Quantum correlations in terms of neutrino oscillation probabilities

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Abstract

Neutrino oscillations provide evidence for the mode entanglement of neutrino mass eigenstates in a given flavour eigenstate. Given this mode entanglement, it is pertinent to consider the relation between the oscillation probabilities and other quantum correlations. In this work, we show that all the well-known quantum correlations, such as the Bell's inequality, are directly related to the neutrino oscillation probabilities. The results of the neutrino oscillation experiments, which measure the neutrino survival probability to be less than unity, imply Bell's inequality violation.
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1. Introduction

The foundations of quantum mechanics are usually studied in optical or electronic systems. In such systems, the interplay between the various measures of quantum correlations is well known. Inspired by the recent technical advances in high energy physics experiments, in particular the meson factories and the long baseline neutrino experiments, attention has also been directed towards subatomic physics [1–10].

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The study of quantum correlations in neutrinos, has been mostly focussed on entanglement [3–6]. Recently, a temporal analogue of Bell's inequality, the Leggett–Garg inequality, has been studied in the context of neutrino oscillations [10]. Here, along with entanglement we also study the other quantum correlations such as Bell's inequality violation, teleportation fidelity and geometric discord in the context of two flavour neutrino oscillations as well as study the interplay between them. In particular, we show that all these quantities are directly related to the neutrino oscillation probabilities.

Neutrino oscillations are experimentally well established [11–17]. Such oscillations are possible if both of the following conditions are satisfied:

- The neutrino flavour state is a linear superposition of non-degenerate mass eigenstates.
- The time evolution of a flavour state is a coherent superposition of the time evolution of the corresponding mass eigenstates.

The coherent time evolution implies that there is *mode entanglement* between the mass eigenstates which make up a flavour state. Such mode entangled states have been the subject of intense discussions over the last two decades [18–23], resulting in the general consensus of subspace entanglement as a generalized feature of inter particle entanglement [23]. It has been the subject of many theoretical and experimental proposals [21] as well as successful experimental realizations [22] in atom-photon systems. Here we use the concept of mode entanglement to relate flavour oscillations to bipartite entanglement of single particle states.

The quest for understanding quantum correlations could be thought to have begun with the efforts of Einstein–Podolsky–Rosen (EPR) [24]. A quantitative understanding of EPR led to the development of Bell's inequality [25], with refinements leading to the Bell-CHSH (Clauser–Horn–Shimony–Holt) inequalities [26]. Violation of Bell's inequality quantifies the non-locality inherent in the system. A weaker, though very popular and widely studied facet of quantum correlations, is entanglement [27]. This has been applied to understand the process of teleportation [28]. A still weaker measure is quantum discord [29,30] and was developed as the difference between the quantum generalizations of two classically equivalent formulations of mutual information. States which are separable and hence have no entanglement could still have non-zero discord. Hence, our present understanding of quantum correlations is that it is a complex entity with many facets. There is now an abundance of measures of quantum correlations such as quantum work deficit [31], measurement induced disturbance [32] and dissonance [33].

In this paper we study a number of quantum correlations in the context of two-flavour neutrino oscillations. Among them are mode non-locality, concurrence, discord and teleportation fidelity. We find that all these quantum correlations are simple functions of the neutrino oscillation probabilities. A non-zero oscillation probability immediately leads to a violation of Bell's inequality and to a teleportation fidelity value of greater than 2/3.

We first provide an introduction to the quantum mechanics of two flavour neutrino oscillations. Here we see that mode entanglement comes in a natural setting. We then discuss and compute different quantum correlations and relate them to the neutrino oscillation probabilities. We finish with our conclusions.

