JOURNAL OF GEOPHYSICAL RESEARCH, VOL. 117, F04002, doi:10.1029/2011JF002211, 2012

Is the von Kármán constant affected by sediment suspension?

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Received 14 September 2011; revised 10 August 2012; accepted 21 August 2012; published 3 October 2012.

[1] Is the von Kármán constant affected by sediment suspension? The presence of suspended sediment in channels and fluvial streams has been known for decades to affect turbulence transfer mechanism in sediment-laden flows, and, therefore, the transport and fate of sediments that determine the bathymetry of natural water courses. This study explores the density stratification effects on the turbulent velocity profile and its impact on the transport of sediment. There is as yet no consensus in the scientific community on the effect of sediment suspension on the von Kármán parameter, κ. Two different theories based on the empirical log-wake velocity profile are currently under debate: One supports a universal value of $\kappa = 0.41$ and a strength of the wake, Π, that is affected by suspended sediment. The other suggests that both κ and Π could vary with suspended sediment. These different theories result in a conceptual problem regarding the effect of suspended sediment on κ, which has divided the research area. In this study, a new mixing length theory is proposed to describe theoretically the turbulent velocity profile. The analytical approach provides added insight defining κ as a turbulent parameter which varies with the distance to the bed in sediment-laden flows. The theory is compared with previous experimental data and simulations using a k-ε turbulence closure to the Reynolds averaged Navier Stokes equations model. The mixing length model indicates that the two contradictory theories incorporate the stratified flow effect into a different component of the log-wake law. The results of this work show that the log-wake fit with a reduced κ is the physically coherent approximation.

Citation: Castro-Orgaz, O., J. V. Giráldez, L. Mateos, and S. Dey (2012), Is the von Kármán constant affected by sediment suspension?, *J. Geophys. Res.*, *117*, F04002, doi:10.1029/2011JF002211.

1. Introduction

[2] Is the von Kármán constant affected by sediment suspension? This is the major question under debate in this paper. Understanding the turbulent velocity profile in sediment-laden flows is of primary importance for understanding the mechanics of sediment transport in fluvial streams and channels. The sediment is kept in suspension due to the fluid turbulence. The transport and final fate of these particles depend on erosion and deposition processes, which finally constrain the bathymetry of the natural water ways. Suspended load is generally the largest fraction of sediment in transport [*Hill et al.*, 1988]. Estimates of the turbulent velocity profile are required to quantify the amount of sediment transported in suspension [*Garcia*, 2008]. Therefore, accurate characterization of the turbulent velocity profile through a physically based theory for prediction of turbulent flow interaction with suspended sediment is relevant for understanding sediment transport [*Smith and McLean*, 1977; *Gust and Southard*, 1983; *Villaret and Trowbridge*, 1991; *Yang*, 2007; *Garcia*, 2008].

[3] Under clear-water turbulent flow conditions, the mean velocity profile is described in the vicinity of the wall by the *logarithmic law of the wall* (henceforth designated log-law), i.e., flow velocity increases logarithmically with height above the channel bed. The shape of this function is characterized by the von Kármán parameter κ, which takes the universal value of 0.41 for clear water [*Nezu and Rodi*, 1986]. However, it has been extensively verified that the log-law does not hold at the outer-region of the boundary layer. For this region, *Coles* [1956] introduced the *wake law*, whose form depends on the strength of the wake parameter, Π. A complete approximation to the time-averaged velocity profile in turbulent flows can be obtained by coupling the log- and wake-laws [*White*, 1991; *Guo and Julien*, 2001]. The law of the wall also breaks down for shallow, steep flows with low particle submergence [*Wiberg and Smith*, 1991; *Lamb et al.*, 2008]. Further, *Gaudio et al.* [2010] showed that the von Kármán constant is not universal in natural waterways. They found that κ is different from 0.41 in flows with either low submergence or with bed and suspended load transport.

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0148-0227/12/2011JF002211

[4] Sediment-laden turbulent velocity profiles have been examined by empirically fitting the log-law function to match experimental data [*Vanoni*, 1946; *Einstein and Chien*, 1955; *Gust and Southard*, 1983; *Gust*, 1984]. This approach indicates that the suspended sediment in the flow induces a damping effect on the turbulence momentum transfer, thereby reducing the von Kármán parameter κ from its clear-water universal value [*Vanoni*, 1946; *Einstein and Chien*, 1955; *Montes*, 1973]. However, the log-law, theoretically, is only valid within the inner- or wall region, leaving without physical significance the previous data-matching in the whole stratified flow depth [*Coleman*, 1981, 1984, 1986]. *Coleman* [1981, 1984, 1986] conducted detailed experiments in a laboratory flume, concluding that velocity profiles in sediment-laden turbulent flow could be described by Coles' log-wake law. The von Kármán parameter κ and the strength of the wake Π must be estimated by profile matching. *Coleman* [1981, 1984, 1986] considered only near-wall data to determine κ. His fitted value of κ was very similar to the universal value of $\kappa = 0.41$ for clear-water flows, whereas fitted Π varied from 0.9, when the sediment concentration was high, to $\Pi = 0.2$ for clear-water flows [*Nezu and Rodi*, 1986]. The analysis by *Coleman* [1981, 1984], although physically reasonable, was severely criticized by *Gust* [1984], who proposed that the classical log-law is universal. One of the reasons for the critique was the lack of data in the near-wall region, typically four data points, which could lead to inaccurate fittings for the corresponding κ values. Subsequent work by *Parker and Coleman* [1986] corroborated the results presented by *Coleman* [1981, 1984, 1986] regarding the strength of the wake, but it did not provide enough additional information to support the value $\kappa = 0.41$. *Itakura and Kishi* [1980] applied the Monin-Obukhov theory for the length scale in sediment-laden flow and found a linear wake profile, which made their approach conceptually similar to *Coleman*'s [1981, 1984, 1986]. As an alternative to the log-wake law, a power law velocity profile for sediment suspension was considered by *Karim and Kennedy* [1987] and *Woo et al.* [1988]. Adopting the entropy concept *Chiu et al.* [2000] proposed yet another expression for the velocity profile.

[5] *Lyn* [1986, 1988] and *Valiani* [1988] further criticized the approach of *Coleman* [1981, 1984, 1986], arguing that more statistically rigorous log-wake fits suggest that both κ and Π are affected by the suspended sediment. *Valiani* [1988] considered the log-wake model for sediment-laden flows and re-analyzed *Coleman*'s data [1986]. He proposed that, rather than determining κ from the near-wall data, the entire velocity profile should be used, with values weighted inversely with respect to distance from the wall. Surprisingly, using this weighting, *Valiani* [1988] detected a strong effect of sediment suspension reducing the value of κ and a lesser effect on Π, which yielded smaller values from those obtained by *Coleman* [1986]. *Guo and Julien* [2001] proposed a novel wake function, which was coupled with the log-law, and they re-analyzed *Coleman*'s [1986] data. This wake function contained a wake parameter, Ω, that is different from Π. Their fitting analysis demonstrated that, in general, both κ and Ω should be affected by sediment suspension. They found that for sediment-laden flows κ is greatly reduced below 0.41 and Ω is slightly larger than its clear-water value. Although the wake functions used by *Valiani* [1988] and *Guo and Julien* [2001] are different, their comprehensive analyses are in general agreement, indicating that for sediment-laden flows $\kappa < 0.41$ and the wake parameter is larger than its clear-water value, regardless of the specific wake function considered. However, both *Valiani* [1988] and *Guo and Julien* [2001] offered a characterization of the velocity profile using empirical fittings of log-wake laws to experiments. The log-wake law contains two free adjustable parameters (κ and Π), which means that the log-wake results cannot add direct physical insight on the turbulent momentum transfer in sediment-laden flows. The results of *Valiani* [1988] and *Guo and Julien* [2001], therefore, relied on an empirical analysis rather than on a theory for the turbulent momentum transfer [*Yang*, 2007]. This unsolved issue is the basis of the present work.

[6] Sediment particles are expected to affect the turbulence structure near the wall [*Smith and McLean*, 1977], but there is no obvious reason to believe that they will greatly affect turbulence parameters in the outer region. The stratified flow depth by sediments may be ideally divided into two zones: near the bottom, where the concentration and its gradient are large and modify the flow characteristics to a large degree, and a main flow area with small sediment concentration where the sediment has little effect [*Einstein and Chien*, 1955; *Montes*, 1973]. Consequently, the debate on the effect of sediment suspension on the turbulent velocity profile remains open [*Yang*, 2007]. A comprehensive review of non-universality of von Kármán κ was given by *Gaudio et al.* [2010, 2011]. However, conflicting conclusions are found depending on the literature source, leading to uncertainty whether the von Kármán parameter is affected by the suspended sediment or not.

