. To regulate the behavior of this system when faced with multiple sources of uncertainty, two different event-based control algorithms are introduced. The first event-triggering rule depends on the error between current and last transmitted state information, whereas the second one uses a nominal model for event generation. Furthermore, for an optimal usage of communication resources in the presence of model uncertainty, a modified optimal control problem has been formulated where both the cost due to the information transmission and system uncertainty are considered. To ensure the closed-loop stability of such systems, a robust control law is computed using the

nominal—or a virtual—dynamics and the prior knowledge of the uncertainty bound. Next, the derived controller gain matrix is used to analyze the closed-loop performance. The ISS theory is applied to derive the event-triggering rule. The key contributions of this work are listed below:

- A class of nonlinear dynamical systems is considered. The nonlinear component and parameter variations of the system model are treated as a source of matched and mismatched uncertainties. Using the optimal control framework for robust controller design, a linear control law is derived by solving a Linear Quadratic Regulator (LQR) problem. The linear robust control law ensures the closed-loop stability of the original nonlinear system.
- Based on the classical input-to-state stability theory, a novel event-triggering rule is developed to reduce the information required to stabilize this class of systems. The triggering law considers the upper bound of uncertainty such that it can withstand a range of variations for the uncertain parameters.
- We propose an event-triggered robust controller for uncertain systems with optimal event-triggering. To solve the robust controller and optimal event-triggering law, a joint optimization problem is formulated by minimizing a cost-function that embodies both control and communication costs for optimal usage of resources. It is shown that the design of robust optimal event-triggered controller using the optimal control framework is split in two sub-problems—the design of robust controller using the linear quadratic regulator (LQR) framework and the optimal event-triggering sequence using dynamic programming.

Organization

The paper is organized as follows. In Section II, we present the problem statement and preliminaries, which will be used subsequently to state the results. The proposed concept considers the infinite-horizon cost and a zero-order-hold (ZOH) at the actuator end to realize the control law. Section III and IV present the key contributions of this work—mainly the event-triggering criterion and stability results. The event-triggering and stability results for mismatched and matched uncertain systems are presented in Sections III and IV respectively. A new ZOH-free robust control law with optimal event-triggering law is also presented in Section IV. The proposed robust control law is derived by minimizing a finite-horizon cost consisting of communication cost and the cost associated with system uncertainty. In Section V, the effectiveness of the developed control algorithm is assessed numerically based on two examples of nonlinear systems. Section VI concludes the paper. Some of the proofs and steps to realize the proposed control laws are included in Appendix.

II. PRELIMINARIES AND PROBLEM FORMULATION

This section mainly presents the problem and briefly describes some preliminaries which are used subsequently in the next sections.

A. NOTATIONS AND DEFINITIONS

The Euclidean norm of a vector $x \in \mathbb{R}^n$ is denoted by $\|x\|$, while $\mathbb{R}^n$ refers to the vector space of real vectors of dimension n, and by extension, $\mathbb{R}^{n \times m}$ is the vector space of real-valued n-by-m matrices. The notation $\mathbb{R}_{\geq 0}$ refers to the set of non-negative real numbers. The symbols $A \leq 0$, A^T and A^{-1} are classically used to specify the negative semi-definite character of a matrix A, its transpose, and its inverse respectively. The symbol I denotes the identity matrix of appropriate dimension. The norm of a matrix $A \in \mathbb{R}^{n \times m}$ is denoted by $\|A\|$ and computed as $\|A\| := \sup\{\|Ax\| : \|x\| = 1\}$. The maximum (resp. minimum) eigenvalue of a symmetric matrix $P \in \mathbb{R}^{n \times n}$ is $\lambda_{\max}(P)$ (resp. $\lambda_{\min}(P)$). A continuous function $f \colon \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ is said to be class $\mathcal{K}_\infty$ if it is strictly increasing and $f(0) = 0$ and $f(s) \to \infty$ as $s \to \infty$. A function $f : \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ is class $\mathcal{K}$, if it is continuous, strictly increasing and $f(0) = 0$. A continuous function $\beta(r, s) : \mathbb{R}_{\geq 0} \times \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ is a $\mathcal{KL}$ function, if it is a class $\mathcal{K}$ function with respect to r for a fixed s, and it is strictly decreasing with respect to s when r is fixed [18]. We remark that the definitions used throughout this paper are identical to those found in the literature [18], [31], [36].

Definition 1 (Input-to-State Stability)**.**
A continuous-time system

$$\dot{x}(t) = f(x(t), u(t)), \qquad (1)$$

is input-to-state stable (ISS) if there exists a solution $x(t)$, $\forall t \geq 0$ satisfying

$$\|x(t)\| \leq \beta(\|x(0)\|, t) + \gamma\left(sup_{\tau \in [0, \infty)}\{\|u_\tau\|\}\right),$$

for all admissible inputs $u(t)$ and for all initial values $x(0)$, with β and γ being a $\mathcal{KL}$ and $\mathcal{K}_\infty$ function, respectively.

Definition 2 (ISS Lyapunov Function)**.**
A continuously differentiable function $V(x(t)) : \mathbb{R}^n \to \mathbb{R}$ is an input-to-state (ISS) Lyapunov function for (1) if there exists class $\mathcal{K}_\infty$ functions $\alpha_1, \alpha_2, \alpha_3$ and a class $\mathcal{K}$ function γ for all $x \in \mathbb{R}^n$ and $u \in \mathbb{R}^m$ satisfying the following conditions:

$$\alpha_1(\|x(t)\|) \leq V(x(t)) \leq \alpha_2(\|x(t)\|), \qquad (2)$$
$$\dot{V}(t) \leq -\alpha_3(\|x(t)\|) + \gamma(\|u(t)\|). \qquad (3)$$

B. PROBLEM DESCRIPTION

This paper considers a feedback control strategy for networked control systems in the presence of bandwidth constraints in feedback path and parametric uncertainty in system dynamics. To tackle channel constraint in feedback loop in the face of model uncertainty, we formulate a novel event-triggered robust control algorithm for a class of nonlinear

systems. Figure 1 shows the block diagram of the proposed robust control technique. In this diagram, the following elements are clearly appearing: (i) system, (ii) controller, and (iii) a communication network interconnecting the previous two components. The states of the system are measured continuously by the sensors at the system end. The information from sensors are shared with the controller through a communication network. In between sensor and controller, an event-monitoring unit monitors continuously the occurrence of an event condition. Specifically, when a predefined triggering event occurs, the monitoring unit ensures the proper transmission of the state variable to the controller. This robust control problem is addressed from an equivalent optimal control strategy based on the linear nominal model or a virtual dynamics of the original nonlinear systems. The gain K of the controller and aperiodic state datum, $x(t_k)$, which is obtained from the nonlinear system serves to compute this event-triggering control rule $u(t_k) = Kx(t_k)$ stabilizing the closed-loop system in the presence of uncertainty. Here, the input function is actuated aperiodically at instants $t_0, t_1, t_2, \cdots, t_k$, where t_k represents the latest such event. A zero-order hold (ZOH) at the actuator end holds the most recent actuated input data until a subsequent triggering event leads to the transmission of another input data. Here, the actuator is assumed to be embedded within the system, with an instantaneous update of the control input at the time of transmission. The primary concern of this paper is to propose
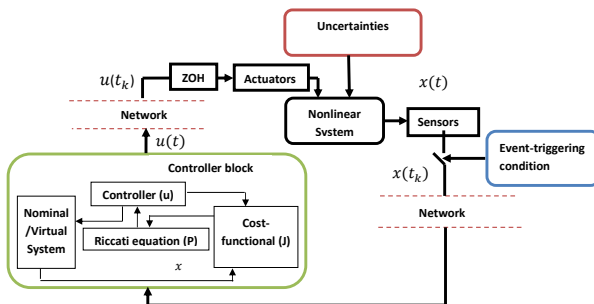

FIGURE 1: Architecture I: Schematic block diagram of the developed event-triggered robust control strategy where sensors, controller and actuators are interconnected through a communication network.

an event-triggered robust control law that can withstand the system nonlinearity and model uncertainty for a class of nonlinear systems. In general, uncertainty in system dynamics is either matched or mismatched (matched: i.e. uncertainty is in the range space of input matrix [5], [17], [19], [34], mismatched: i.e. uncertainty is not in the range space of input matrix). In this section, we consider mismatched system first and then results of matched systems are reported in Section IV as a special case of the mismatched case.

Remark 1. *This paper considers two architectures for sensors, controller and actuators network [39]. In the first architecture, shown in Fig. 1, we assume that sensors and actuators are collocated but the controller is not collocated and it is interconnected through a communication network. In the second architecture, shown in Fig. 2, we consider actuators and controller to be collocated but sensors are spatially distributed and interconnected with the controller via a communication network. This type of NCS architecture has been considered in [27], [28]. A detailed discussion of the second architecture is given in Section IV.*

System description: Consider a class of nonlinear systems with uncertainty characterized by the following dynamical law

$$\dot{x}(t) = Ax(t) + \underbrace{D\Phi(x)}_{\Delta_1(x)} + \big(B + \underbrace{Bh(x)}_{\Delta_2(x)}\big)u_{\text{mis}}(t), \quad (4)$$

where $x \in \mathbb{R}^n$, $u_{\text{mis}} \in \mathbb{R}^m$ are the state and input vectors respectively. The matrices A, B and D are constant matrices with appropriate dimensions. The matrix pair (A, B) is controllable. Two unknown nonlinear functions $\Delta_1(x) = D\Phi(x)$ and $\Delta_2(x) = Bh(x)$ are treated as uncertainty sources. Specifically, $h(x)$ corresponds to the uncertainty at the input level, while $\Phi(x)$ embodies the uncertainty at the system's level. In general, uncertainty in system dynamics is either matched or mismatched [5], [19]. The system (4) suffers from matched uncertainty if both uncertainties

$$\begin{cases} \Delta_1(x) = D\Phi(x), \\ \Delta_2(x) = Bh(x), \end{cases} \quad (5)$$

are in the range space of the nominal input matrix B. However in (4), the nonlinear function $\Delta_1(x)$ does not hold the matching condition as $D \neq B$, thereby yielding a mismatched case. The uncertainty $\Delta_1(x)$ in (4) can be decomposed into matched and mismatched components:

$$D\Phi(x) = \underbrace{BB^+ D\Phi(x)}_{\text{matched}} + \underbrace{(I - BB^+)D\Phi(x)}_{\text{mismatched}}. \quad (6)$$

The notation B^+ is used to represent the pseudoinverse [14] of input matrix B. Unknown functions $\Phi(x)$ and $h(x)$ satisfy the following assumptions:

Assumption 1. *The function $\Phi(x)$ is bounded $\forall x$ and the following inequality holds*

$$\Phi(x)^T[D^T B^{+T} B^+ D + I]\Phi(x) \leq x^T F_{mis}x. \quad (7)$$

where the positive semi-definite matrix F_{mis} is a priori known.