2. Quantum mechanics of two flavour neutrino oscillations

It is well known that there are three flavour states of neutrinos, ν_e, ν_μ and ν_τ [34,35]. In the oscillation formalism, it is assumed that they mix via a 3×3 unitary matrix to form the three mass

eigenstates ν_1, ν_2 and ν_3. Neutrino oscillations occur only if the three corresponding masses, m_1, m_2 and m_3, are non-degenerate. Of the three mass-squared differences $\Delta_{kj} = m_k^2 - m_j^2$ (where $j, k = 1, 2, 3$ with $k > j$), only two are independent. Oscillation data tells us that $\Delta_{21} \approx 0.03 \times \Delta_{32}$, hence $\Delta_{31} \approx \Delta_{32}$. One of the three mixing angles parametrizing the mixing matrix, θ_{13}, is measured to be quite small (about 0.14 radians) [36–39].

In considering neutrino oscillations, in general, one should use the full three flavour oscillation formulae. A number of studies do this, fitting all the available neutrino oscillation data to the three flavour formulae [40–42]. In the following three important experimentally relevant cases, the three flavour formula reduces to an effective two flavour formula:

1. Long baseline reactor experiments: Reactors emit electron anti-neutrinos with energies of a few MeV. In long baseline reactor neutrino experiments, the baseline is expected to be greater than 50 km. For example, in KamLAND experiment [12,13] the baseline is approximately 180 km. These experiments measure the anti-neutrino survival probability $P(\bar{\nu}_e \to \bar{\nu}_e)$. In the limit of neglecting θ_{13}, this probability reduces to the effective two flavour formula $[1 - \sin^2 2\theta_{12} \sin^2(\Delta_{21}L/4E)]$.

2. Short baseline reactor experiments: These experiments have baselines of about a km [36–39]. Given this short baseline, they are not capable to observing the oscillations induced by the smaller mass-square difference Δ_{21}. Setting this quantity equal to zero in the expression for $P(\bar{\nu}_e \to \bar{\nu}_e)$, an effective two flavour formula $[1 - \sin^2 2\theta_{13} \sin^2(\Delta_{31}L/4E)]$ is obtained once again.

3. Long baseline accelerator experiments: Accelerator neutrino beams consist of muon neutrinos (or anti-neutrinos) with energies ranging from hundreds of MeV [17] to a few GeV [15]. They have baselines of hundreds of km. In the expression of the muon neutrino survival probability $P(\nu_\mu \to \nu_\mu)$ for these experiments, both the small parameters, Δ_{21} and θ_{13}, can be set to zero in the leading order. In this approximation, once again an effective two flavour formula, $[1 - \sin^2 2\theta_{23} \sin^2(\Delta_{32}L/4E)]$, is obtained. Then the problem reduces to that of two flavour mixing of ν_μ and ν_τ to form two mass eigenstates ν_2 and ν_3. The corresponding oscillations are described by one mixing angle θ_{23} and one mass-squared difference Δ_{32}.

In the case of two flavour mixing, the relation between the flavour and the mass eigenstates is described by a 2×2 rotation matrix, $U(\theta)$,

$$\begin{pmatrix} \nu_\alpha \\ \nu_\beta \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_j \\ \nu_k \end{pmatrix}, \tag{1}$$

where $\alpha, \beta = e, \mu, \tau$ and $j, k = 1, 2, 3$. Therefore, each flavour state is given by a superposition of mass eigenstates,

$$|\nu_\alpha\rangle = \sum_j U_{\alpha j} |\nu_j\rangle. \tag{2}$$

The time evolution of the mass eigenstates $|\nu_j\rangle$ is given by

$$|\nu_j(t)\rangle = e^{-iE_j t} |\nu_j\rangle, \tag{3}$$

where $|\nu_j\rangle$ are the mass states at time $t = 0$. Thus, we can write

$$|\nu_\alpha(t)\rangle = \sum_j e^{-iE_j t} U_{\alpha j} |\nu_j\rangle. \tag{4}$$