[7] This work proposes a physical interpretation of the turbulent momentum transfer in sediment-laden flows using a modified form of Prandtl's mixing length theory. This approach allows a more physically based characterization of the sediment-laden turbulent velocity profile. In this model, the von Kármán parameter is not a universal constant for a turbulent velocity profile in sediment-laden flows. It is a variable in terms of distance to the wall, which is determined by the damping of the mixing length in the turbulent momentum transfer, caused by the suspended sediment. This is the major difference between this proposal and either the log-wake model [*Coleman*, 1981; *Valiani*, 1988; *Guo and Julien*, 2001] or the log-law model [*Vanoni*, 1946; *Montes*, 1973; *Gust*, 1984; *Wright and Parker*, 2004a, 2004b], in which the von Kármán parameter is constant for a given velocity profile and determined by its adjustment by profile matching. The proposed theoretical model permits explanation of the turbulent momentum transfer mechanism in sediment-laden flows. When this theoretical profile is confronted to log-wake laws, it reveals the effects of the suspended sediment on the fitted parameters κ and Π of the latter. The proposed theory has a major advantage over log-wake laws in that it does not require experimental data to adjust any empirical constant. It is a theoretical profile based on a physical interpretation of the turbulent momentum transfer. The present model was compared with RANS simulations and experimental observations, producing similar results. The proposed theory can therefore be used instead of the log-wake law to characterize the velocity profile with suspended sediment,

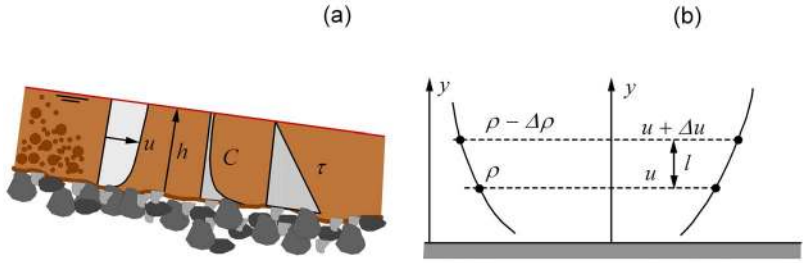

Figure 1. (a) Definition sketch of sediment-laden free surface flow, where u is the flow velocity, τ is the shear stress, C is the concentration and h is the flow depth. (b) Mass density $\rho(y)$ and velocity $u(y)$ profiles in a sediment-laden flow as components of the momentum, $M = \rho u$, transferred between two layers separated by the mixing length l.

thereby substituting fits to experimental data by a theoretical consideration on the turbulent momentum transfer.

2. Wall-Wake Velocity Profile for Clear-Water Flows

[8] The complete time-averaged velocity profile in wall-bounded turbulent boundary layers can be expressed in velocity defect form as [*White* 1991; *Cebeci and Cousteix* 2005]:

$$\frac{U-u}{u_*} = -\frac{1}{\kappa}\ln\left(\frac{y}{\delta}\right) + \frac{2\Pi_o}{\kappa}[1-\omega] \tag{1}$$

where u is the time-averaged streamwise velocity component at elevation y above the channel bed, U is the free stream velocity, u_* the shear velocity, $u_* = (\tau_o/\rho_o)^{1/2}$, τ_o the boundary shear stress, ρ_o the mass density of clear-water, $\omega = \omega(y/\delta)$ the wake function, δ the boundary layer thickness, which, for fully developed free surface flows is equal to the water depth h, and Π_o the wake parameter for clear-water flow.

[9] Equation (1) was analyzed by *Nezu and Rodi* [1986] for clear-water flows in open channels. They obtained mean values of $\kappa = 0.41$ and $\Pi_o = 0.2$. Several forms for the wake function ω have been proposed in the literature [*Cebeci and Cousteix*, 2005]. A widely used wake function is the *law* of *Coles* [1956]:

$$\omega\left(\frac{y}{\delta}\right) = \sin^2\left(\frac{\pi}{2}\frac{y}{\delta}\right) \tag{2}$$

This law is adopted in this work. The turbulent momentum transfer can be expressed in terms of Prandtl's mixing length l_o [*White*, 1991] as

$$\tau = \rho_o l_o^2 \left|\frac{du}{dy}\right| \frac{du}{dy} \tag{3}$$

where τ is the Reynolds shear stress. For a steady-uniform two-dimensional flow, neglecting the viscous sublayer, τ is expressed as:

$$\tau = -\rho_o \overline{u'v'} = \tau_o(1-\eta) \tag{4}$$

where u' and v' are the fluctuations of the velocities from their time-averaged values in the x and y directions, respectively, and $\eta = y/h$ is the normalized elevation. Solving equations (3) and (4) for τ, and computing du/dy using equation (1), results in a normalized mixing length as

$$\frac{l_o}{h} = \frac{\kappa\eta(1-\eta)^{1/2}}{(1+2\Pi_o y\, d\omega/dy)} \tag{5}$$

Equation (5) yields l_o in clear-water flows, once the values of $\kappa = 0.41$ and $\Pi_o = 0.2$ are assumed. If $\Pi_o = 0$, equation (5) reduces to the mixing length distribution for the log-law, as indicated in the semi-empirical expression of *Umeyama and Gerritsen* [1992]. Herein, equation (5) is used as a basis for a turbulent momentum transfer approach in sediment-laden flows.

3. Effect of Suspended Sediment on Turbulent Momentum Transfer

[10] In a steady two-dimensional turbulent flow, with a mean velocity profile $u = u(y)$ (Figure 1a), the sediment suspension induces a mass density profile $\rho = \rho(y)$ (Figure 1b). The momentum of the flow $M = \rho u$ at a certain elevation y above the channel bed can be expanded as a Taylor series, retaining just the first order term [*Montes*, 1973]:

$$M(y+\Delta y) = M(y) + \Delta y \frac{dM}{dy} \tag{6}$$

Similarly to Prandtl's theory, it is assumed that Δy in sediment-laden flows is the length scale l called the *mixing length* for the sediment-laden flow. This mixing length is associated with the distance traveled by an elementary control volume in a direction normal to the wall, due to the fluctuations v', until its momentum is absorbed by the other layer. Thus, from equation (6), one can write:

$$\Delta M = M(y+\Delta y) - M(y) = l\frac{d}{dy}(\rho u) = lu\frac{d\rho}{dy} + l\rho\frac{du}{dy} \tag{7}$$

Further, ΔM can be evaluated from the velocity and mass density profiles to the first order as (see Figure 1b) [*Montes*, 1973]:

$$\Delta M = (\rho - \Delta\rho)(u + \Delta u) - \rho u = \rho\Delta u - u\Delta\rho \tag{8}$$

where Δu is the velocity variation between sediment-laden layers separated by a distance l (Figure 1b), and the second order term $\Delta u \Delta \rho$ has been neglected. The density gradient (Figure 1b) modifies the momentum transfer by its interaction with the velocity profile. Thus, from equations (7) and (8):

$$\Delta u = l \frac{u}{\rho} \frac{d\rho}{dy} + l \frac{du}{dy} + u \frac{\Delta \rho}{\rho} \tag{9}$$

It can be seen that equation (9) includes both the velocity and density gradient effects in the momentum transfer. For constant density, equation (9) simplifies to the classical theory proposed by Prandtl. The mass density variation could also be expanded in terms of l as:

$$\Delta \rho = l \frac{d\rho}{dy} + \frac{l^2}{2} \frac{d^2\rho}{dy^2} \tag{10}$$

Substituting equation (10) into equation (9) produces:

$$\Delta u = l \frac{du}{dy} \left[1 + \frac{u}{\rho} \left(\frac{du}{dy} \right)^{-1} \left(2 \frac{d\rho}{dy} + \frac{l}{2} \frac{d^2\rho}{dy^2} \right) \right] \tag{11}$$

Equation (11), originally developed by *Montes* [1973], includes the effect of the suspension mass density on the momentum transfer. The turbulent momentum transfer is affected not only by the local values of density ρ and velocity u, but also by their gradients, indicating that in sediment-laden flows it is determined by a transport relationship. Therefore, equation (11) suggests that the effect of suspended particles must be relevant in regions with large gradients of mass and momentum transfer, such as near the bed, where both du/dy and $d\rho/dy$ are large.

[11] The mixture mass density is given by:

$$\rho = \rho_s C + \rho_o (1 - C) = \rho_o [1 + (s - 1) C] \tag{12}$$

where C is the volumetric concentration of sediment, ρ_s is the density of solids and $s = \rho_s / \rho_o$. Differentiation of equation (12) with respect to y yields:

$$\frac{d\rho}{dy} = \rho_o (s - 1) \frac{dC}{dy}, \quad \frac{d^2\rho}{dy^2} = \rho_o (s - 1) \frac{d^2C}{dy^2} \tag{13}$$

The added-mass force or the virtual mass force is the inertia added to the particles because an accelerating or decelerating solid must displace some volume of the surrounding fluid as it moves through it. Therefore, the density of a solid particle ρ_p is modified as [*Montes*, 1973; *Liggett*, 1994, section 3.11.2]

$$\rho_p = \rho_s + K \rho_o \tag{14}$$

with K as the added-mass coefficient. Equation (12) can be modified using equation (14) as:

$$\rho = (\rho_s + K\rho_o) C + \rho_o (1 - C) = \rho_o [1 + (s - 1 + K) C] = \rho_o [1 + (s - \beta) C] \tag{15}$$

where $\beta = 1 - K$. Sediment particles are maintained in suspension due to fluid turbulence. The vertical velocity has a random variation with time, provoking the particles to move upward and downward. This random unsteady particle movement associated with turbulent flow displaces the fluid surrounding the sediment. Therefore, a characteristic feature of the turbulent flow with suspended sediment is that the particles undergo a virtual mass force during their instantaneous unsteady movement. This effect is accounted for by the virtual mass coefficient β. It should be realized that the mixing length model developed here is a mixture flow model. Therefore, equation (15) is an improved definition accounting for the movement of the particles inside the fluid. Neglecting β is equivalent to assuming that the particles are static inside the mixture, which is by definition against the concept of turbulent flow.