Assumption 2. *The function $h(x)$ is positive semi-definite, $h(x) \geq 0$ and there exists a known non-negative function $h_{max}(x)$ such that for all x, $h(x)$ satisfies*

$$0 \leq h(x) \leq h_{max}(x). \quad (8)$$

The matrix F_{mis} and function $h_{\text{max}}(x)$ in (7) and (8) are related with the upper-bound on uncertainties $\Phi(x)$ and $h(x)$.

In the subsequent sections, these Assumptions will be used to derive the controller gain matrices and stability results.

From [37], the closed-loop system (4) with event-triggered control input $u_{\mathrm{mis}}(t_k)$ can be written as

$$\dot{x}(t) = Ax(t) + D\Phi(x) + (B + Bh(x))u_{\mathrm{mis}}(t_k), \quad (9)$$

$$u_{\mathrm{mis}}(t_k) = K_{\mathrm{mis}}x(t_k) = K_{\mathrm{mis}}(x(t) + e(t)). \quad (10)$$

where K_{mis} is the controller gain and $x(t_k)$ is the state information of (9) at the kth event-triggering instant. To tackle aperiodic information $x(t_k)$, an error variable $e(t)$ is defined

$$e(t) = x(t_k) - x(t), \quad t \in [t_k, \ t_{k+1}). \quad (11)$$

To stabilize (9) in the presence of uncertainty and aperiodic feedback information, the following problem is formulated.

$\mathbf{P}_1-$ **Problem Statement:** Design the robust state feedback control law (10) to regulate the closed-loop behavior of the event-triggered system (9) such that it is input-to-state stable (ISS) with respect to its measurement error $e(t)$, in the presence of uncertainty (5).

Proposed solution: To solve the proposed problem, two different steps are adopted. First, results from the optimal control theory are used to develop a robust control strategy. As a next step, an event-triggering criterion is established to ensure input-to-state stability of (9). This criterion is obtained from assuming the existence of an input-to-state stable Lyapunov function $V(x) = x^T P x$, $P \geq 0$. The specific details about the derivation of this criterion are presented in the following Sections. The method to derive the robust controller gains to tackle uncertainty and event-triggering rule to deal with aperiodic feedback are presented next.

III. EVENT-TRIGGERED ROBUST CONTROL

This section describes the steps involved in designing the robust controller and event-triggering law. The controller design steps are discussed first, followed by the theorem associated with the event-triggering condition.

A. CONTROLLER DESIGN

To determine the state feedback gain, this paper adopts the emulation approach. That is, initially the gain matrices are derived assuming that feedback information is available continuously. Next, some techniques are developed to take into consideration some network effects. In the following, the controller design process is discussed.

Aim: Design the state feedback controller K_{mis} such that system (4) remains stable in the presence of bounded uncertainties (5).

To solve the above-mentioned robust control problem, an optimal control approach is adopted. The central idea is to design the optimal control input for the linear virtual (or nominal) system that minimizes a modified cost function. The term "modified" is used here to characterize the cost function given its dependence on the maximum variation (i.e. upper bound) of uncertainty. Then, it is shown that

this derived optimal input is also a robust solution to the original system in the presence of uncertainty. Now, we derive the corresponding virtual system and cost function of the uncertain system (4).

- The virtual dynamical law for system (4) reads

$$\dot{x} = Ax + Bu_{\mathrm{mis}} + (I - BB^+)Dv, \quad (12)$$

and the cost function for the mismatched uncertain systems (4) is given by

$$J_{\mathrm{mis}} = \int_0^\infty (x^T(F_{\mathrm{mis}} + \eta^2 I)x + u_{\mathrm{mis}}^T u_{\mathrm{mis}} + \rho^2 v^T v)dt, \quad (13)$$

where the matrix F_{mis} is selected such that the inequality (7) holds.

The state feedback control input $u_{\mathrm{mis}} = K_{\mathrm{mis}}x$ and virtual input $v = Lx$ serve to stabilize (12). The virtual control input v is introduced to consider the mismatched part of the uncertainty. To obtain a robust controller in this optimal control approach, we use the following Lemma stated in [1], [22], [23].

Lemma 1. *The optimal control solutions for virtual system* (12) *with a modified cost function* (13) *is robust for the original system* (4) *in the presence of all bounded variations of uncertainties* (5).

A proof for Lemma 1 can be found in [1], [22], [23]. Based on this Lemma, the robust controller gain matrices can be obtained by solving a linear-quadratic regulator (LQR) problem. According to the optimal control theory [29], the optimal control signals for (12) minimizing the cost function (13) are given by

$$u_{\mathrm{mis}} = \underbrace{-B^T P_1}_{K_{\mathrm{mis}}} x = K_{\mathrm{mis}}x, \quad (14)$$

$$v = \underbrace{-\rho^{-2}D^T(I - BB^+)^T P_1}_{L} x = Lx, \quad (15)$$

where P_1 satisfies the following Riccati equation

$$P_1 A + A^T P_1 - P_1 B B^T P_1 + F_{\mathrm{mis}} + \eta^2 I$$
$$-\rho^2 P_1(I - BB^+)DD^T(I - BB^+)^T P_1 = 0. \quad (16)$$

The aperiodic state information $x(t_k)$ and controller gain matrices are used to derive the event-triggered control law, which is discussed next.

B. DESIGN OF EVENT-TRIGGERING LAW

This subsection presents the event-triggering condition and stability results for (9), in the presence of uncertainties (5). The solution of problems $\mathbf{P}_1$ is described below in the form of a theorem.

Theorem 1. *Let $\sigma \in (0,1)$ and η, $\beta \in \mathbb{R}$, the event-triggered control input (10) with the controller gains K_{mis}*

and L defined in (14) *and* (15) *guarantees asymptotic stability of the closed-loop system* (9), *if* (10) *is executed according to the following sequence of events*

$$t_0 = 0, \ t_{k+1} = inf\{t \in \mathbb{R} | t > t_k \wedge \mu_1 \|x\|^2 - \|e\|^2 \le 0\}, \tag{17}$$

and the following condition holds for L

$$2\rho^2 L^T L \le \beta^2 I < \eta^2 I, \tag{18}$$

with μ_1 defined as

$$\mu_1 = \frac{\sigma(\eta^2 - \beta^2)^2}{8(1 + \|h_{max}(x)\|^2)\|K_{mis}^T K_{mis}\|^2}. \tag{19}$$

Proof. Let $V(x) = (x^T P_1 x)$ be the Lyapunov function for (9). Then, $\dot{V}(x)$ along the direction of (9) is

$$
\begin{aligned}
\dot{V}(x) &= V_x^T (Ax + Bu_{\text{mis}} + (I - BB^+)Dv) \\
&\quad - V_x^T (I - BB^+)Dv + V_x^T D\Phi(x) + V_x^T BK_{\text{mis}}e \\
&\quad + V_x^T Bh(x)K_{\text{mis}}(x + e) \\
&= x^T (P_1 A + A^T P_1 - 2P_1 BB^T P_1 - 2\rho^2 P_1 (I - BB^+) \\
&\quad DD^T (I - BB^+)^T P_1)x + 2x^T P_1 Bh(x)K_{\text{mis}}x \\
&\quad - 2x^T P_1 (I - BB^+)Dv + 2x^T P_1 D\Phi(x) \\
&\quad + 2x^T P_1 BK_{\text{mis}}e + 2x^T P_1 Bh(x)K_{\text{mis}}e, \tag{20}
\end{aligned}
$$

where matrix V_x is used to represent $\frac{\partial V}{\partial x} = (2P_1 x)$. Using Eqs. (14), (15) and, (16), Eq. (20) is simplified as

$$
\begin{aligned}
\dot{V}(x) &= -\{x^T F_{\text{mis}} x + \eta^2 x^T x + u_{\text{mis}}^T u_{\text{mis}} + \rho^2 v^T v\} + 2\rho^2 v^T v \\
&\quad + 2x^T P_1 BB^+ D\Phi(x) + 2x^T P_1 (I - BB^+)D\Phi(x) \\
&\quad - 2x^T K_{\text{mis}}^T K_{\text{mis}} e - 2x^T K_{\text{mis}} h(x)K_{\text{mis}} e. \tag{21}
\end{aligned}
$$

Simplifying (21), the following expression is obtained

$$
\begin{aligned}
\dot{V}(x) &= -\{x^T F_{\text{mis}} x + \eta^2 x^T x + u_{\text{mis}}^T u_{\text{mis}} + \rho^2 v^T v\} + 2\rho^2 v^T v \\
&\quad - 2u_{\text{mis}}^T B^+ D\Phi(x) - 2\rho^2 v^T \Phi(x) \\
&\quad - 2x^T K_{\text{mis}}^T K_{\text{mis}} e - 2x^T K_{\text{mis}} h(x) K_{\text{mis}} e, \\
&\le -[x^T F_{\text{mis}} x - \Phi(x)^T (D^T B^{+T} B^+ D + I)\Phi(x)] \\
&\quad - x^T (\eta^2 I - 2\rho^2 L^T L)x - 2x^T K_{\text{mis}}^T K_{\text{mis}} e - \\
&\quad 2x^T K_{\text{mis}} h(x) K_{\text{mis}} e. \tag{22}
\end{aligned}
$$

Now, using (7) and (18), Eq. (22) reduces to

$$
\begin{aligned}
\dot{V}(x) &\le -(\eta^2 - \beta^2)\|x\|^2 - 2x^T K_{\text{mis}}^T K_{\text{mis}} e - 2x^T K_{\text{mis}} h(x) K_{\text{mis}} e, \\
&\le -\frac{q}{2}\|x\|^2 + \frac{4}{q}\|K_{\text{mis}}^T K_{\text{mis}}\|^2 (1 + \|h_{\max}(x)\|^2)\|e\|^2, \tag{23}
\end{aligned}
$$

where $q = (\eta^2 - \beta^2) > 0$. Based on Definition 2, the inequality (23) ensures the ISS of (9). The event-triggering rule (17) is also derived from (23). ☐

Algorithm 1 reported in Appendix B presents a procedure to realize the proposed control law.