The evolving flavour neutrino state $|\nu_\alpha\rangle$ can also be projected on to the flavour basis in the form

$$|\nu_\alpha(t)\rangle = \tilde{U}_{\alpha\alpha}(t)\,|\nu_\alpha\rangle + \tilde{U}_{\alpha\beta}(t)\,\big|\nu_\beta\big\rangle , \tag{5}$$

where $|\nu_\alpha\rangle$ is the flavour state at time $t = 0$ and $|\tilde{U}_{\alpha\alpha}(t)|^2 + |\tilde{U}_{\alpha\beta}(t)|^2 = 1$. We introduce occupation number states as [3,4]

$$|\nu_\alpha\rangle \equiv |1\rangle_\alpha \otimes |0\rangle_\beta \equiv |10\rangle , \quad \big|\nu_\beta\big\rangle \equiv |0\rangle_\alpha \otimes |1\rangle_\beta \equiv |01\rangle . \tag{6}$$

Eq. (5) can therefore be rewritten as

$$|\nu_\alpha(t)\rangle = \tilde{U}_{\alpha\alpha}(t)\,|1\rangle_\alpha \otimes |0\rangle_\beta + \tilde{U}_{\alpha\beta}(t)\,|0\rangle_\alpha \otimes |1\rangle_\beta , \tag{7}$$

where,

$$\begin{aligned}
\tilde{U}_{\alpha\alpha}(t) &= \cos^2\theta e^{-iE_j t} + \sin^2\theta e^{-iE_k t} , \\
\tilde{U}_{\alpha\beta}(t) &= \sin\theta\cos\theta (e^{-iE_k t} - e^{-iE_j t}) .
\end{aligned} \tag{8}$$

Now the state in Eq. (7) has the form of a mode entangled single particle state [18–22]. The corresponding density matrix is given by

$$\rho_\alpha(t) = \begin{pmatrix}
0 & 0 & 0 & 0 \\
0 & |\tilde{U}_{\alpha\alpha}(t)|^2 & \tilde{U}_{\alpha\alpha}(t)\tilde{U}^*_{\alpha\beta}(t) & 0 \\
0 & \tilde{U}_{\alpha\beta}(t)\tilde{U}^*_{\alpha\alpha}(t) & |\tilde{U}_{\alpha\beta}(t)|^2 & 0 \\
0 & 0 & 0 & 0
\end{pmatrix} , \tag{9}$$

where

$$|\tilde{U}_{\alpha\alpha}(t)|^2 = c^4 + s^4 + 2s^2c^2\cos\left(\frac{\Delta t}{2E}\right) = P_{\text{sur}} , \tag{10}$$

$$|\tilde{U}_{\alpha\beta}(t)|^2 = 4s^2c^2\sin^2\left(\frac{\Delta t}{4E}\right) = P_{\text{osc}} , \tag{11}$$

$$\tilde{U}_{\alpha\alpha}(t)\tilde{U}^*_{\alpha\beta}(t) = s\,c\left(s^2 - c^2 + c^2 e^{i\frac{\Delta t}{2E}} - s^2 e^{-i\frac{\Delta t}{2E}}\right) , \tag{12}$$

$$\tilde{U}_{\alpha\beta}(t)\tilde{U}^*_{\alpha\alpha}(t) = s\,c\left(s^2 - c^2 + c^2 e^{-i\frac{\Delta t}{2E}} - s^2 e^{i\frac{\Delta t}{2E}}\right) , \tag{13}$$

with $c \equiv \cos\theta$ and $s \equiv \sin\theta$. In the above equations, θ is a generic two flavour mixing angle and Δ is the corresponding mass-square difference. Since the neutrino masses are very small (less than 1 eV), the neutrinos are assumed to be ultra relativistic. Hence the time of travel t is equal to the distance of travel L and the difference in energies of the mass eigenstates ($E_k - E_j$) can be set equal to $\Delta/2E$. The quantities in eqs. (10) and (11), $|\tilde{U}_{\alpha\alpha}(t)|^2$ and $|\tilde{U}_{\alpha\beta}(t)|^2$, are the two flavour survival and oscillation probabilities, respectively. Note that $P_{\text{sur}} < 1$, immediately implies $P_{\text{osc}} > 0$.