[12] Differentiation with respect to y of equation (15) gives:

$$\frac{d\rho}{dy} = \rho_o (s - \beta) \frac{dC}{dy}, \quad \frac{d^2\rho}{dy^2} = \rho_o (s - \beta) \frac{d^2C}{dy^2} \tag{16}$$

Then, inserting equation (16) into equation (11) leads to:

$$\Delta u = l \frac{du}{dy} \left[1 + \frac{u \rho_o (R + 1 - \beta)}{\rho} \left(\frac{du}{dy} \right)^{-1} \frac{dC}{dy} \left(2 + \frac{l}{2} \frac{d^2C/dy^2}{dC/dy} \right) dy \right] \tag{17}$$

where the submerged specific gravity, $R = s - 1$, is introduced. On the other hand, the diffusion equation for the concentration of suspended sediments [*Hunt*, 1954] is

$$\varepsilon_s \frac{dC}{dy} + C(1 - C) w = 0 \tag{18}$$

where ε_s is the sediment diffusion coefficient and w the settling velocity. Differentiation of equation (18) with respect to y yields:

$$\varepsilon_s \frac{d^2C}{dy^2} + \frac{dC}{dy} (1 - 2C) w = 0 \tag{19}$$

Inserting equation (19) into equation (17) produces:

$$\Delta u = l \frac{du}{dy} \left\{ 1 + \frac{u \rho_o (R + 1 - \beta)}{\rho} \left(\frac{du}{dy} \right)^{-1} \frac{dC}{dy} \left[2 - \frac{lw}{2\varepsilon_s} (1 - 2C) \right] \right\} \tag{20}$$

The suspended sediment is stratified within the fluid according to the concentration gradient, dC/dy, which affects the density of the water–sediment mixture and the turbulent transfer of momentum. Coarser particles have larger values of w and, therefore, from equation (20), they will provide a greater damping on Δu. Equation (20) is a mixing length relationship which generalizes the classical theory of Prandtl for clear water. The density stratification effects are predominant in the region near the bed [*Smith and McLean*, 1977]. In order to simplify equation (20), ε_s may be approximated near the bed by [*Montes*, 1973]:

$$\varepsilon_s = l u_* \tag{21}$$

Despite this near-bed simplification, it is reasonable to postulate its application to the whole water depth layer, because in the upper part of the water layer the transport of mass and momentum is less affected by suspended particles [*Smith and McLean*, 1977]. Although the sediment-laden mixing length

l differs from the clear-water value l_o, $l = l_o$ will be assumed. Whenever $C \rightarrow 0$, then $dC/dy \rightarrow 0$ as indicated by equation (18). Then, equation (20) tends to the classical Prandtl equation, $\Delta u = l_o du/dy$, if $l \rightarrow l_o$. Therefore, l_o will be considered as an estimator of l in equation (20). Using this approximation, as well as that given by equation (21), equation (20) is rewritten as:

$$\Delta u = l_o \frac{du}{dy}\left\{1 + \frac{u\rho_o(R+1-\beta)}{\rho}\left(\frac{du}{dy}\right)^{-1}\frac{dC}{dy}\left[2 - \frac{w}{2u_*}(1-2C)\right]\right\} \quad (22)$$

Equation (22) can be presented alternatively as:

$$\Delta u = \psi l_o \frac{du}{dy} \quad (23)$$

where ψ is a sediment damping factor defined by:

$$\psi = 1 + \frac{u\rho_o(R+1-\beta)}{\rho}\left(\frac{du}{dy}\right)^{-1}\frac{dC}{dy}\left[2 - \frac{w}{2u_*}(1-2C)\right] \quad (24)$$

In equation (23), ψ modifies the suspended mixing length l_o and their product defines a modified mixing length for the sediment suspension, determined by $l_m = l_o\psi$. Inspection of equation (24) indicates that ψ is a function of the velocity, $u(y)$, the concentration $C(y)$, and their gradients.

[13] The present model for the turbulent momentum transfer, expressed by equations (23) and (24), implies a damping of the clear-water mixing length due to the suspended sediment concentration. This result agrees with previous studies in the literature, where the clear-water mixing length was modified by empirical damping functions [*De Vantier and Narayanaswamy*, 1989; *Umeyama and Gerritsen*, 1992; *Kovacs*, 1998; *Mazumder and Ghoshal*, 2006; *Yang*, 2007]. *Umeyama and Gerritsen* [1992] and *Mazumder and Ghoshal* [2006] used the empirical function

$$\psi = (1-\eta)^{0.5\lambda C/C_a} \quad (25)$$

where λ is a fitting parameter (positive) and C_a is a reference concentration. Equation (25) implies that $\psi < 1$, so that, indeed, ψ is a damping effect on the mixing length. It is an empirical relationship, whereas equation (24) was developed from physical principles. *Kovacs* [1998] proposed the empirical function:

$$\psi = (1-C)^{1/3}. \quad (26)$$

Yang [2007] presented another empirical relationship for ψ as a function of C.

[14] The present model is based on a theoretical approximation for density stratification effects expressed by equation (11). Previous work in the literature also considered the inclusion of density stratification effects on turbulent momentum transfer [*Smith and McLean*, 1977; *Villaret and Trowbridge*, 1991; *Mazumder and Ghoshal*, 2006; *Wright and Parker*, 2004a, 2004b]. In these studies, the momentum diffusivity ε_m was empirically modified to account for sediment damping as:

$$\varepsilon_m = \varepsilon_{mo}[1 - \alpha \mathsf{R}_i] \quad (27)$$

and

$$\mathsf{R}_i = -\frac{g}{\rho}\frac{d\rho}{dy}\left[\frac{du}{dy}\right]^{-2} \quad (28)$$

where α is a fitting parameter and R_i is the Richardson number. *Smith and McLean* [1977] indicated that ε_{mo} generally depends on R_i, but, however, it can be estimated by the clear-water eddy diffusivity [*Smith and McLean*, 1977; *Villaret and Trowbridge*, 1991; *Mazumder and Ghoshal*, 2006; *Wright and Parker*, 2004a, 2004b]. Equation (11) can be rewritten as

$$\begin{aligned}\Delta u &= l\frac{du}{dy}\left[1 + \frac{u}{\rho}\left(\frac{du}{dy}\right)^{-1}\left(2\frac{d\rho}{dy} + \frac{l}{2}\frac{d^2\rho}{dy^2}\right)\right] \\ &= l\frac{du}{dy}\left[1 - \frac{u}{g}\frac{du}{dy}\left(2 + \frac{l}{2}\frac{d^2\rho/dy^2}{d\rho/dy}\right)\mathsf{R}_i\right]\end{aligned} \quad (29)$$

or

$$\Delta u = \psi l\frac{du}{dy}, \quad \psi = 1 - \frac{u}{g}\frac{du}{dy}\left[2 - \frac{w}{2u_*}(1-2C)\right]\mathsf{R}_i \quad (30)$$

with the consequence that the damping in our model is also proportional to R_i. The mixing length l_m and the momentum diffusivity ε_m are generally linked by $\varepsilon_m = l_m^2 du/dy$ [*White*, 1991; *Cebeci and Cousteix*, 2005]. Using the previous definition of $l_m = \psi l_o$ then:

$$\varepsilon_m = \psi^2 l_o^2 \frac{du}{dy} = \psi^2 \varepsilon_{mo} \quad (31)$$

Thus, this mixing length model also implies that momentum diffusivity is damped by the suspended sediment. Using the second of equations (30), equation (31) can be rewritten as

$$\begin{aligned}\varepsilon_m &= \varepsilon_{mo}\left[1 - \frac{u}{g}\frac{du}{dy}\left[2 - \frac{w}{2u_*}(1-2C)\right]\mathsf{R}_i\right]^2 \\ &\approx \varepsilon_{mo}\left[1 - \frac{2u}{g}\frac{du}{dy}\left[2 - \frac{w}{2u_*}(1-2C)\right]\mathsf{R}_i\right]\end{aligned} \quad (32)$$

after using a first order Taylor series expansion. Equation (32) is the same as equation (27) by defining $\alpha = 1 - (2u/g)(du/dy)[2 - w(1 - 2C)/(2u_*)]$. The damping of the mixing length by suspended sediment is equivalent to a damping of the diffusivity, and, thus, our current approach is in agreement with arguments used in previous stratified models [*Smith and McLean*, 1977; *Gelfenbaum and Smith*, 1986; *Villaret and Trowbridge*, 1991; *Mazumder and Ghoshal*, 2006; *Wright and Parker*, 2004a, 2004b].

4. Proposed Mixing Length Model for Sediment-Laden Flows

[15] Based on the mixing length for clear-water flows, given by equation (5), consideration of the sediment-laden effect on the momentum transfer results in the normalized mixing length as

$$\frac{l_m}{h} = \psi\frac{l_o}{h} = \frac{\psi\kappa\eta(1-\eta)^{1/2}}{1 + 2\Pi_o y(d\omega/dy)} \quad (33)$$

where $d\omega/dy$ is taken from equation (2). Therefore, a von Kármán parameter for sediment-laden flows can be defined by analogy with clear-water flows based on equation (33) as

$$\kappa_m = \kappa\psi \tag{34}$$

This equation indicates that the variable distribution $\kappa_m = \kappa_m(y)$ depends on the function $\psi = \psi(y)$. Therefore, ψ may be physically interpreted as a correction factor to the von Kármán constant for clear-water flows, $\kappa = 0.41$, introducing the concentration $C = C(y)$ and velocity $u = u(y)$ gradient effects in sediment-laden flows.