The minimum inter-event time which is the minimum time between two consecutive events

$$\tau = \min_k \ (t_{k+1} - t_k),$$

has to be always greater than zero; otherwise, the so-called Zeno effect [16] can occur within the system dynamics. In order to prove that τ is always greater than zero, one has to derive its expression. In the following Lemma, we consider the mismatched system (9) and prove that τ is always greater than zero for the event-triggered rule derived in Theorem 1.

Lemma 2. *Consider the uncertain system* (9). *The minimum inter-event time τ for the event-triggered law* (17) *is*

$$\tau = \frac{2}{(\kappa_1 - \kappa_2)} \ln \left\| \frac{(1 + \mu_1)}{(1 + \frac{\kappa_2}{\kappa_1}\mu_1)} \right\|, \forall \kappa_1 > \kappa_2, \tag{24}$$

where $\kappa_1 = \left(\|A\| + \|BK_{mis} + B\bar{h}_{max}K_{mis}\| + \|D\| \frac{\|F_{mis}\|^{\frac{1}{2}}}{\|D^T B^+ B^+ D + I\|^{\frac{1}{2}}} \right)$ *and*
$\kappa_2 = \|BK_{mis} + B\bar{h}_{max}K_{mis}\|$, *with $\bar{h}_{max}$ being the upper bound of the known function $h_{max}(x)$.*

Proof. See Appendix. ☐

It is well-known that a system with mismatched uncertainty is difficult to control. In particular, it is hard to ensure the existence of a stabilizing controller satisfying all the conditions stated in Theorem 1. In the next section, we consider the matched uncertain system where uncertainty is in the range space of the input matrix B. These systems form a special case of the mismatched one. The main distinguishing feature is that there always exists a stabilizing controller for matched system while this is not the case for mismatched systems.

IV. NONLINEAR SYSTEM WITH MATCHED UNCERTAINTY

In (4), we consider the uncertainty description (6) which consists of both matched and mismatched components. Now, for a selection of matrix $D = B$, (4) reduces to a matched system with the following state-space representation

$$\dot{x} = Ax + B\Phi(x) + (B + Bh(x))u_{\text{mat1}}. \tag{25}$$

The notations x and u_{mat1} represent the state vector and control input for (25) respectively. Here, the nonlinear function $\Phi(x)$ satisfies the following assumption.

Assumption 3. *The uncertainty $\Phi(x)$ satisfies*

$$\Phi(x)^T \Phi(x) \le x^T F_{mat} x, \tag{26}$$

where F_{mat} is a positive semi-definite matrix.

From (25), it appears that this problem is afflicted by matched uncertainty since both $\Phi(x)$ and $h(x)$ are associated with the nominal input matrix B. Using [37], the closed-loop system (25) with event-triggered control input $u_{\text{mat1}}(t_k)$ can be written as

$$\dot{x}(t) = Ax + B\Phi(x) + (B + \Delta B)u_{\text{mat1}}(t_k), \tag{27}$$

$$u_{\text{mat1}}(t_k) = K_{\text{mat1}} x(t_k) = K_{\text{mat1}}(x(t) + e(t)), \tag{28}$$

where K_{mat1} is the controller gain and error variable $e(t)$ as defined in (11).

Example 1. *Euler–Lagrange (EL) systems [7], [33] can be represented as (25), given that its dynamics is governed by*

$$M(q)\ddot{q} + N(q, \dot{q}) = \tau, \qquad (29)$$

where $N(q, \dot{q}) = V(q, \dot{q}) + F(\dot{q}) + G(q)$. The vectors $q \in \mathbb{R}^n$ and $\tau \in \mathbb{R}^n$ denote the state variables and generalized forces, respectively. The inertia matrix, Coriolis vector, gravity vector and friction vector are also denoted by $M(q) \in \mathbb{R}^{n \times n}$, $V(q, \dot{q})$, $G(q)$ and $F(\dot{q}) \in \mathbb{R}^n$, respectively. As a result of uncertain load variations and unmodeled dissipative effects, the terms $M(q)$ and $N(q, \dot{q})$ in (29) carry some levels of uncertainty. With uncertainty accounted for and letting the state vector $x = [q, \dot{q}]^T$, the state-space representation reads as (25) with $A = \begin{bmatrix} 0 & I \\ 0 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & I \end{bmatrix}^T$. For EL systems, the two sources of nonlinearity at the input and system level are given by

$$h(x) = M(q)^{-1} M_0(q) - I \geq 0, \qquad (30)$$

$$\Phi(x) = M(q)^{-1}(N_0(q, \dot{q}) - N(q, \dot{q})). \qquad (31)$$

$\square$

To regulate the closed-loop behavior of (27), the following problem is formulated.

P$_2$− Problem Statement: Design a robust state feedback control law (28) to regulate the closed-loop behavior of the event-triggered system (27) such that it is input-to-state stable with respect to its measurement error $e(t)$ in the presence of matched uncertainty. The problem is solved using a method similar to the one adopted in Problem P$_1$. To this end, we state the following nominal dynamics for system (25) in the presence of uncertainty

$$\dot{x} = Ax + Bu_{\text{mat1}}, \qquad (32)$$

and the modified cost function for this matched uncertain system (25) is given by

$$J_{mat} = \int_0^\infty (x^T F_{\text{mat}} x + x^T Q x + u_{\text{mat1}}^T u_{\text{mat1}}) dt, \qquad (33)$$

with $Q \geq 0$. The matrix $F_{\text{mat}} \geq 0$ is the upper bound of the uncertainty defined in (26). Similarly, based on Lemma 1, the robust controller gain matrices can be obtained by solving the LQR problem. According to the optimal control theory [29], the optimal control signal for (32) minimizing the cost function (33) is

$$u_{\text{mat1}} = -\underbrace{B^T P_2}_{K_{\text{mat1}}} x, \qquad (34)$$

where P_2 satisfies the following Riccati equation

$$P_2 A + A^T P_2 - P_2 B B^T P_2 + F_{\text{mat}} + Q = 0. \qquad (35)$$

To establish the triggering law for (27), we propose the following Corollary.

Corollary 1. *Let $\sigma \in (0, 1)$ and the optimal controller gain K_{mat1} derived for the nominal system (32) with cost function (33). The event-triggered control law (28) ensures*

asymptotic stability of the uncertain system (27) if the control input actuation instant satisfies the following sequence

$$t_0 = 0, \ t_{k+1} = \inf\{t \in \mathbb{R} | t > t_k \wedge \mu_2 \|x\|^2 - \|e\|^2 \leq 0\}, \qquad (36)$$

where the variable μ_2 is defined as

$$\mu_2 = \frac{\sigma \lambda_{min}^2(Q)}{8(1 + \|h_{max}(x)\|^2) \|K_{mat1}^T K_{mat1}\|^2}. \qquad (37)$$

Proof: The proof of this Corollary is included in Appendix A.

The procedure to realize the control law designed for Problem 2 is presented in Algorithm 1 (see Appendix B).

In the following Lemma, we prove that the event-triggering law (36) ensures that the minimum inter-event time τ is always grater than zero thereby no Zeno effect can occur in the closed-loop system.

Lemma 3. *Consider the uncertain system (27). The minimum inter-event time τ for the event-triggered law (36) is*

$$\tau = \frac{2}{(\kappa_1 - \kappa_2)} \ln \left\| \frac{(1 + \mu_2)}{(1 + \frac{\kappa_2}{\kappa_1} \mu_2)} \right\|, \forall \, \kappa_1 > \kappa_2 \qquad (38)$$

where $\kappa_1 = (\|A\| + \|BK + B\bar{h}_{max}K\| + \|B\| \|F\|^{\frac{1}{2}})$ and $\kappa_2 = (\|BK + B\bar{h}_{max}K\|)$.

Proof. The proof follows very similar steps as the proof of Lemma 2 and hence is omitted. $\square$

A. FINITE-HORIZON ROBUST CONTROL WITH OPTIMAL EVENT-TRIGGERING

So far, the controller design and communication constraint problems have been addressed separately using an emulation-based approach. We first formulated an infinite-horizon optimal control problem and designed the state feedback controller gain. Then, to deal with communication constraints within the feedback loop, an event-triggering law has been derived using the ISS theory. Recently, A. Molin et al. [26] and J. Wu et al. [46] addressed the co-design problem for discrete-time linear event-triggered systems to derive the controller and an event-triggering law simultaneously. Inspired by the results proposed in [26], [46], in this section, we consider both communication cost and system uncertainty, and propose an optimal control framework jointly optimizing both costs—communication cost and the cost associated with system uncertainty.