3. Quantum correlations in two flavour neutrino oscillations

In this section, we discuss and compute various quantum correlations inherent in the state given in Eq. (7). In all our subsequent calculations, the states considered are represented by 4×4 density matrices.

Bell's inequality is used to study the non-locality of a given system. Its physical content is that a system that can be described by a local realistic theory will satisfy this inequality. Quantum

mechanics provides many examples where this inequality gets violated [43]. However, here we do not propose to derive a Bell's inequality from local realism. Instead we make use of a very interesting result obtained in [44] which facilitates quantitative statements about Bell inequality violations just by making use of the parameters of the density operator describing the system.

The density matrix ρ, in general, can be expanded in the form

$$\rho = \frac{1}{4}[I \otimes I + (r.\sigma) \otimes I + I \otimes (s.\sigma) + \sum_{n,m=1}^{3} T_{mn}(\sigma_m \otimes \sigma_n)]. \tag{14}$$

The elements of the correlation matrix T are given by $T_{mn} = Tr[\rho(\sigma_m \otimes \sigma_n)]$. Let u_i ($i = 1, 2, 3$) be the eigenvalues of the matrix $T^{\dagger}T$. Then the Bell-CHSH inequality can be written as $M(\rho) \leq 1$, where $M(\rho) = \max(u_i + u_j)$ ($i \neq j$) [44]. For the state (7), $M(\rho)$ is given by

$$M(\rho) = 1 + \left[3 + \cos 4\theta + 2\cos\left(\frac{\Delta t}{2E}\right)\sin^2 2\theta\right]\sin^2 2\theta \sin^2\left(\frac{\Delta t}{4E}\right),$$
$$= 1 + 4P_{\text{sur}}P_{\text{osc}}. \tag{15}$$

Thus we see that $M(\rho)$ is directly related to the neutrino oscillation probabilities and a measurement of $P_{\text{sur}} < 1$ leads to a violation of Bell-CHSH inequality. We also note that the maximal violation occurs when $P_{\text{sur}} = 1/2 = P_{\text{osc}}$.

Non-locality is the strongest aspect of quantum correlations. A weaker, though popular and extensively studied feature, is entanglement. For the case of entangled two-level systems it is synonymous with concurrence. For a state with density matrix ρ, the concurrence is [27]

$$C = \max(\lambda_1 - \lambda_2 - \lambda_3 - \lambda_4, 0), \tag{16}$$

where λ_i are the square roots of the eigenvalues of $\rho\tilde{\rho}$ in decreasing order, where $\tilde{\rho} = (\sigma_y \otimes \sigma_y)\rho^*(\sigma_y \otimes \sigma_y)$ and is obtained by applying the spin flip operation on ρ. Here, concurrence can be shown to be

$$C = 2\sqrt{\sin^4\theta + \cos^4\theta + 2\cos^2\theta\sin^2\theta\cos\left(\frac{\Delta t}{2E}\right)}\sin 2\theta\sin\left(\frac{\Delta t}{4E}\right),$$
$$= 2\sqrt{P_{\text{sur}}P_{\text{osc}}} \tag{17}$$

Obviously, entanglement is non-zero if the oscillation probability is non-zero.

A still weaker measure of quantum correlations is quantum discord which points out that classicality and separability are not synonymous. To obtain an analytical formula for quantum discord is a very difficult task as it involves an optimization over local measurements, requiring numerical methods. To overcome this difficulty, another measure of quantum correlation called geometric discord was introduced in [45] which quantifies the amount of non-classical correlation, of an arbitrary quantum composite system, in terms of its minimal distance from the set of classical states. For ρ, geometric discord can be shown to be

$$D_G(\rho) = \frac{1}{3}[\|\vec{y}\|^2 + \|T\|^2 - \lambda_{max}], \tag{18}$$

where T is the correlation matrix defined above, $\vec{y}$ is the vector whose components are $y_m = \text{Tr}(\rho(\sigma_m \otimes I))$, and λ_{max} is the maximum eigenvalue of the matrix $(\vec{y}\vec{y}^{\dagger} + TT^{\dagger})$ [45]. It is not difficult to show that $D_G(\rho)$, here, is