[16] For sediment-laden flow, equation (3) becomes [*Umeyama and Gerritsen*, 1992]:

$$\tau = \rho l_m^2 \left|\frac{du}{dy}\right| \frac{du}{dy} \tag{35}$$

The Reynolds stress distribution, allowing for density stratification effects [*Lyn*, 1986; *Mazumder and Ghoshal*, 2006], is deduced from a vertical momentum balance as:

$$\tau = \tau_o \left(1 - \eta + R\int_\eta^1 C d\eta\right) \tag{36}$$

Combining equations (35) and (36) produces a differential equation for the turbulent velocity profile:

$$\frac{du^+}{d\eta} = \frac{h}{l_m}\left(\frac{1 - \eta + R\int_\eta^1 C d\eta}{1 + RC}\right)^{1/2} \tag{37}$$

where u^+ is the normalized velocity, $u^+ = u/u_*$.

[17] Many experimental studies have attempted to explain the effects of sediment suspension on the momentum transfer by a reduction of κ, that is, by the slope of the best fit line of the velocity profile over the entire water depth using semilogarithmic scales. However, this technique is only a rough estimation of density stratification, which is not constant over the depth [*Wright and Parker*, 2004a, 2004b], as confirmed by ψ in equation (33). The velocity profile fitting technique with semilogarithmic scales, therefore, provides no insight into the mechanics of density stratification or any mechanistic interpretation of the reason underlying the decrease of fitted κ values. The present model introduces a local function given by equation (34), involving density stratification effects, in agreement with *Wright and Parker* [2004a, 2004b]. Therefore, this new formulation of the turbulent momentum transfer can physically explain the reduced fitted values of κ with semilogarithmic scaling. For illustrative purposes, consider flow near the bed, say $\eta < 0.2$ and assume $\Pi_o \approx 0$. Then, equation (33) yields the simplified form $l_m \approx \psi\kappa y$. If the term in brackets on the right hand side of equation (37) is neglected, its integration yields an estimate of the velocity profile in the wall region as:

$$u^+ - u_a^+ = \int_{\eta_a}^{\eta} \frac{d\eta}{\psi\kappa\eta} \tag{38}$$

where subscript a refers to an arbitrary point in the wall region. Note that for $\psi = 1$ the clear-water log-law profile (with $\kappa = 0.41$) is regained from equation (38). However, in general $\psi < 1$, which means that if a log-law exists for u^+, then the following relationship is satisfied:

$$\frac{1}{\phi\kappa}\ln\left(\frac{\eta}{\eta_a}\right) \equiv \int_{\eta_a}^{\eta} \frac{d\eta}{\psi\kappa\eta} \tag{39}$$

provided that the fitting parameter $\phi < 1$. Note that given η_a, for different values of η, ϕ is different as well, implying that a different log-law would strictly exist as an estimator for each point u^+ of the velocity profile near the bed. Equation (39) shows that a unique log-law for a velocity profile with suspended sediment can only exist in a mean sense, that is, a mean value of ϕ, $\bar{\phi}$, can be fitted to estimate the velocity profile in a certain layer thickness. The discussion can be made more general by considering the complete equation (37), applicable to the whole water depth thickness h, rather than its simplified version of equation (38). However, the conclusions with regard to ϕ and the existence of log-laws are identical. A mean $\bar{\phi}$ was proposed by *Wright and Parker* [2004a, 2004b] to fit log-laws to the whole water depth of sediment-laden flows. *Cantero et al.* [2009] used DNS (direct numerical simulation) to investigate density stratification effects in sediment-laden flows. They fitted log-laws in the wall layer to their DNS data and determined by best fit, an "apparent von Kármán constant" less than 0.41, $\bar{\phi}\kappa < 0.41$. On the basis of equation (39), the proposals of *Wright and Parker* [2004a, 2004b] and *Cantero et al.* [2009] agree with this model. The variation of density stratification with distance to the bed, $\psi = f(y)$, can be measured in average with the damping parameter $\bar{\phi}$ in a log-law that is fitted to modeled or measured velocity data. Nevertheless this approximation using $\bar{\phi}$ is only an empirical, indirect method to represent density stratification effects.

[18] From the works of *Engelund* [1970] and *Montes* [1973], pure suspension with w/u_* much larger than 1 is not possible. Thus, for a sediment suspension with $w/u_* < 1$ one can make the approximation $w(1 - 2C)/(2u_*) \ll 2$. Using this approximation, ψ of equation (24) is simplified to

$$\psi = 1 + 2u^+\left(\frac{R + 1 - \beta}{1 + RC}\right)\frac{dC}{d\eta}\left(\frac{du^+}{d\eta}\right)^{-1} \tag{40}$$

Current knowledge on sediment suspension and velocity profiles is mainly based on the hypothesis that sediment suspension affects the value of κ relative to that for the clear water [*Gust*, 1984]. However, the introduction of a modified von Kármán parameter can take account of variations in sediment concentration, C. This mathematical approach provides

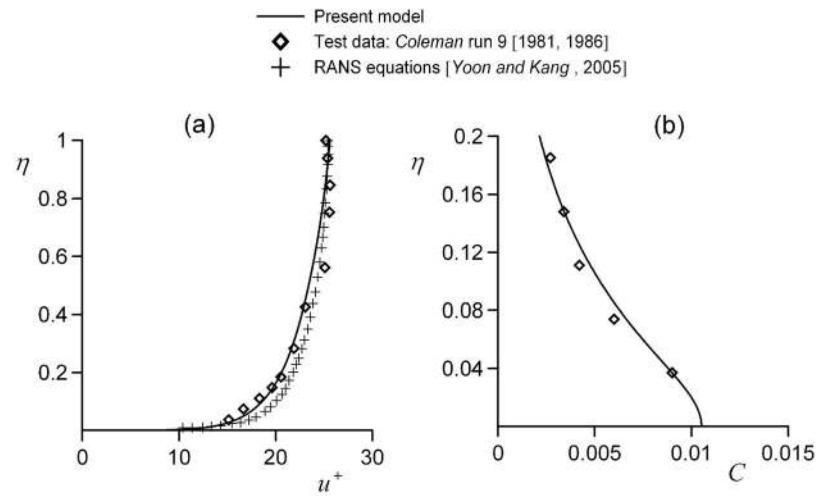

Figure 2. (a) Comparison of results obtained from equation (37) for the velocity profile $u^+ = u^+(\eta)$ with test data by *Coleman* [1981], RUN 9, and with CFD results by *Yoon and Kang* [2005]. (b) Comparison of fitted equation (41) for the concentration profile $C = C(\eta)$ with test data by *Coleman* [1981].

an explanation of the phenomena observed experimentally by other researchers, who fitted test data of velocity profiles to a log-law [*Vanoni*, 1946; *Einstein and Chien*, 1955; *Montes*, 1973; *Gust and Southard*, 1983; *Gust*, 1984; *Wright and Parker*, 2004a, 2004b]. They obtained diminished means of the von Kármán parameter as functions of any parameters indicating a bulk measure of density stratification, for example the depth-averaged concentration or a mean Richardson number. Furthermore, this model is based on a Taylor series expansion, and if needed, more terms of the series can be considered. A relation $C = C(y)$ needs to be introduced into equation (40). Since the aim of this study is to describe the turbulent velocity profile, the precise definition of the concentration profile $C(y)$ has not been considered. An exponential-type function corrected for the slope effects near the wall [*Montes*, 1973] is adopted here:

$$C = C_b e^{-A\eta}\left(\frac{1 + e^{-2D\eta}}{2}\right)^{-A/D} \qquad (41)$$

where C_b is the maximum concentration, i.e., the concentration at the interface between the bed load and the suspended load, $\eta = 0$, and A and D are empirical coefficients. According to *Montes* [1973], D is approximately 30, while A can be estimated by fitting equation (41) to measured data of concentration profiles. The classical Ippen-Rouse equation [*Montes*, 1973] was first considered by the authors, following *Umeyama and Gerritsen* [1992]. However, this equation produced a poor approximation of the concentration gradients near the wall. It should be realized that the first factor of equation (41) forces it to get a null concentration gradient, $dC/d\eta = 0$, at the wall, $\eta = 0$, which appears to be a reasonable boundary condition. The Ippen-Rouse equation induces the unreal wall boundary condition $dC/d\eta \rightarrow \infty$. For a more accurate description of the concentration profile gradient near the wall, a finite-mixing length model is necessary [*Nielsen and Teakle*, 2004; *Absi*, 2010]. However, this considerably increases the complexity of the model, and, therefore, for the purpose of examining the turbulent velocity profile, the empirical characterization of the concentration profiles as given by equation (41) has been adopted in this work.

5. Evaluation of the Proposed Model Based on Experimental Data

5.1. *Coleman* [1981, 1984, 1986] Data Set

[19] *Coleman* [1981, 1984, 1986] conducted experiments in a laboratory flume 0.356 m wide with sand grains of diameter $d = 0.105$, 0.21 and 0.42 mm. His experimental results were reanalyzed using the proposed model. In *Coleman*'s [1986] experiments, the streamline of the maximum velocity lay at a certain distance below the free surface, due to so-called dip-phenomena, indicating the presence of a three-dimensional effect induced by the secondary current in the flow section. The proposed approach is restricted to two-dimensional uniform sediment-laden flows, so it cannot predict dip phenomena. No attempt has been made as yet to analyze this effect, thus a qualitative comparison was performed here, using a value of 0.2 for Π_o based on data from *Coleman*'s [1986] clear-water tests.