To derive the results, a finite-horizon optimal control problem for linear systems is proposed. Such a finite-horizon control is considered as it constitutes a more realistic scenario in practical problems. In addition, the approach presented in Section II considered a zero-order hold (ZOH) at the actuator end, such that the last transmitted state and control input were held constant until new information was transmitted (see Figure 1). This forces the system to operate in an open-loop manner in between two consecutive events. To avoid this issue, this subsection proposes a ZOH-free robust control
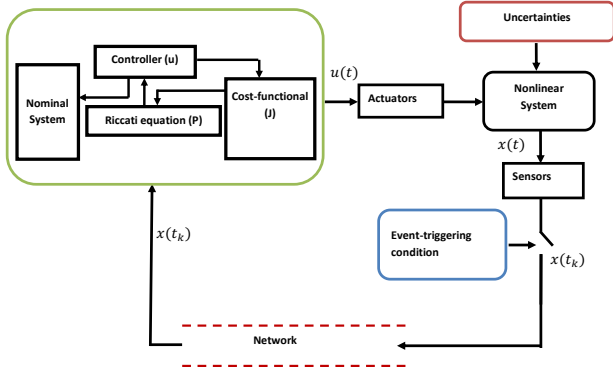
FIGURE 2: Architecture II: Schematic block diagram of the proposed finite-horizon robust control technique with event-triggered feedback where controller and actuators are collocated but sensors are spatially distributed.

technique with optimal event-triggered feedback. The block diagram of the proposed control technique is shown in Figure 2. The state of the uncertain system is measured by the sensors and each sensor has a copy of the nominal model. Originally, the concept of such sensors has been proposed by P. J. Antsaklis et al. [8], [27]. The presence of nominal model at the sensor end helps to compute the error between actual state $x(t)$ and nominal state $x_n(t)$:

$$\hat{e}(t) = x_n(t) - x(t). \qquad (39)$$

The variable $\hat{e}(t)$ measures the deviation of the actual closed-loop performance from the nominal behavior of the system. The event-triggering unit computes $\hat{e}(t)$ and solves an optimization problem considering the communication cost to obtain the optimal transmission sequence. Based on the obtained optimal transmission sequence, the actual state is transferred through the communication channel. A dynamic programming based technique is used to solve the associated optimization problem. In the previous event-triggered control approach stated in Section II, the triggering condition depends on the growth of the error $e(t)$. Here, the time instant t_k represents the event-triggering instants as mentioned in Section II. The measurement transmitted to the controller-end remains fixed until new information is received. Yet, here, the nominal model is available at the controller-end, and is used to estimate the nominal behavior of the system. At the event-triggering instant t_k, the state of the nominal model within the controller is replaced by the new measurement $x(t_k)$ available from the original uncertain system. The nominal system state is used to compute the control law $u_{\text{mat2}}(t) = K_{\text{mat2}}x_n(t)$, where K_{mat2} is the controller gain. Hence, between two consecutive event-triggering instants, the control input is generated by using the nominal model

$$\dot{x}_n(t) = (A + BK_{\text{mat2}})x_n(t), \ \forall \, t \in \, [t_k, \, t_{k+1}). \qquad (40)$$

Now, applying the control input u_{mat2} in (25), it reduces to

$$\dot{x}(t) = Ax + B\Phi(x) + B\{u_{\text{mat2}}(t) + h(x)u_{\text{mat2}}(t)\}, \quad (41)$$
$$u_{\text{mat2}}(t) = K_{\text{mat2}}x_n(t) = K_{\text{mat2}}(x(t) + \hat{e}(t)), \qquad (42)$$

where $\hat{e}(t)$ is defined in (39). In (40), at every event-triggering instant t_k, the nominal state $x_n(t)$ is replaced by the original state $x(t)$ and it resets the error $\hat{e}(t)$ to zero.

Remark 2. *Here, we have used two error variables: $e(t)$ and $\hat{e}(t)$. The variable $e(t)$ is used to compute the difference between the last transmitted state $x(t_k)$ and current state $x(t)$, that is $e(t) = x(t_k) - x(t)$ where $t \in [t_k, t_{k+1})$. On the other hand, $\hat{e}(t)$ measures the difference between the nominal state $x_n(t)$ and the state of uncertain system $x(t)$, that means $\hat{e}(t) = x_n(t) - x(t)$.*

In order to describe the network constraints, we consider a variable δ_t, which decides whether the state information is transmitted or not. The variable δ_t is defined as

$$\delta_t = \begin{cases} 1 & \text{when } x(t) \text{ is transmitted,} \\ 0 & \text{no information transmitted.} \end{cases} \qquad (43)$$

The switch of the binary decision variable δ_t from 0 to 1 depends on the selection of a particular event-triggering law. Let Ξ be a triggering law whose evolution depends on the error variable $\hat{e}(t)$. The design objective is to define the robust controller K_{mat2} and the event-triggering law Ξ that minimizes a certain cost-functional. With this aim, this paper considers the following cost-functional

$$J_{\text{mat2}} = \int_0^T (x^TQx + x^TF_{\text{mat}}x + u_{\text{mat2}}^Tu_{\text{mat2}} + \lambda\delta_t)dt, \quad (44)$$

where $\lambda > 0$ is a penalty due to any exchange of information between sensor, controller and actuator over the transmission network, and T denotes the final time of execution. To regulate the state of (41) by event-triggered feedback with the transmission cost $\int_0^T \lambda\delta_t dt$, the following problem is introduced.

P_3- **Problem statement:** Design a finite-horizon, linear, robust state feedback control law $u_{\text{mat2}}(t) = K_{\text{mat2}}x_n(t)$ and an optimal event-triggering law $\Xi^*(\hat{e}(t))$ for (41) that ensures the stability in the presence of uncertainties (8), (26).

Proposed solution: The solution to this problem is derived in two steps. First, a robust controller gain is designed for (41), and subsequently an optimal event-triggering law is introduced to reduce the number of data transmission over the network.

Robust control law: To design the robust controller gain for (41), we adopt the optimal control framework where a finite-horizon optimal control problem is solved for (40) while considering the cost function (44). The robust controller gain K_{mat2} can be obtained by solving a finite-horizon LQR problem for (40) with the cost-functional (44). Using

the optimal control theory [29], the control input is computed as

$$u_{\text{mat2}}(t) = \underbrace{-B^T P(t)}_{K_{\text{mat2}}(t)} x_n(t), \qquad (45)$$

where $P(t)$ is the solution of the following differential Riccati equation (DRE)

$$-\dot{P} = A^T P + PA - PBB^T P + Q + F_{\text{mat}}. \qquad (46)$$

For simplicity of notation, in what follows, we omit the argument t from $P(t)$. The steps to obtain the numerical solution of (46) are discussed in [30], [32].

Optimal event-triggering law: From the event-triggering law $\Xi(\hat{e}(t))$, it can be stated that the variable $\hat{e}(t)$ influences the number of transmissions over the network. In order to design the optimal event-triggering law, it is necessary to define the dynamics of $\hat{e}(t)$. Using (40) and (41), $\hat{e}(t)$ evolves based on the following dynamics

$$\dot{\hat{e}}(t) = A(x_n(t) - x(t)) - B\Phi(x) - Bh(x)u_{\text{mat2}}.$$

Neglecting the uncertain terms $f(x)$ and $h(x)$, the nominal error dynamics reads

$$\dot{\hat{e}}(t) = A\hat{e}(t), \ \forall \ t \in [t_k, \ t_{k+1}). \qquad (47)$$

At the event-triggering instant t_k, $\hat{e}(t)$ is zero as the nominal state $x_n(t)$ is replaced by actual state $x(t)$. To obtain the optimal event-triggering, the following optimization problem is solved:

$$\delta_t^* = \arg\min_{\delta_t} J(\hat{e}(t), \delta_t) = \int_0^T \{(1-\delta_t)\hat{e}^T K_{\text{mat2}}^T K_{\text{mat2}}\hat{e} + \lambda\delta_t\}dt,$$
$$(48)$$

subject to: (47) and $\hat{e}(t) \in \Omega$,

where

$$\Omega = \{\hat{e}(t) \in \mathbb{R}^n |\, \|\hat{e}(t)\|^2 \leq \xi\}. \qquad (49)$$

The state-dependent variable $\xi > 0$ is computed from the stability results. The optimization problem defined in (48) can be solved using dynamic programming with discrete approximations [29] which converges to the optimal solution [20], [47].

Remark 3. *The term $u_{mat2}^T(t)u_{mat2}(t)$ in (44), can be rewritten as $(K_{mat2}x + K_{mat2}\hat{e}(t))^T(K_{mat2}x + K_{mat2}\hat{e}(t))$ using (39). This helps to rewrite the cost-functional (44). To compute the optimal controller $u_{mat2}(t)$ for the nominal system, the terms δ_t and $\hat{e}(t)$ can be neglected from the minimization, since δ_t is constant and the controller gain design is independent of error $\hat{e}(t)$. However, the triggering condition design depends on the variable δ_t and $\hat{e}(t)$, which help to consider the cost-functional (48) to design the triggering law $\Xi^*(e\hat{}(t))$.*

To obtain the robust controller and optimal event-triggering law, the following Theorem is proposed.

Theorem 2. *The optimal state feedback gain K_{mat2} derived in (45) remains robust for the original uncertain system (41)*

if control inputs are actuated based on the optimal event triggering sequence δ_t^* obtained from (48).