$$D_G(\rho) = \frac{2}{3} \sin^2 2\theta \sin^2 \left(\frac{\Delta t}{4E} \right) \left[3 + \cos 4\theta + 2 \cos \left(\frac{\Delta t}{2E} \right) \sin^2 2\theta \right],$$

$$= \frac{8}{3} P_{\mathrm{sur}} P_{\mathrm{osc}}. \tag{19}$$

$D_G(\rho) \neq 0$ for $P_{\mathrm{osc}} \neq 0$, taking it away from the classically allowed value of geometric discord [46].

Apart from the above foundational measures of various aspects of quantum correlations, a need was felt to have a measure that defines the practical use of quantum correlations. This was supplied by teleportation. Since neutrinos interact only through weak interactions, the effect of decoherence is minimal, when compared to other particles such as electrons and photons that are widely used in quantum information processing. Hence it has the potential to impact practical quantum information processing.

The classical fidelity of teleportation in the absence of entanglement is 2/3. Whenever the maximum teleportation fidelity, $F_{\mathrm{max}} > 2/3$, quantum teleportation is possible. F_{max}, is easily computed in terms of the eigenvalues $\{u_i\}$ of $T^\dagger T$ mentioned above and is given by $F_{\mathrm{max}} = \frac{1}{2} \left(1 + \frac{1}{3} N(\rho) \right)$ where $N(\rho) = \left(\sqrt{u_1} + \sqrt{u_2} + \sqrt{u_3} \right)$ [47]. This expression allows for a useful interplay between teleportation fidelity and $M(\rho)$. This is so because $N(\rho) \geq M(\rho)$. Hence $M(\rho) > 1$ automatically implies $F_{max} > 2/3$. For the case of two flavour neutrino oscillations, F_{max} is given by

$$F_{\mathrm{max}} = \frac{2}{3} + \frac{1}{3} \sqrt{3 + \cos 4\theta + 2 \sin^2 2\theta \cos \left(\frac{\Delta t}{2E} \right)} \sin 2\theta \sin \left(\frac{\Delta t}{4E} \right),$$

$$= \frac{2}{3} \left(1 + P_{\mathrm{sur}} P_{\mathrm{osc}} \right). \tag{20}$$

For non-zero P_{osc}, $F_{\mathrm{max}} > 2/3$, where 2/3 is the classical value of teleportation fidelity. Thus the usual relation between Bell's inequality violation and teleportation fidelity [47], as seen in electronic and photonic systems, is obeyed here. This is in contrast to the unstable oscillating neutral mesons [7].

From the above analysis, it is obvious that all the quantum correlations are directly related to the neutrino oscillation probability. A measurement of the neutrino survival probability which is less than unity directly leads to the conclusion that all the quantum correlations take classically forbidden values.

4. Conclusions

In this work we have computed four facets of quantum correlations for the two flavour neutrino oscillations. We find that all these correlations are simple functions of the product of neutrino survival and oscillation probabilities. They acquire classically forbidden values when the oscillation probability is non-zero. In that case, the Bell's inequality is always violated and teleportation fidelity is always greater than 2/3. Since the three types of neutrino experiments discussed in section 2, long and short baseline reactor and long baseline accelerator, have all measured the neutrino survival probabilities to be less than unity, we can conclude that they have also demonstrated the non-trivial quantum correlations in each case.