[20] Figures 2–4 compare results obtained from the proposed model with data from three experiments with reference concentrations $C_a = 9{\cdot}10^{-3}$, $9.5{\cdot}10^{-3}$ and $2.6{\cdot}10^{-3}$ m^3m^{-3}, where C_a is the concentration at a given reference elevation above the channel bed in the experiments. The computed velocity profile $u^+ = u^+(\eta)$ from equation (37) is plotted in Figures 2a, 3a, and 4a for runs 9, 29 and 40 of *Coleman* [1981, 1986]. Figures 2b, 3b and 4b contain the fitted equation (41) for runs 9, 29 and 40. Equation (37) is solved in velocity-defect form, $dZ/d\eta = -du^+/d\eta$, where $Z = (U - u)/u_*$, using equation (33) for the mixing length distribution in sediment-laden flows and equation (40) for ψ. $C(y)$ and dC/dy are estimated from equation (41), once A is determined by fitting to concentration test data. The term $d\omega/dy$ in equation (33)

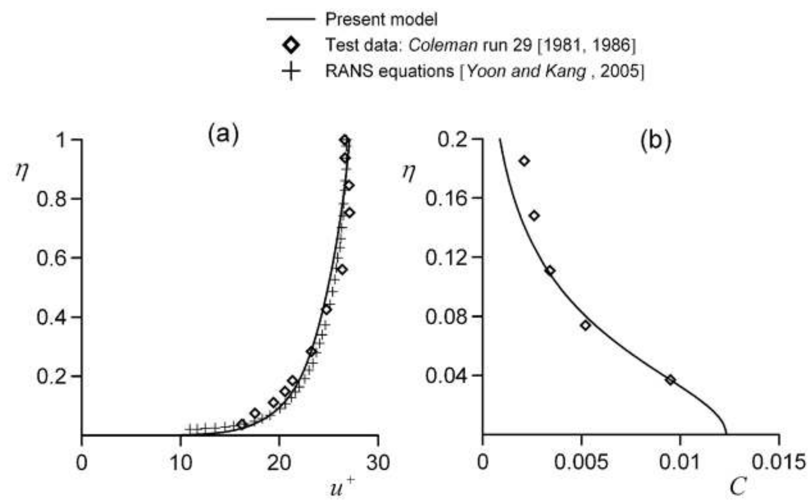

Figure 3. (a) Comparison of results obtained from the equation (37) for the velocity profile $u^+ = u^+(\eta)$ with test data by *Coleman* [1981], RUN 29, and with CFD results by *Yoon and Kang* [2005]. (b) Comparison of fitted equation (41) for the concentration profile $C = C(\eta)$ with test data by *Coleman* [1981].

is evaluated using equation (2). The approximate value $\beta = 0.5$ is taken from *Montes* [1973]. Equation (37) is solved numerically using a standard 4th-order Runge–Kutta method, with the free surface $Z(\eta = 1) = 0$ as boundary condition. To avoid a singularity, $\eta \approx 0.999$ is used as the initial value for the computation. Equation (37) is implicit in $du^+/d\eta$. The $du^+/d\eta$ value in a given computational step for the evaluation of ψ is used from the previous step. The computational step is successively reduced until no significant variations are detected between the assumed $du^+/d\eta$ in ψ and the value computed from equation (37). To transform the defect profiles into absolute velocities, the relation $u^+ = (U/u_*) - Z$ is used, where U/u_* is obtained from the corresponding experiment. Equation (41) was found to provide a good fit to the test data for $\eta < 0.2$, that is, within the wall region. The proposed model is not able to recognize the dip-phenomena, as expected. However, in general, there is good agreement between predictions and observations. To further assess the robustness of the proposed theory, recent computational fluid dynamics (CFD) results have been considered, which were obtained by *Yoon and Kang* [2005] solving the two-dimensional Reynolds averaged Navier–Stokes (henceforth RANS) equations with a k-ε turbulence closure for sediment-laden flows. Corresponding results for the same test cases are included in Figures 2–4. It is apparent that the results of the proposed model are also close to those of the RANS equations. Thus, the proposed model is a good estimator of turbulent velocity profiles with suspended sediments given its general qualitative agreement with experimental evidence and CFD results.

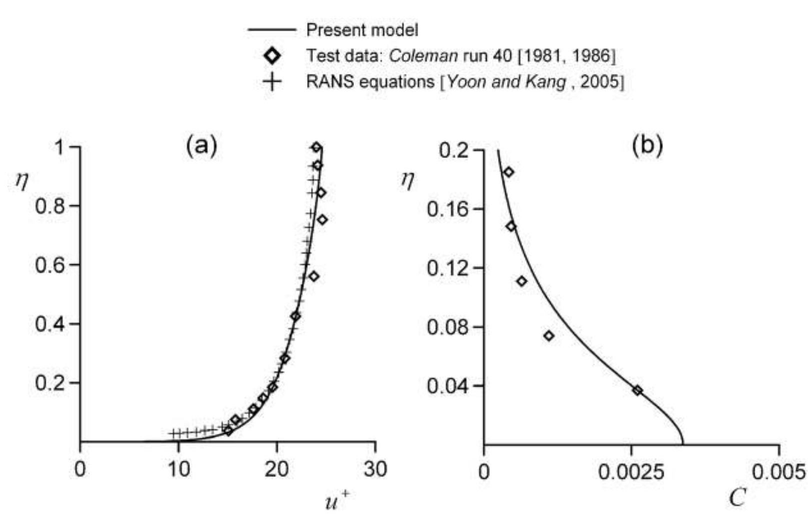

Figure 4. (a) Comparison of results obtained from equation (37) for the velocity profile $u^+ = u^+(\eta)$ with test data by *Coleman* [1981], RUN 40, and with CFD results by *Yoon and Kang* [2005]. (b) Comparison of fitted equation (41) for the concentration profile $C = C(\eta)$ with test data by *Coleman* [1981].

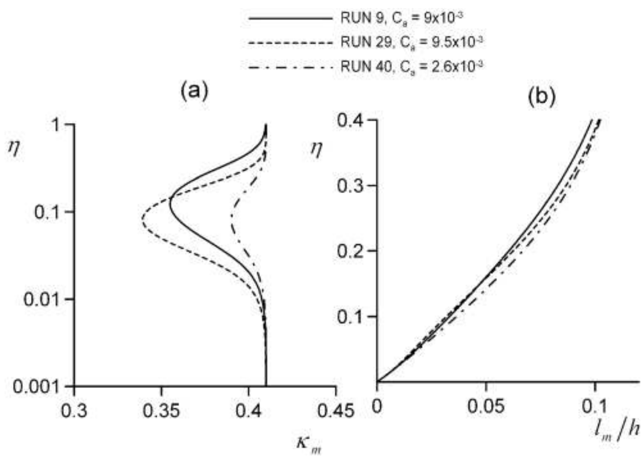

Figure 5. Depth-distribution of: (a) von Kármán parameter $\kappa_m = \kappa_m(\eta)$ and (b) mixing length $l_m/h = l_m/h(\eta)$, for RUNS 9, 29 and 40 of *Coleman* [1981], based on the numerical solution of equation (37).

[21] To explore the influence of sediment suspension on the velocity profile, the functions $\kappa_m = \kappa_m(\eta)$ and $l_m = l_m(\eta)$ for these runs are plotted in Figures 5a and 5b. The first relevant result is the sediment damping effect on κ_m. The parameter κ_m was far from constant, less than 0.41, and its distribution greatly affected by the suspended sediment. The κ_m function has a semi-normal distribution that declines toward a minimum value as one moves away from the bed, followed by a recovery of the near-bed value $\kappa_m \approx 0.41$ as one approaches $\eta = 1$. This specific distribution is fixed by $u^+(\eta)$, $C(\eta)$, $du^+/d\eta(\eta)$ and $dC/d\eta(\eta)$ in equation (40). Of particular importance in the evaluation of ψ is the function $dC/d\eta(\eta)$. Note from equation (41) that $dC/d\eta \rightarrow 0$ for $\eta \rightarrow 0$, and, thus, $\kappa_m \rightarrow 0.41$. For $\eta \rightarrow 1$, equation (41) indicates that $dC/d\eta$ is proportional to C. In the usual case $C(\eta \rightarrow 1) \approx 0$ and, therefore, $\psi \rightarrow 1$ and $\kappa_m \rightarrow 0.41$. The C_a value is inversely related to the minimum of κ_m, designated κ_{mM} (see depth distributions $\kappa_m = \kappa_m(\eta)$ in Figure 5a). For run 29, in which $C_a = 9.5 \times 10^{-3}$ m^{3}m^{-3}, $\kappa_{mM} \approx 0.34$. The fact that κ_m tends toward 0.41 near the bed (Figure 5a) may initially appear to be consistent with *Coleman*'s [1981, 1986] promotion of the universal use of $\kappa = 0.41$ in log-wake models. However, there is a large variation of κ_m in the wall-region ($\eta < 0.2$) (Figure 5a). The reduced κ values found by *Valiani* [1988] in sediment-laden flows are an indirect bulk estimation of the flow density stratification in the wall zone, dictated by the curve $\kappa_m = \kappa_m(\eta)$. Any fitted κ in a log-wake model will be influenced by the density stratification effect given by $\kappa_m = \kappa_m(\eta)$. Density stratification effects may increase away from the wall-layer as in RUN 9 ($C_a = 9.5 \times 10^{-3}$ m^{3}m^{-3}) of Figure 5a. Two observations are of interest: (i) If the fitting technique in a log-wake law results in $\kappa \approx 0.41$ the log-law component of the log-wake law catches only the flow conditions very near the wall. This clear-water log-term leaves the main density stratification effects given by $\kappa_m = \kappa_m(\eta)$ for the wake component; (ii) If the fitting technique results in $\kappa < 0.41$, the density stratification effect indicated by $\kappa_m = \kappa_m(\eta)$ is partially incorporated into the log-law component, whereas the remainder needs to be incorporated, of necessity, into the wake component. The reduction of the mixing length caused by suspended sediment (Figure 5b) agrees with results obtained by *Umeyama and Gerritsen* [1992], *Kovacs* [1998] and *Yang* [2007]. Measurements in both clear water and sediment-laden flows [*Nikora and Goering*, 2000] indicate that the Reynolds stress matches the linear shear stress distribution only for η greater than about 0.1. Using the linear Reynolds stress distribution $(1 - \eta)$ in equation (37) therefore introduces some error close to the wall in the variation predicted for $\kappa_m = \kappa_m(\eta)$.