Proof. Consider the Lyapunov function $V(x) = x^T P(t)x$. Then $\dot{V}$ is computed as

$$\dot{V}(x) = x^T(A^T P + PA + \dot{P})x - 2u_{\text{mat2}}^T u_{\text{mat2}} - u_{\text{mat2}}^T h^T u_{\text{mat2}}$$
$$- u_{\text{mat2}}^T hu_{\text{mat2}} - \Phi(x)^T u_{\text{mat2}} - u_{\text{mat2}}^T \Phi(x) + \hat{e}^T K_{\text{mat2}}^T B^T Px$$
$$+ x^T PBK_{\text{mat2}}\hat{e} + x^T PBhK_{\text{mat2}}\hat{e} + \hat{e}^T K_{\text{mat2}}^T h^T B^T Px.$$

Using (45) and (46), the above equality gives the following inequality:

$$\dot{V}(x) \leq -x^T Qx - (x^T F_{\text{mat}}x - \Phi(x)^T \Phi(x)) - (u_{\text{mat2}} + \Phi(x))^T$$
$$(u_{\text{mat2}} + \Phi(x)) + \hat{e}^T K_{\text{mat2}}^T K_{\text{mat2}}x + x^T K_{\text{mat2}}^T hK_{\text{mat2}}\hat{e}$$
$$+ x^T K_{\text{mat2}}^T K_{\text{mat2}}\hat{e} + \hat{e}^T K_{\text{mat2}}^T h^T K_{\text{mat2}}x. \qquad (50)$$

Using (8) and (26), the inequality (50) reduces to

$$\dot{V}(x) \leq -\frac{\lambda_{\min}(Q)}{2}\|x\|^2 + \frac{4}{\lambda_{\min}(Q)}\left(\left\|K_{\text{mat2}}^T K_{\text{mat2}}\right\|^2\right.$$
$$\left. + \left\|K_{\text{mat2}}^T K_{\text{mat2}}\right\|^2 \|h_{\max}(x)\|^2\right)\|\hat{e}\|^2. \qquad (51)$$

This ensures that the closed-loop system (41) is ISS with the event-triggering law Ξ^*. The threshold ξ in (49) can be computed from (51) as

$$\xi \leq \mu_3\|x\|^2. \qquad (52)$$

where $\mu_3 = \frac{\sigma\lambda_{\min}^2(Q)}{8(1+\|h_{\max}(x)\|^2)\left\|K_{\text{mat2}}^T K_{\text{mat2}}\right\|^2}$ and $\sigma \in (0, 1)$. $\square$

The steps to realize the robust control law for (41) with optimal event-triggering law $\Xi^*(\hat{e}(t))$ are detailed in Algorithm 3 presented in Appendix B.

Remark 4. *Computation of $\delta^*(t)$ is done by solving the optimization problem (48). The symbol T in (48) is used to represent the final time which is selected to be larger than the minimum time between two consecutive events. Furthermore, the variable ξ is not a constant and evolves based on (52).*

Remark 5. *A similar method as the one mentioned in Appendix A (proof of Lemma 2) can be applied to derive the lower bound of inter-event time for the controller stated in Theorem 2. For matched systems, the expression of the lower bound of inter-event time τ will be similar to the one stated in Lemma 3; but, the coefficients κ_1 κ_2 and scalar μ_2 will be different.*

V. SIMULATIONS

This section tests the theoretical results derived in previous sections for two classical nonlinear systems.

A. EXAMPLE 1

Let us consider a system (25) with state and system matrices given by $x = [x_1 \ x_2]^T$,

$$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \text{ and } B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

TABLE 1: Event-triggered control vs. continuous control

Control Strategy	$\tau_{\max}$ (sec)	$\tau_{\min}$ (sec)	u_{total}
Continuous control	0.008	0.008	500
Event-triggered control	0.27	0.008	316

The nonlinearities correspond to $h(x) = \frac{2w_1 x_1^2}{(x_1^2+1)}$ and $\Phi(x) = 2w_2 x_1 \sin^2(x_1) \cos(x_2)$, with w_1 and w_2 being uncertain scalar parameters whose uncertainty can vary in the interval $[0, 1]$. The upper bound of $h_{\max}(x)$ is considered as $\|h_{\max}\| = 2$. The controller gain is computed using (34), which minimizes (33). We consider the matrices $F_{\text{mat1}} = 4I$ and $Q = 10I$. To compute K_{mat1}, the Riccati equation (35) is solved. The positive definite solution P_2 of (35) is used to compute the optimal input

$$u_{\text{mat1}} = - \begin{bmatrix} 10 & 10.4 \end{bmatrix} x.$$

To realize the event-triggering sequence (36), the design parameter σ is selected to be 0.6. The numerical simulation runs for 4 time units with the initial condition $[0.1, -0.1]^T$.

For all simulations, we extracted 100 random samples of w_1 and w_2 within the interval $[0, 1]$ and tested the performance of the designed controller. Figure 3a shows the convergence of state trajectories for different values of w_1 and w_2. As it can be seen from Fig. 3a, all states converge to zero for various samples extracted from the set of uncertainty which confirms the robustness of the designed controller. Figure 3c shows the inter-event time of execution instants, and reveals that the number of computed control inputs is drastically reduced, thereby confirming the reduction in the ensuing communication cost. Figure 4 shows that Assumption 1 always holds during the entire run time. A comparative study with the conventional continuous control approach is shown in Table 1. It confirms that the total number of actuations u_{total} for the event-triggered case is far less than that of the continuous control technique. The symbols $\tau_{\max}$ and $\tau_{\min}$ denote the maximum and minimum inter-event time of event generation. We have calculated the lower bound of inter-event time $\tau_{\min}$ for Example 1 using (38). The calculated value of $\tau_{\min}$ is 0.016 sec which is very close to the numerical one.

To realize the optimal event-triggered control approach proposed in Section IV-A, we consider the same example discussed above. The control law is computed for a finite-horizon $T = 4$ seconds. The control law (42) is computed numerically using the solution of the DRE (46). To obtain the optimal event-triggering law Ξ^*, the dynamic programming based optimization problem is formulated which generates the optimal triggering instants δ_t^*. Sensors at the system end transmit state x based on δ_t^*. The convergence of states with the optimal triggering law Ξ^* is shown in Fig. 3b. The scalar λ is selected to be 0.4. Figure 3d shows the evolution of the switching variable δ_t^* for a given run-time. Table 2

TABLE 2: Comparison of event-triggered robust control with optimal triggering vs. continuous control

Control Strategy	$\tau_{\max}$(sec.)	$\tau_{\min}$(sec.)	u_{total}
Continuous control	0.04	0.04	100
Finite-horizon event-triggered control	1.8	0.04	36

TABLE 3: Comparison of event-triggered control vs. continuous control for for mismatched nonlinear systems.

Control Strategy	$\tau_{\max}$(sec.)	$\tau_{\min}$(sec.)	u_{total}
Continuous control	0.02	0.02	400
Event-triggered control	0.78	0.02	289

compares the total number of transmission between event-triggered control technique with optimal triggering and the conventional continuous approach. Again, we observe that the total number of transmissions is significantly reduced thereby confirming the efficacy of the proposed approach.

B. EXAMPLE 2:

Consider the state-space form of a one-link robot manipulator with revolute joints [35] as an example of a class of nonlinear system (9). It is expressed in the form of (4), with the matrices

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -48.9 & -1.25 & 48.6 & 0 \\ 0 & 0 & 0 & 1 \\ 19.5 & 0 & -19.5 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 21.6 \\ 0 \\ 0 \end{bmatrix}, D = I,$$

and uncertainty $\Phi(x) = \begin{bmatrix} 0 & 0 & 0 & \gamma \sin x_3 \end{bmatrix}^T$, such that the the property (7) holds. For simulation purposes, the scalar γ is selected as 0.33. This numerical simulations run on Matlab for 30 seconds with the following state vector $\begin{bmatrix} 0.1 & 0.01 & 0.2 & 0.3 \end{bmatrix}^T$ as initial condition. The controller gain matrices K_{mis} and L are calculated as

$$K_{\text{mis}} = \begin{bmatrix} -14.81 & -3.96 & 9.89 & -2.03 \end{bmatrix},$$

$$L = \begin{bmatrix} -0.67 & 0 & 0.47 & -0.08 \\ 0 & 0 & 0 & 0 \\ 0.47 & 0 & -0.58 & 0.04 \\ -0.08 & 0 & 0.04 & -0.04 \end{bmatrix},$$

respectively. The design matrix $F_{\text{mis}} = I$ is selected. The design parameters $\eta = 2.2$, $\beta = 2$ and $\rho = 0.1$ are selected such that the condition (18) is met. To realize the event-triggering law (17), the scalar $\mu = 0.018$ is computed based on (19).