References

[1] P.H. Eberhard, Nucl. Phys. B 398 (1993) 155;
A. Di Domenico, Nucl. Phys. B 450 (1995) 293;
F. Uchiyama, Phys. Lett. A 231 (1997) 295;
F. Selleri, Phys. Rev. A 56 (1997) 3493;
F. Benatti, R. Floreanini, Eur. Phys. J. C 13 (2000) 267;
B. Ancochea, A. Bramon, M. Nowakowski, Phys. Rev. D 60 (1999) 094008;
R.A. Bertlmann, W. Grimus, B.C. Hiesmayr, Phys. Lett. A 289 (2001) 21;
M. Genovese, C. Novero, E. Predazzi, Found. Phys. 32 (2002) 589;
P. Caban, J. Rembielinski, K.A. Smolinski, Z. Walczak, M. Wlodarczyk, Phys. Lett. A 357 (2006) 6;
B.C. Hiesmayr, Eur. Phys. J. C 50 (2007) 73;
B. Ananthanarayan, K. Garg, Curr. Sci. 93 (2007) 1055;
L. Lello, D. Boyanovsky, R. Holman, J. High Energy Phys. 1311 (2013) 116;
N. Nikitin, V. Sotnikov, K. Toms, Phys. Rev. D 92 (2015) 016008.
[2] A. Go, Belle Collaboration, J. Mod. Opt. 51 (2004) 991;
A. Go, et al., Belle Collaboration, Phys. Rev. Lett. 99 (2007) 131802;
G. Amelino-Camelia, F. Archilli, D. Babusci, D. Badoni, G. Bencivenni, J. Bernabeu, R.A. Bertlmann, D.R. Boito, et al., Eur. Phys. J. C 68 (2010) 619.
[3] M. Blasone, F. Dell'Anno, S. De Siena, F. Illuminati, Europhys. Lett. 85 (2009) 50002.
[4] M. Blasone, F. Dell'Anno, S. De Siena, F. Illuminati, Phys. Rev. D 77 (2008) 096002.
[5] M. Blasone, F. Dell'Anno, S. De Siena, F. Illuminati, J. Phys. Conf. Ser. 237 (2010) 012007.
[6] M. Blasone, F. Dell'Anno, S. De Siena, F. Illuminati, Europhys. Lett. 106 (2014) 30002;
M. Blasone, F. Dell'Anno, S.D. Siena, F. Illuminati, Adv. High Energy Phys. 2014 (2014) 359168.
[7] S. Banerjee, A.K. Alok, R. MacKenzie, Eur. Phys. J. Plus 131 (5) (2016) 1–8, arXiv:1409.1034 [hep-ph].
[8] S. Banerjee, A.K. Alok, R. Srikanth, B.C. Hiesmayr, Eur. Phys. J. C 75 (10) (2015) 487, arXiv:1508.03480 [hep-ph].
[9] M. Blasone, F. Dell'Anno, S. De Siena, F. Illuminati, Europhys. Lett. 112 (2) (2015) 20007, http://dx.doi.org/10.1209/0295-5075/112/20007, arXiv:1510.06761 [quant-ph].
[10] J.A. Formaggio, D.I. Kaiser, M.M. Murskyj, T.E. Weiss, arXiv:1602.00041 [quant-ph].
[11] J.N. Bahcall, M.C. Gonzalez-Garcia, C. Pena-Garay, J. High Energy Phys. 0408 (2004) 016.
[12] K. Eguchi, et al., KamLAND Collaboration, Phys. Rev. Lett. 90 (2003) 021802.
[13] T. Araki, et al., KamLAND Collaboration, Phys. Rev. Lett. 94 (2005) 081801.
[14] Y. Ashie, et al., Super-Kamiokande Collaboration, Phys. Rev. Lett. 93 (2004) 101801.
[15] D.G. Michael, et al., MINOS Collaboration, Phys. Rev. Lett. 97 (2006) 191801.
[16] K. Abe, et al., T2K Collaboration, Phys. Rev. Lett. 112 (2014) 061802.
[17] K. Abe, et al., T2K Collaboration, Phys. Rev. Lett. 111 (21) (2013) 211803.
[18] S.J. van Enk, Phys. Rev. A 72 (2005) 064306.
[19] C.C. Gerry, Phys. Rev. A 53 (1996) 4583.
[20] M.O. Terra Cunha, J.A. Dunningham, V. Vedral, Proc. R. Soc. A 463 (2007) 2277.
[21] A.I. Lvovsky, H. Hansen, T. Aichele, O. Benson, J. Mlynek, S. Schiller, Phys. Rev. Lett. 87 (2001) 050402.
[22] E. Lombardi, F. Sciarrino, S. Popescu, F. De Martini, Phys. Rev. Lett. 88 (2002) 070402.
[23] H. Barnum, et al., Phys. Rev. Lett. 92 (2004) 107902.
[24] A. Einstein, et al., Phys. Rev. 47 (1935) 777.
[25] J.S. Bell, Physics 1 (1964) 195.
[26] J.F. Clauser, A. Shimony, Rep. Prog. Phys. 41 (1978) 1881.
[27] S. Hill, W.K. Wooters, Phys. Rev. Lett. 78 (1997) 5022;
W.K. Wootters, Phys. Rev. Lett. 80 (1998) 2245.
[28] C.H. Bennett, G. Brassard, C. Crepeau, R. Jozsa, A. Peres, W.K. Wootters, Phys. Rev. Lett. 70 (1993) 1895.
[29] H. Ollivier, W.H. Zurek, Phys. Rev. Lett. 88 (2001) 017901.
[30] L. Henderson, V. Vedral, J. Phys. A 34 (2001) 6899.
[31] J. Oppenheim, et al., Phys. Rev. Lett. 89 (2002) 180402;
M. Horodecki, et al., Phys. Rev. Lett. 90 (2003) 100402.
[32] S. Luo, Phys. Rev. A 77 (2008) 022301.
[33] K. Modi, et al., Phys. Rev. Lett. 104 (2010) 080501.
[34] S. Schael, et al., ALEPH Collaboration, DELPHI Collaboration, L3 Collaboration, OPAL Collaboration, SLD Collaboration, LEP Electroweak Working Group, SLD Electroweak Group, SLD Heavy Flavour Group, Phys. Rep. 427 (2006) 257.