[22] The numerical predictions of equation (37) for u^+ are now used to fit a log-wake law following the method of *Coleman* [1981, 1986]. The asymptotic value of equation (1) in the wall-region is:

$$U^+ - u^+ = -\frac{1}{\kappa_{fit}} \ln\eta + \frac{2\Pi_s}{\kappa_{fit}} \tag{42}$$

where Π_s is the strength of the wake for a flow with sediment suspension, U^+ is U/u_* and κ_{fit} the von Kármán coefficient fitted in a log-wake model. κ_{fit} is a constant whereas κ_m is a complex turbulent variable. Equation (42) was fitted with data computed with equation (37) near the bed, $0.001 < \eta < 0.01$. Plotting $U^+ - u^+$ against $\ln\eta$ the value of κ_{fit} is fixed by the slope of the regression line using equation (42), and Π_s by its intersection with the $(U^+ - u^+)$ axis. This computation should not be considered as a rigorous attempt to investigate conditions near the wall, since the proposed theoretical approach considers neither the viscous nor the buffer layer, usually found when $\eta < 0.015$ [*Coleman*, 1992; *Nezu and Azuma*, 2004]. Instead, it is intended to add insight to, and extend, *Coleman*'s [1986] findings.

[23] Figures 6a and 6b show κ_{fit} and κ_{mM} values obtained for $d = 0.105$ mm, together with κ_{fit} computed by *Coleman* [1981, 1984, 1986], *Valiani* [1988] and *Guo and Julien* [2001], respectively, against the mean Richardson number. The mean Richardson number $\overline{\mathsf{R}}_i$ is a bulk measure of density stratification effects and is given by [*Coleman*, 1981, 1984, 1986; *Valiani*, 1988]:

$$\overline{\mathsf{R}}_i = \frac{gh}{u_*^2}\left[\frac{\rho_{\eta=0} - \rho_{\eta=1}}{\bar{\rho}}\right] \tag{43}$$

where $\bar{\rho}$ is the mean density in the flow depth h. Inspecting Figure 6a the values of κ_{fit} are close to 0.41, as observed by *Coleman* [1981, 1984, 1986]. Further, in Figure 6b, κ_{mM} follows essentially the same trend as the κ_{fit} values obtained by *Valiani* [1988]. *Valiani* [1988] included the complete velocity profile to fit log-wake models, but used weighting factors that are inversely proportional to the distance from the wall. He did not consider solely the influence of data restricted to the wall-region (as *Coleman* [1986] did), thereby obtaining a value of κ_{fit} smaller than 0.41 due to the incorporation of density stratification effects by fitting κ to the entire flow depth. The fact that κ_{fit} varies with $\overline{\mathsf{R}}_i$ (Figure 6b) further emphasizes that *Valiani*'s [1988]

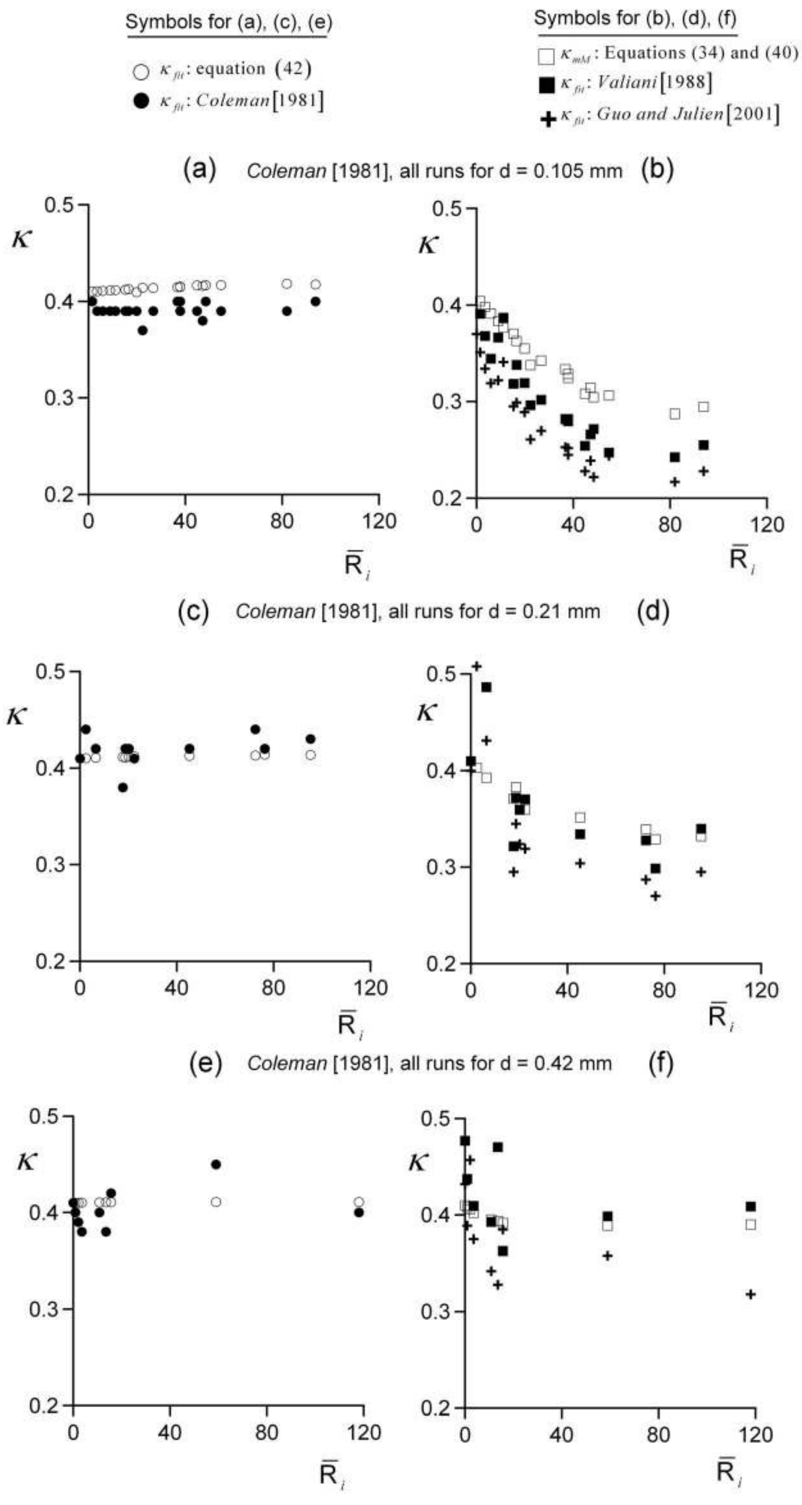

Figure 6. (a, c, e) Comparisons of κ_{fit} obtained by fitting equation (42) to the numerical results of equation (37) with the κ_{fit} obtained by *Coleman* [1981] (the results for the 40 runs in Coleman's paper are displayed) for d = 0.105 mm (Figure 6a), 0.211 mm (Figure 6c), and 0.42 mm (Figure 6e). (b, d, f) Comparisons of κ_{mM} obtained from the numerical solution of equation (37) with the κ_{fit} obtained by *Valiani* [1988] and by *Guo and Julien* [2001] for d 0.105 mm (Figure 6b), 0.211 mm (Figure 6d), and 0.42 mm (Figure 6f). The independent variable is the mean Richardson number, $\overline{R}_i$, which is a measure of the density gradient effect in a given run (the results for the 40 runs in Coleman's paper are displayed).

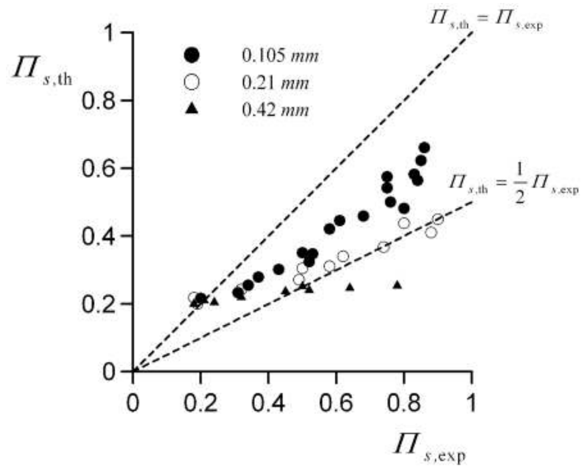

Figure 7. Predicted strength of the wake ($\Pi_{s,\mathrm{th}}$) based on equation (42) fitted to the numerical results for $u^{+}(\eta)$ obtained using equation (37), and fitted values obtained by *Coleman* [1981] in his experiments ($\Pi_{s,\mathrm{exp}}$) using equation (42).

approach for fitting the log-wake law incorporates density stratification effects in the wall-zone; that is, the damping of the mixing length as indicated by the distribution of $\kappa_m(\eta)$ (Figure 5a). Moreover, these results suggest that κ values in log-wake laws should be determined from detailed statistical analysis across the complete water depth, instead of using only near-bed data. Otherwise, there is no guarantee that the κ value will correctly reflect the density stratification within the fluid. Because κ_{fit} and κ_{mM} are both influenced by the degree of density stratification within the flow, a correlation between the two is expected, as shown in Figure 6b. The results for sand with grain sizes of $d = 0.21$ and 0.42 mm, are presented in Figures 6c–6e, respectively. The trends are essentially as previously discussed on the basis of data shown in Figures 6a and 6b, supporting the present deliberation. The values of κ_{fit} obtained by *Guo and Julien* [2001] are included in Figure 6. They found $\kappa_{fit} < 0.41$ in agreement with *Valiani* [1988], although the drop below 0.41 in their log-wake model was larger. This may be attributed to the different fitting technique and wake function. However, both *Valiani* [1988] and *Guo and Julien* [2001] are in agreement that $\kappa_{fit} < 0.41$, which, in turn, arises from density stratification effects, as displayed from our model results of κ_{mM}.