Figure 5a shows the convergence of state trajectories with the event-triggered actuation. The aperiodic variation of control inputs are shown in Fig. 5b. A zoomed-in view of Fig. 5b is also shown in the same figure to visualize the aperiodic variation of inputs more clearly. The condition
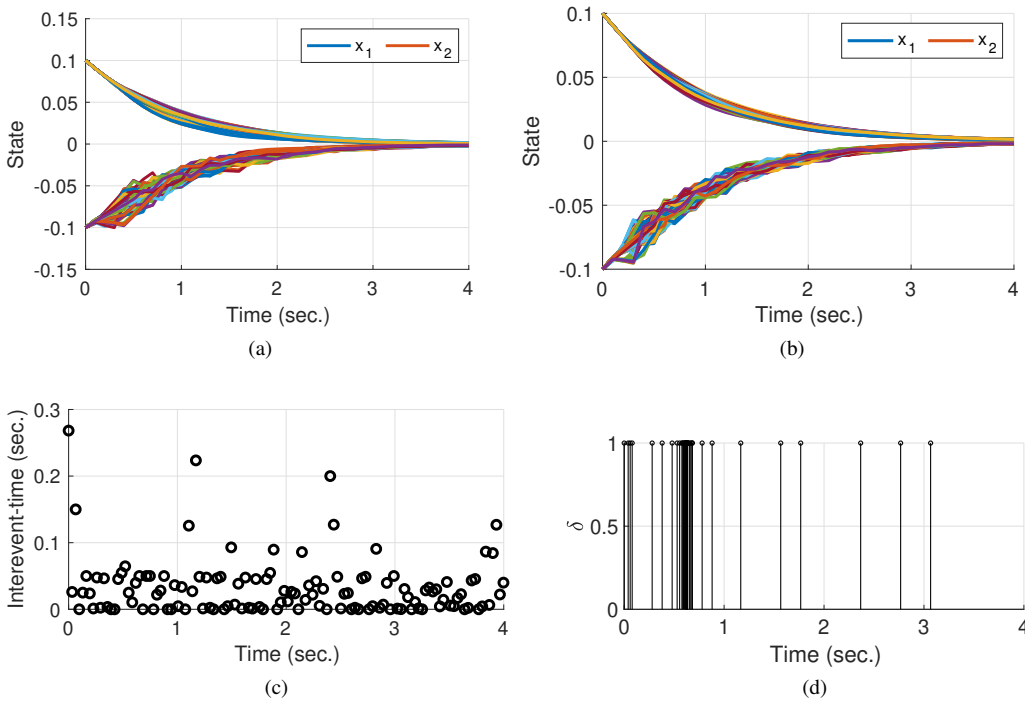
FIGURE 3: (a): Stabilization of states for 100 random uncertain samples of w_1 and w_2. (b): Convergence of states with uncertainty for 100 random uncertain samples of w_1 and w_2 using optimal event-triggered control for $T = 4$ sec. (c): Inter-event time for $w_1 = 0.5$, $w_2 = 0.5$. (d): Evolution of δ with time for $w_1 = 0.3$ and $w_2 = 0.3$.
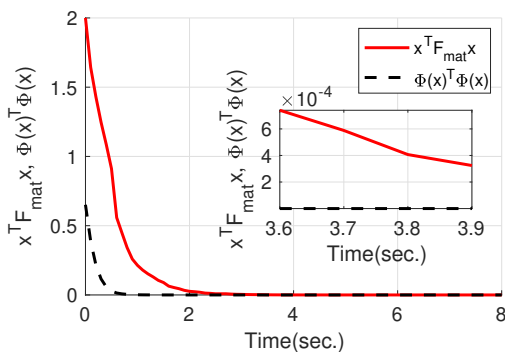

FIGURE 4: Numerical verification of condition (26) in Example 1.

(7) is also verified and shown in Figure 6. This proves that Assumption 1 holds for Example 2. A comparative study between continuous and event-triggered control techniques is shown in Table 3. It shows the efficacy of the proposed event-triggering technique over the continuous one in terms of total number of actuations for a given run time.

VI. CONCLUSIONS

In this paper, we consider a class of nonlinear systems afflicted with matched and mismatched uncertainty. To design adequate and effective event-triggered control laws, we consider both the nonlinearity and parameter variations as a sources of uncertainty. The controller—whose design is based on the linear part of the system—remains robust in the presence of these sources of uncertainty. We propose a linear robust control law derived within the optimal control framework with an infinite horizon cost. Furthermore, the corresponding event-triggering law is also derived while regulating aperiodic feedback information with the goal of saving the network bandwidth. Specifically, for matched uncertain systems, we solve a finite-horizon robust control problem with optimal event-triggering which constitutes a more realistic scenario in practical problems. To this end, we assume that each sensor has a copy of the nominal dynamics and can form an error signal corresponding to the difference between actual and nominal states. To compute the optimal event-triggering law, an optimization problem is solved using dynamic programming. The effectiveness of the designed control laws is illustrated through numerical simulations of two distinct problems.

There are numerous challenges for future research based on the work reported in this article. In particular, considering network-induced uncertainties such as time delays, data packet dropouts, and noise in the transmission channel would be an interesting extension to the current contribution. Furthermore, an output-feedback control law—instead of state-feedback—results in a controller more suitable for practical applications.
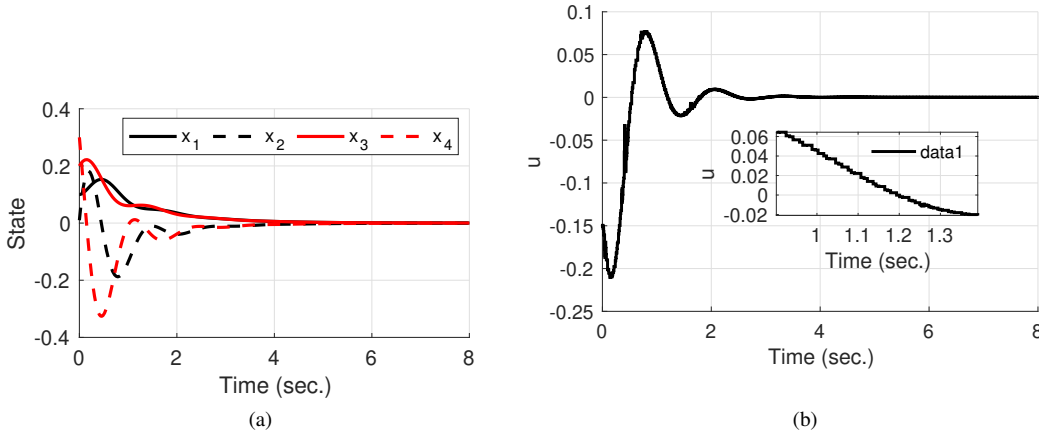
FIGURE 5: (a): Convergence of states using event-triggered control for mismatched nonlinear systems. (b): Convergence of event-triggered control input $u(t_k)$ with mismatched uncertainties.
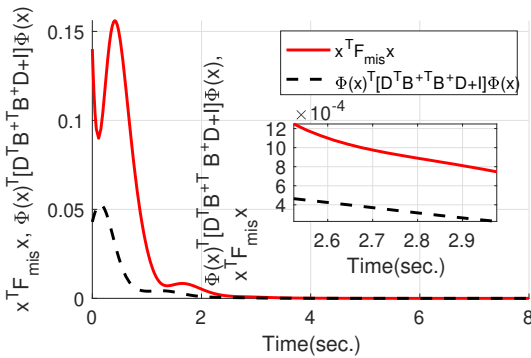

FIGURE 6: Numerical validation of condition (7) in Example 2.

A. PROOFS

1) Proof of Corollary 1

To prove the ISS-stability of uncertain system (27) with control input (28), it is necessary to reformulate $\dot{V}(x)$ such that it satisfies (3). Consider the Lyapunov function for (27) in the form of a positive smooth function $V(x) = x^T P_2 x$. To ensure the stability of (27), $\dot{V}(x)$ is recast as

$$\dot{V}(x) = \left(\frac{\partial V}{\partial x}\right)^T \left(Ax + B\{(K_{\text{mat1}}x + K_{\text{mat1}}e) + h(x)(K_{\text{mat1}}x + K_{\text{mat1}}e)\} + B\Phi(x)\right). \quad (53)$$

The function $V(x)$ is a Lyapunov function for (32) that satisfies the Hamilton–Jacobi–Bellman (HJB) equation

$$\min_{u_1} \left(x^T F_{mat1}x + x^T Qx + u_{\text{mat1}}^T u_{\text{mat1}} + V_x^T(Ax + Bu_{\text{mat1}})\right) = 0, \quad (54)$$

where matrix V_x denotes $\frac{\partial V}{\partial x}$. For a selection of Lyapunov function $V(x) = x^T P_2 x$, the HJB equation (54) reduces to a Riccati equation (35). The optimal input u_{mat1} must satisfy (54); that means

$$x^T F_{\text{mat}}x + x^T Qx + u_{\text{mat1}}^T u_{\text{mat1}} + V_x^T(Ax + Bu_{\text{mat1}}) = 0, \quad (55)$$

$$2u_{\text{mat1}}^T = -V_x^T B. \quad (56)$$

Using (55) and (56), Eq. (53) is simplified as

$$\dot{V}(x) \leq - x^T F_{\text{mat}}x + \Phi(x)^T\Phi(x) - x^T Qx - 2u_{\text{mat1}}^T K_{\text{mat1}}e - 2u_{\text{mat1}}^T hu_{\text{mat1}} - 2u_{\text{mat1}}^T hK_{\text{mat1}}e - (u_{\text{mat1}} + \Phi(x))^T(u_{\text{mat1}} + \Phi(x)) \quad (57)$$

Now applying (26) in (57) and after further simplification following is achieved

$$\dot{V}(x) \leq -\frac{\lambda_{\min}(Q)}{2}\|x\|^2 + \frac{4\|K_{\text{mat1}}^T K_{\text{mat1}}\|^2}{\lambda_{\min}(Q)}\left(1 + \|h_{\max}(x)\|^2\right)\|e\|^2 \quad (58)$$

The inequality (58) ensures the ISS of (27) with respect to measurement error e. From (3) and (58), it is observed that the actuation of control input is solely required upon violation of the event-triggering criterion (36).