[35] G. Alexander, et al., LEP Collaboration, ALEPH Collaboration, DELPHI Collaboration, L3 Collaboration, OPAL Collaboration, Phys. Lett. B 276 (1992) 247.
[36] F.P. An, et al., DAYA-BAY Collaboration, Phys. Rev. Lett. 108 (2012) 171803.
[37] J.K. Ahn, et al., RENO Collaboration, Phys. Rev. Lett. 108 (2012) 191802.
[38] Y. Abe, et al., Double Chooz Collaboration, Phys. Rev. D 86 (2012) 052008.
[39] D.A. Dwyer, Daya Bay Collaboration, Nucl. Phys. Proc. Suppl. 235–236 (2013) 30.
[40] D.V. Forero, M. Tortola, J.W.F. Valle, Phys. Rev. D 86 (2012) 073012, arXiv:1205.4018 [hep-ph].
[41] G.L. Fogli, E. Lisi, A. Marrone, D. Montanino, A. Palazzo, A.M. Rotunno, Phys. Rev. D 86 (2012) 013012.
[42] M.C. Gonzalez-Garcia, M. Maltoni, J. Salvado, T. Schwetz, Global fit to three neutrino mixing: critical look at present precision, J. High Energy Phys. 1212 (2012) 123.
[43] A. Aspect, J. Dalibard, G. Roger, Phys. Rev. Lett. 49 (1982) 1804;
A. Aspect, P. Grangier, G. Roger, Phys. Rev. Lett. 49 (1982) 91;
A. Aspect, P. Grangier, G. Roger, Phys. Rev. Lett. 47 (1981) 460.
[44] R. Horodecki, P. Horodecki, M. Horodecki, Phys. Lett. A 200 (1995) 340.
[45] B. Dakic, et al., Phys. Rev. Lett. 105 (2010) 190502.
[46] S. Adhikari, S. Banerjee, Phys. Rev. A 86 (2012) 062313.
[47] R. Horodecki, P. Horodecki, M. Horodecki, Phys. Lett. A 222 (1996) 21.