[24] In addition, we used the near-bed data to analyze the hypothesis of a constant value of $\kappa = 0.41$, independent of density stratification, in the wall region [*Coleman*, 1981, 1984, 1986]. Because if $\kappa_{fit} \approx 0.41$, any actual variation $\kappa_m = \kappa_m(\eta)$ in the wall region will be incorporated into the parameter Π_s when equation (42) is applied. However, this effect does not necessarily imply that the suspended sediment affects the outer-layer turbulence structure of the boundary layer. It is an effect of the fitting technique. Thus, this critical perspective calls for caution when concluding that the sediment suspension results in an alteration of the turbulence structure of the outer-layer ($\eta > 0.2$), typically associated with Π_s. It can be noted that on the basis of our mixing length model, Π_s is determined by application of equation (42), and its numerical value is influenced by the effect of suspended sediment on the wall region, but not necessarily by its effects on the outer-layer.

[25] The proposed approach results in values of κ_{mM} which are similar in order of magnitude to κ_{fit} (Figure 6). The κ_{mM} values are obtained from a theoretical velocity profile based on an improved definition of the mixing length. By contrast, κ_{fit} is the result of a fit to experimental data, without any physical support. Therefore, the comparison of these results (Figure 6) has shed light on the empirical approach of other researchers. Turbulent momentum transfer reduces the mixing length, which introduces the new theoretical parameter $\kappa_m = \kappa_m(\eta)$ investigated here.

[26] Modeled values of Π_s for $d = 0.105$, 0.21 and 0.42 mm are plotted in Figure 7 against the values obtained by *Coleman* [1986] applying equation (42) to his experimental velocity profiles. The Π_s values of *Coleman* [1986] are higher than the values obtained here, although the trends observed in Figure 7 for $d = 0.105$ mm are very similar. The discrepancies may be attributed to dip-phenomena, because *Coleman* [1986] considered the elevation of the maximum velocity streamline as the boundary layer thickness to calibrate his wall-wake model. However, the differences, although significant for Π_s, appear to be irrelevant for practical considerations, given the reasonable agreement between the predictions and the observations of velocity profiles (see Figures 3 and 4 for runs 29 and 40 and sand diameters 0.21 and 0.42 mm, respectively). The results of the proposed model are not far from *Coleman*'s [1986] data, despite the evident deviation of Π_s predictions in Figure 7. In conclusion, *Coleman*'s [1986] Π_s values include the effect of the damping of turbulence near the wall, because the log-law is assumed to be that of a clear-water flow, whereas the analyses of *Valiani* [1988] and *Guo and Julien* [2001] incorporate this effect mainly in κ_{fit}. Both parameters reflect the same effect, and the differences are based on a predominant effect of applying the fitting method to a different term of the log-wake model. In addition to its physical significance, an additional advantage of the proposed velocity profile is that it does not require any experimental measurements to adjust the value of empirical constants. Further, the computation based on the ODE equation (37) is simpler than RANS or DNS solutions.

5.2. *Montes* [1973] and *Lyn* [1986] Data Sets

[27] To further check the proposed model, experiments from two additional and independent data sets available in the literature [*Montes*, 1973; *Lyn*, 1986] are considered. Runs 20 and 23 of *Montes* [1973] are included in Figure 8. Montes's experiments were conducted in a flume of 0.487 m width. In the experiments described by *Montes* [1973] there were no secondary currents, thus the flow could be considered two-dimensional, and in his clear-water control tests the approximate values of κ and Π_o were = 0.41 and 0, respectively. Run 20 of *Montes* [1973] is plotted in Figures 8a and 8b. For this experiment the interface values were $C_a = 0.0246$ and $\eta_a = 0.0308$. The fitted concentration profile $C(\eta)$ using equation (41) is plotted in Figure 8b and the prediction of $u^{+}(\eta)$ using equation (37) is plotted in Figure 8a. The modeled velocity profile approaches closely to experimental data. The function $\kappa_m(\eta)$ obtained from equation (40) is

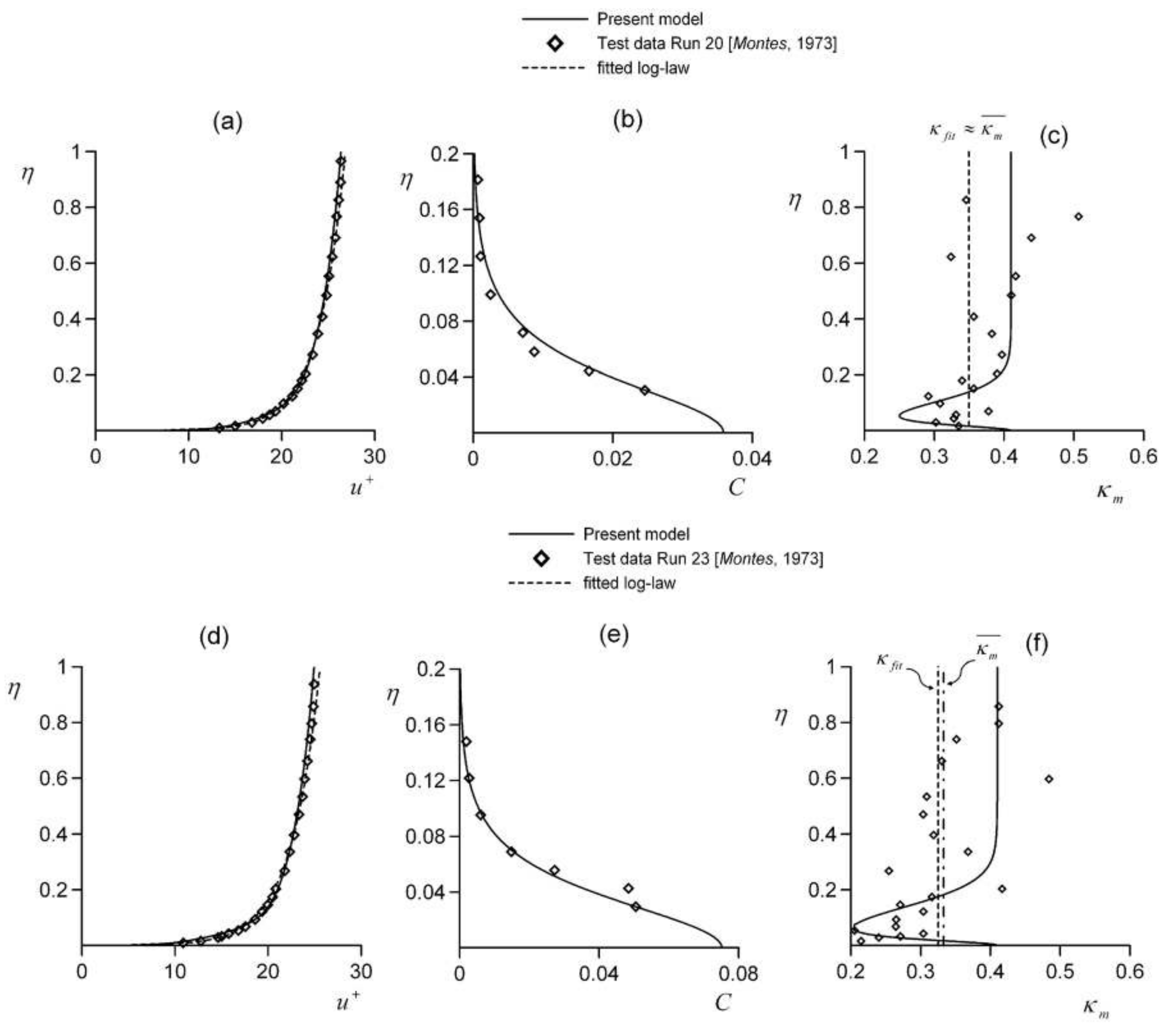

Figure 8. (a) Comparison of results obtained from the equation (37) for the velocity profile $u^+ = u^+(\eta)$ with test data by *Montes* [1973], RUN 20. (b) Comparison of fitted equation (41) for the concentration profile $C = C(\eta)$ with test data by *Montes* [1973]. (c) Comparison of $\kappa_m = \kappa_m(\eta)$ from equation (40) with test data by *Montes* [1973]; (d, e, f) Analogous to Figures 8a–8c but for RUN 23.

represented in Figure 8c. As previously mentioned, the variation of $\kappa_m(\eta)$ is influenced by some of the simplifying assumptions of the theory. Therefore, an approximate experimental estimate of $\kappa_m(\eta)$ was sought from equation (38) as

$$\kappa_m = \left(\eta \frac{du^+}{d\eta}\right)^{-1} \tag{44}$$

Using the measured velocity profiles, equation (44) was evaluated and the corresponding experimental variation of $\kappa_m(\eta)$ is depicted in Figure 8c. Despite data scatter, the reduction of κ_m toward the wall can be appreciated. The theoretical prediction using equation (40) is in fair agreement with the experimental estimates. The present model provides, therefore, an advanced mechanistic interpretation of stratified velocity profiles. To further prove the improvement of this theory over previous statistical analysis, a log-law $u^+ = U^+ + (1/\kappa_{fit})\ln\eta$ deduced from equation (42) was fitted to experimental data. The statistical value obtained for κ_{fit} is 0.349, which is marked in Figure 8c, and the fitted log-law is plotted in Figure 8a as a dotted line. The fitted log-law is close to the theoretical prediction using equation (37). $\kappa_{fit} = 0.349$ is only an empirical, at best, average and indirect indication of the density stratification effects.