Proof of Lemma 2. From (9), $\|\dot{x}\|$ can be written as

$$\|\dot{x}\| \leq \|(A + BK_{\text{mis}} + Bh_{\max}(x)K_{\text{mis}})\|\|x\| + \|D\Phi(x)\| + \|BK_{\text{mis}} + Bh_{\max}(x)K_{\text{mis}}\|\|e\|. \quad (59)$$

Using (7), the upper-bound of $\|\Phi(x)\|$ is derived as

$$\|\Phi(x)\| \leq \frac{\|F_{\text{mis}}\|^{\frac{1}{2}}}{\|D^T B^{+T}B^+D + I\|^{\frac{1}{2}}}\|x\| \quad (60)$$

Now applying (8) and (60), (59) can be simplified as

$$\|\dot{x}\| \leq \left(\|A\| + \|BK_{\mathrm{mis}} + Bh_{\max}(x)K_{\mathrm{mis}}\| + \|D\| \frac{\|F_{\mathrm{mis}}\|^{\frac{1}{2}}}{\|D^T B^{+T} B^+ D + I\|^{\frac{1}{2}}} \right)\|x\| + \|BK_{\mathrm{mis}} + Bh_{\max}(x)K_{\mathrm{mis}}\|\|e\|.$$

(61)

From [37] and [38], the computation of inter-event time depends on the evolution of $\frac{\|e\|}{\|x\|}$. Now considering [37] and using the relation (11), $\frac{d}{dt}\left(\frac{\|e\|}{\|x\|}\right)$ can be computed as

$$\frac{d}{dt}\left(\frac{\|e\|}{\|x\|}\right) = \left(1 + \frac{\|e\|}{\|x\|}\right)\frac{\|\dot{x}\|}{\|x\|}.$$

(62)

Applying (61) in (62) and denoting $z = \frac{\|e\|}{\|x\|}$, (62) reduces to

$$\frac{dz}{dt} \leq \left(\|A\| + \|BK_{\mathrm{mis}} + Bh_{\max}(x)K_{\mathrm{mis}}\| + \|D\| \frac{\|F_{\mathrm{mis}}\|^{\frac{1}{2}}}{\|D^T B^{+T} B^+ D + I\|^{\frac{1}{2}}} \right) + \left(\|A\| + 2(\|BK_{\mathrm{mis}} + Bh_{\max}(x)K_{\mathrm{mis}}\|) + \|D\| \frac{\|F_{\mathrm{mis}}\|^{\frac{1}{2}}}{\|D^T B^{+T} B^+ D + I\|^{\frac{1}{2}}} \right)z + (\|BK_{\mathrm{mis}} + Bh_{\max}(x)K\|)z^2.$$

(63)

Using, comparison Lemma from [18], the inequality (63) reduces to following equality

$$\frac{dz}{dt} = (\|BK_{\mathrm{mis}} + Bh_{\max}(x)K\|)z^2 + \left(\|A\| + 2(\|BK_{\mathrm{mis}} + Bh_{\max}(x)K_{\mathrm{mis}}\|) + \|D\| \frac{\|F_{\mathrm{mis}}\|^{\frac{1}{2}}}{\|D^T B^{+T} B^+ D + I\|^{\frac{1}{2}}} \right)z + \left(\|A\| + \|BK_{\mathrm{mis}} + Bh_{\max}(x)K_{\mathrm{mis}}\| + \|D\| \frac{\|F_{\mathrm{mis}}\|^{\frac{1}{2}}}{\|D^T B^{+T} B^+ D + I\|^{\frac{1}{2}}} \right).$$

(64)

From the definition of inter-event time, it should be always bounded by a positive unit of time that means in between two consecutive events (say t_k to t_{k+1}), the ratio of $\frac{\|e\|}{\|x\|}$ evolves from 0 to $\mu_1 \in \mathbb{R}^+$. This evolution will take a finite amount of time unit. Now to show $\tau > 0$, (64) is solved with a initial condition $z(0, z_0) = z_0$ and the solution $z(t, z_0)$ must holds the inequality $\frac{\|e\|}{\|x\|} \leq z(t, z_0)$. To derive τ, (64) is written as

$$\int_0^{\mu_1} \frac{dz}{az^2 + bz + c} = \int_{t_k}^t dt$$

(65)

where $a = (\|BK_{\mathrm{mis}} + Bh_{\max}(x)K\|)$, $b = \left(\|A\| + 2(\|BK_{\mathrm{mis}} + Bh_{\max}(x)K_{\mathrm{mis}}\|) + \|D\| \frac{\|F_{\mathrm{mis}}\|^{\frac{1}{2}}}{\|D^T B^{+T} B^+ D + I\|^{\frac{1}{2}}} \right)$ and

$$c = \left(\|A\| + \|BK_{\mathrm{mis}} + Bh_{\max}(x)K_{\mathrm{mis}}\| + \|D\| \frac{\|F_{\mathrm{mis}}\|^{\frac{1}{2}}}{\|D^T B^{+T} B^+ D + I\|^{\frac{1}{2}}} \right).$$

Since, a, b and c are function of $h_{\max}(x)$, the integration (65) is not trivial to compute. We also observe that the maximum value of the known function $h_{\max}(x)$, denoted as $\bar{h}_{\max}$, leads to the minimum value of the inter-event time τ. Hence, after certain simplification, and considering above mentioned point, the expression of inter-event time τ can be derived as

$$\tau = \frac{2}{(\kappa_1 - \kappa_2)} \ln \left\| \frac{(1 + \mu_1)}{(1 + \frac{\kappa_2}{\kappa_1}\mu_1)} \right\|, \forall\, \kappa_1 > \kappa_2,$$

(66)

where $\kappa_1 = \left(\|A\| + \|BK_{\mathrm{mis}} + B\bar{h}_{\max}K_{\mathrm{mis}}\| + \|D\| \frac{\|F_{\mathrm{mis}}\|^{\frac{1}{2}}}{\|D^T B^{+T} B^+ D + I\|^{\frac{1}{2}}} \right)$ and $\kappa_2 = \|BK_{\mathrm{mis}} + B\bar{h}_{\max}K_{\mathrm{mis}}\|$. From (66), it is observed that $\kappa_1 > \kappa_2$ and this proves that $\tau > 0$.

□

B. ALGORITHMS

Algorithm 1 Event-Triggered Robust Control for Problem $\mathbf{P}_1$ and $\mathbf{P}_2$

1: Initialization: $x \leftarrow x(0)$, $x(t_k) \leftarrow x(0)$.
2: Using A, B, F_{mis} [or F_{mat}], σ, β, η compute K_{mis} and L[or K_{mat}] from (14) and (15), [or (34)].
3: Compute $\|x(t)\|$, $\|e(t)\|$ and μ_1 using (19) [or μ_2 using (37)].
4: **if** $\|e\|^2 \geq \mu_1\|x\|^2$ [or $\|e\|^3 \geq \mu_2\|x\|^2$] **then**
5:    Send $x(t_k)$ from sensor to controller.
6:    Compute and update the control laws (10) —for the system (9)—and (28)—for the system (27).
7: **else**
8:    Hold the previous input
9: **end if**
10: Return to line 3

Algorithm 2 Event-Triggered Robust Control with Optimal Triggering

---

1: Initialization: $x \leftarrow x(0)$, $t \leftarrow 0$.
2: Given: A, B, F_{mat}, T,
3: Compute K_{mat2} using (45), (46).
4: Compute $\|\hat{e}(t)\|$, ξ using (47) and (52) and solve optimization problem (48) to obtain δ_t^*.
5: **if** $\delta_t^* = 1$ **then**
6:    Send $x(t)$ from sensor to controller.
7:    Replace $x_n(t)$ with $x(t)$ in (40).
8:    Compute and update the control laws (45) using (40)—for the system (41).
9: **else**
10:    Compute and update the control laws (45) using (40) —for the system (41).
11: **end if**
12: Return to line 3

Algorithm 3 Dynamic Programming

1: Select the counter $k = 1$, initial value $\hat{e}(0)$, time step Δt, and integer N such that $T = N \times \Delta t$ with T being the final execution time.

2: Discretize the continuous-time system (47) and cost-functional (48).

3: **while** $k \neq N$ **do**

4:      Solve the finite-dimension (discretized) version of optimization problem defined by (48) and (49)—using principle of optimality [29]—to find optimal event-triggering δ^*.

5:      Set $k = k + 1$

6: **end while**

REFERENCES

[1] D. Adhyaru, I. N. Kar, and M. Gopal. Bounded robust control of nonlinear systems using neural network–based hjb solution. Neural Computing and Applications, 20(1):91–103, 2011.

[2] A. Al-Tamimi, F. Lewis, and M. Abu-Khalaf. Discrete-time nonlinear hjb solution using approximate dynamic programming: Convergence proof. IEEE Transactions on Systems, Man, and Cybernetics, Part B (Cybernetics), 38(4):943–949, 2008.

[3] A. Anta and P. Tabuada. To sample or not to sample: Self-triggered control for nonlinear systems. IEEE Transactions on automatic control, 55(9):2030–2042, 2010.

[4] K.J. Astrom and B.M. Bernhardsson. Comparison of riemann and lebesgue sampling for first order stochastic systems. In Proceedings of the 41st IEEE Conference on Decision and Control, 2002., volume 2, pages 2011–2016. IEEE, 2002.

[5] B Barmish and G Leitmann. On ultimate boundedness control of uncertain systems in the absence of matching assumptions. IEEE Transactions on Automatic Control, 27(1):153–158, 1982.

[6] Z. Chen and S. Jagannathan. Generalized hamilton–jacobi–bellman formulation-based neural network control of affine nonlinear discrete-time systems. IEEE Transactions on Neural Networks, 19(1):90–106, 2008.

[7] J. Craig. Introduction to robotics: mechanics and control, 3rd ed. Pearson Education India, 2009.

[8] E. Garcia and P. Antsaklis. Model-based event-triggered control for systems with quantization and time-varying network delays. IEEE Transactions on Automatic Control, 58(2):422–434, 2012.

[9] M. Ghodrat and H. J. Marquez. On the event-triggered controller design. IEEE Transactions on Automatic Control, pages doi: 10.1109/TAC.2019.2955660,, 2019.

[10] M. Ghodrat and H. J. Marquez. Robust stabilization of input-affine nonlinear systems under network constraints. IEEE Transactions on Control of Network Systems, pages 1–1, 2019.

[11] W. Heemels and M. Donkers. Model-based periodic event-triggered control for linear systems. Automatica, 49(3):698–711, 2013.

[12] W. Heemels, M. Donkers, and A. Teel. Periodic event-triggered control for linear systems. IEEE Transactions on Automatic Control, 58(4):847–861, 2012.

[13] W. Heemels, K.H. Johansson, and P. Tabuada. An introduction to event-triggered and self-triggered control. In Proceedings of the 51st IEEE Conference on Decision and Control (CDC), pages 3270–3285. IEEE, 2012.

[14] R. Horn and C. Johnson. Matrix analysis. Cambridge university press, 2012.

[15] O. C. Imer and T. Basar. Optimal control with limited controls. In 2006 American Control Conference, pages pp.–298–303, 2006.

[16] K.H. Johansson, M. Egerstedt, J. Lygeros, and S. Sastry. On the regularization of zeno hybrid automata. Systems & control letters, 38(3):141–150, 1999.

[17] I. N. Kar. Quadratic stabilization of a collection of linear systems. International Journal of Systems Science, 33(2):153–160, 2002.

[18] H. Khalil. Nonlinear systems, volume 3rd ed. Prentice hall Upper Saddle River, NJ, 2002.

[19] P. P Khargonekar, I. Petersen, and K. Zhou. Robust stabilization of uncertain linear systems: quadratic stabilizability and h_∞ control theory. IEEE Transactions on Automatic control, 35(3):356–361, 1990.

[20] H. Kushner and P. Dupuis. Numerical methods for stochastic control problems in continuous time, volume 24. Springer Science & Business Media, 2013.

[21] L. Zhang and D. Hristu-Varsakelis. Lqg control under limited communication. In Proceedings of the 44th IEEE Conference on Decision and Control, pages 185–190, 2005.

[22] F. Lin. An optimal control approach to robust control design. International journal of control, 73(3):177–186, 2000.

[23] F. Lin and R. Brandt. An optimal control approach to robust control of robot manipulators. IEEE Transactions on robotics and automation, 14(1):69–77, 1998.

[24] T. Liu and Z. Jiang. A small-gain approach to robust event-triggered control of nonlinear systems. IEEE Transactions on Automatic Control, 60(8):2072–2085, 2015.

[25] W. Liu and J. Huang. Event-triggered global robust output regulation for a class of nonlinear systems. IEEE Transactions on Automatic Control, 62(11):5923–5930, 2017.

[26] A. Molin. Optimal event-triggered control with communication constraints. PhD thesis, Technische Universität München, 2014.

[27] L. A. Montestruque and P. Antsaklis. On the model-based control of networked systems. Automatica, 9(10):1837 – 1843, 2003.

[28] L. A. Montestruque and P. Antsaklis. Stability of model-based networked control systems with time-varying transmission times. IEEE Transactions on Automatic Control, 49(9):1562–1572, 2004.

[29] D. Naidu. Optimal control systems. CRC press, 2002.

[30] J. Nazarzadeh, M. Razzaghi, and K.Y. Nikravesh. Solution of the matrix riccati equation for the linear quadratic control problems. Mathematical and computer modelling, 27(7):51–55, 1998.

[31] D Nešić and A. Teel. Input-to-state stability of networked control systems. Automatica, 40(12):2121–2128, 2004.

[32] T. Nguyen and Z. Gajic. Solving the matrix differential riccati equation: a lyapunov equation approach. IEEE Transactions on Automatic Control, 55(1):191–194, 2009.

[33] R. Ortega, J. Perez, P. Nicklasson, and H. Sira-Ramirez. Passivity-based control of Euler-Lagrange systems: mechanical, electrical and electromechanical applications. Springer Science & Business Media, 2013.

[34] I. Petersen. Structural stabilization of uncertain systems: Necessity of the matching condition. SIAM journal on control and optimization, 23(2):286–296, 1985.

[35] G. Phanomchoeng and R.h Rajamani. Observer design for lipschitz nonlinear systems using riccati equations. In Proceedings of the 2010 American Control Conference, pages 6060–6065. IEEE, 2010.

[36] E. Sontag. Input to state stability: Basic concepts and results. In Nonlinear and optimal control theory, pages 163–220. Springer, 2008.

[37] P. Tabuada. Event-triggered real-time scheduling of stabilizing control tasks. IEEE Transactions on Automatic Control, 52(9):1680–1685, 2007.

[38] P. Tabuada and X. Wang. Preliminary results on state-trigered scheduling of stabilizing control tasks. In Proceedings of the 45th IEEE Conference on Decision and Control, pages 282–287, 2006.

[39] P. Tallapragada and N. Chopra. Event-triggered dynamic output feedback control of lti systems over sensor-controller-actuator networks. In 52nd IEEE Conference on Decision and Control, pages 4625–4630, 2013.

[40] P. Tallapragada and N. Chopra. On event triggered tracking for nonlinear systems. IEEE Transactions on Automatic Control, 58(9):2343–2348, 2013.

[41] S. Trimpe and R. D'Andrea. Event-based state estimation with variance-based triggering. IEEE Transactions on Automatic Control, 59(12):3266–3281, 2014.

[42] N. S. Tripathy, I. N. Kar, and K. Paul. Model based robust control law for linear event-triggered system. Asian Journal of Control, 18(5):1765–1780, 2016.

[43] N. S. Tripathy, I. N. Kar, and K. Paul. Stabilization of uncertain discrete-time linear system with limited communication. IEEE Transactions on Automatic Control, 62(9):4727–4733, 2016.

[44] D. Wang, D. Liu, and H. Li. Policy iteration algorithm for online design of robust control for a class of continuous-time nonlinear systems. IEEE Transactions on Automation Science and Engineering, 11(2):627–632, 2014.

[45] X. Wang and M. Lemmon. Self-triggered feedback control systems with finite-gain l_2 stability. IEEE transactions on automatic control, 54(3):452–467, 2009.

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/ACCESS.2020.3018733, IEEE Access

IEEE Access

Tripathy *et al.*: Robust Stabilization of a Class of Nonlinear Systems via Aperiodic Sensing and Actuation

[46] J. Wu, Q. Jia, K.H. Johansson, and L. Shi. Event-based sensor data scheduling: Trade-off between communication rate and estimation quality. IEEE Transactions on automatic control, 58(4):1041–1046, 2012.

[47] Y. Xu and J.P Hespanha. Optimal communication logics in networked control systems. In 2004 43rd IEEE Conference on Decision and Control (CDC)(IEEE Cat. No. 04CH37601), volume 4, pages 3527–3532. IEEE, 2004.

[48] X. Yang and H. He. Self-learning robust optimal control for continuous-time nonlinear systems with mismatched disturbances. Neural Networks, 99:19–30, 2018.

[49] X. Yang and H. He. Event-triggered robust stabilization of nonlinear input-constrained systems using single network adaptive critic designs. IEEE Transactions on Systems, Man, and Cybernetics: Systems, DOI: 10.1109/TSMC.2018.2853089, 2018.

[50] H. Yu and F. Hao. Input-to-state stability of integral-based event-triggered control for linear plants. Automatica, 85:248 – 255, 2017.

MOHAMMADREZA CHAMANBAZ received his BSc in Electrical Engineering from Shiraz University of Technology. In 2014 he received his PhD from the Department of Electrical & Computer Engineering, National University of Singapore in control science. Dr. Chamanbaz was with Data Storage Institute, Singapore as research scholar from 2010 to 2014. From 2014 to 2017, he was postdoctoral research fellow in Singapore University of Technology and Design. He was Assistant Professor in Arak University of Technology, from Jan 2017 to Jan 2019. Dr. Chamanbaz is now senior research fellow in iTrust Center for Research in Cyber Security, Singapore.

His research activities are mainly focused on probabilistic and randomized algorithms for analysis and control of uncertain systems, robust and distributed optimization, and secure control of cyber-physical systems.

ROLAND BOUFFANAIS received his BSc and MSc in Physics from École Normale Supérieure (ENS Lyon) in 1997, and 1999 respectively. He also received a MSc from UPMC Paris Sorbonne University in 1999. In 2007, he received his PhD in Engineering from École Polytechnique Fédérale de Lausanne (EPFL). Dr. Bouffanais was a post-doctoral fellow and associate at the Massachusetts Institute of Technology (MIT) in the department of mechanical engineering. He joined the faculty of the Singapore University of Technology and Design (SUTD) in 2011, and he is now an Associate Professor of Engineering and the Principal Investigator of the Applied Complexity Group. Dr. Bouffanais is an IEEE Member.

His interdisciplinary research spans a vast breadth of areas, and is focused on the design and control of decentralized complex systems, multi-agent systems, leader-follower consensus dynamics, and nonlinear dynamical systems.

NILADRI SEKHAR TRIPATHY completed his Bachelor Degree in Electronics and Communication Engineering and Master Degree in Mechatronics Engineering in 2009 and 2011, respectively. After completion of master degree he worked as a Senior Research Fellow (SRF) in IIT Delhi from 2011 to2012. In 2017, he received his Ph.D. (Electrical Engg.) degree from IIT Delhi. From 2018 to 2019, he worked as a post-doctoral researcher in Singapore University of Technology and Design (SUTD). Presently, he is working as an Assistant Professor at Indian institute of Technology Jodhpur, India.

His research interests include Control Techniques for Cyber-Physical System, Cybersecurity, Mechatonics and Application of Control Theory in Computing.

INDRA NARAYAN KAR received the B.E. degree in electrical engineering from Bengal Engineering College (currently, IIEST), Shibpur, India, in 1988, and the M.Tech. and Ph.D. degrees in electrical engineering from the Indian Institute of Technology Kanpur, Kanpur, India, in 1991 and 1997, respectively.

From 1996 to 1998, he was a Research Student with Nihon University, Tokyo, Japan, under the Japanese Government Monbusho scholarship program. He joined the Department of Electrical Engineering, Indian Institute of Technology Delhi, New Delhi, India, in 1998, where he is currently a Professor and the Institute Chair Professor. He has published over 150 papers in international journals and conferences. His current research interests include nonlinear control, time-delayed control, incremental stability analysis, cyber-physical system, application of control theory in power network and robotics.