[28] In addition, a depth averaged value of κ_m was obtained from equation (37) as

$$\overline{\kappa_m} = \int_0^{\eta_{\max}} \kappa_m d\eta = \int_0^{\eta_{\max}} 0.41\left[1 + 2u^+\left(\frac{R+1-\beta}{1+RC}\right)\frac{dC}{d\eta}\left(\frac{du^+}{d\eta}\right)^{-1}\right]d\eta. \tag{45}$$

For run 20, the variation of κ_s is significant up to $\eta_{\max} = 0.3$ (Figure 8c). Taking this limit for depth-averaging, $\overline{\kappa_m} = 0.35$ was found, a value very close to $\kappa_{fit} = 0.349$. This computation further reveals that the fitted κ_{fit} incorporates

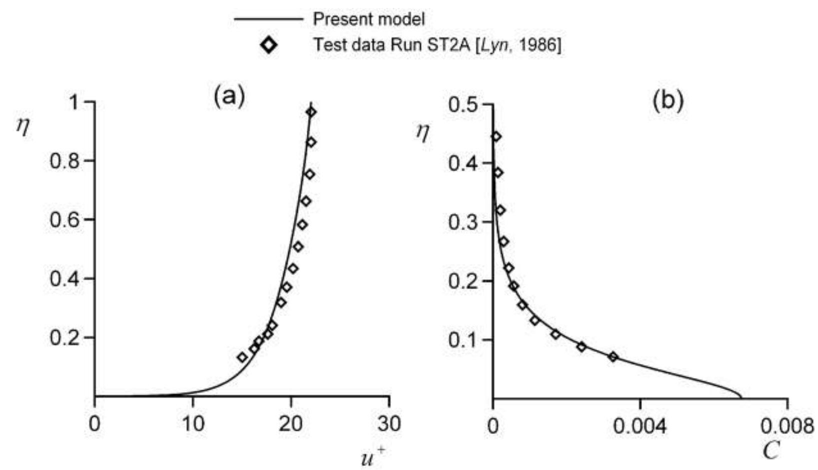

Figure 9. (a) Comparison of results obtained from the equation (37) for the velocity profile $u^+ = u^+(\eta)$ with test data by *Lyn* [1986], ST2A. (b) Comparison of fitted equation (41) for the concentration profile $C = C(\eta)$ with test data by *Lyn* [1986].

empirically a mean density stratification effect. Run 23 of *Montes* [1973] is plotted in Figures 8d–8f. For this experiment, C_a = 0.05 and η_a = 0.0296. Taking $\eta_{\max}$ = 0.4 (Figure 8f) a value $\overline{\kappa_m} = 0.333$ results comparable to κ_{fit} = 0.325 (Figure 8f). The results are analogous to run 20 and therefore confirm the interpretation offered here.

[29] The experiment ST2A from *Lyn* [1986] is plotted in Figures 9a and 9b. For this experiment, C_a = 0.0032 and η_a = 0.071, and the value Π_o = 0.2 was used. The concentration profile is plotted in Figure 9b, whereas Figure 9a compares predicted and measured velocity profiles. The predicted velocity profile is very close to the observations, supporting the predictions of the proposed model.

6. Effect of Velocity Profile on Suspended Load

[30] The turbulent velocity profile in flows with sediment suspension has been a subject of major theoretical and practical interest in the past 50 years. One of the major reasons is that the velocity and concentration profiles allow the prediction of the sediment discharge, q_s, [*McLean*, 1992; *Garcia*, 2008]

$$q_s = \int_0^h uCdy \tag{46}$$

Since the velocity u affects the computation of q_s, equation (46), a precise definition of the turbulent velocity profile of sediment-laden flows is relevant for the description of earth surface processes. For example, river bed structures like ripples, dunes or antidunes are influenced by the action of the turbulent stream. Therefore, an improved description of u not only provides insights into the mechanics of sediment suspension but also permits a more accurate and rational computation of q_s. The normalized suspended load q_N is

$$q_N = \frac{\Delta q_s}{u^* C_o h} = \int_{n_2}^{n_1} u^+ \frac{C}{C_o} d\eta \tag{47}$$

The elevations are η_1 and η_2, and Δq_s is the suspended sediment load between them. To investigate the effect of u^+ on q_N, representative runs from the data sets of *Coleman* [1986], *Montes* [1973] and *Lyn* [1986] were selected (Table 1, runs 29, ST2A and 23, respectively). For each experiment, the layer η_1 was selected as the first elevation at which experimental measurements were taken, η_a. The upper limit of integration η_2 was selected in order to have simultaneous measurements of velocity and concentration profiles. Equation (47) was evaluated numerically using the measured u^+ and C/C_o for each run. The computation are presented in Table 1 and labeled as q_{Nexp}. Computations using velocity profiles u^+ predicted by equation (37) are also shown in Table 1 as $q_{N\text{th}}$. The clear-water log-law velocity profile

$$u^+ = \frac{1}{0.41} \ln\left(\frac{yu^*}{\nu}\right) + 5.5 \tag{48}$$

is frequently used for computation of q_s [*Guo and Wood*, 1995]. Equation (48) was used into equation (47) for the same 3 experiments and the results are presented in Table 1. Results presented in Table 1 confirm that the proposed velocity profile model produces suspended load estimations reasonably close to the experimental values, with a maximum deviation of −4.11%. In contrast, the log-law model induces a maximum deviation of −26.66%.

7. Discussion of Results in the Context of Velocity Profile Modeling

[31] Early studies [*Vanoni*, 1946; *Einstein and Chien*, 1955] proposed that suspended sediment induces a damping of turbulence based on indirect evidence. An indirect bulk measure of the turbulence damping was given by the von Kármán parameter κ, understanding it as the inverse of the slope of the best fit line to experimental data in the plane u^+-lnη. However, *Coleman* [1981, 1986], based on his experiments, concluded that κ = 0.41 and that sediments affect the outer layer turbulence structure by an increased wake function. Currently, the research community is divided on this point. A more rational framework to investigate the

Table 1. Effect of Velocity Profile u^+ in Suspended Load Computations

Run	$q_{N\mathrm{exp}}$	$q_{N\mathrm{th}}$		Error (%)	
		equation (37)	equation (48)	equation (37)	equation (48)
29 [*Coleman*, 1986]	2.88	2.85	3.15	1.04	−9.47
ST2A [*Lyn*, 1986]	1.29	1.34	1.65	−4.11	−26.66
23 [*Montes*, 1973]	0.55	0.53	0.64	2.23	−16.64

turbulent velocity profile relies on understanding the turbulent momentum transfer in sediment-laden flow [*Yang*, 2007]. Consideration of the density gradient effects on the turbulence momentum transfer by a generalized mixing length theory enables the formulation of a more general turbulent velocity profile for sediment-laden flows without any pre-assumed hypothesis about κ. The mixing length formulation proposed in this work allows the definition of a function which is variable with its distance to the wall and dependent on density stratification. This κ function can be defined as a modified von Kármán parameter, κ_m. This does not mean that a priori it is assumed that κ varies with distance to the wall, and it is a theoretical result different from previous log-wake studies. Note that the log-law [*Montes*, 1973] or the log-wake law [*Guo and Julien*, 2001] are velocity profile functions that are empirically fitted to experimental data. In contrast, the proposed equation (37) is a mechanistic model based on the turbulent momentum transfer with suspended sediment. It therefore introduces rationality in the computations based on physical arguments. Further, the use of the log-law [*Montes*, 1973] or the log-wake law [*Guo and Julien*, 2001] requires experimental measurements. In contrast, the modeled velocity profile for sediment-laden flows uses the concentration profile as an input but its equation (37) is free from other fitting parameters. The model provides a theoretical computation of the velocity defect profiles Z with no need for velocity data. An estimation of U^+ from flow resistance equations permits the determination of the absolute velocity $u^+ = U^+ - Z$ without resorting to experimental measurements. Thereafter, the proposed model permits the theoretical simulation of the velocity profile, thereby simplifying practical computations. In addition, previous stratified flow models [*Smith and McLean*, 1977; *Villaret and Trowbridge*, 1991; *Mazumder and Ghoshal*, 2006; *Wright and Parker*, 2004a, 2004b] obtained the parameter α for the damping of eddy-viscosity in equation (27) by fitting it to experimental data. In contrast, our model provides an analytical predictor for it (see equation (32)).

8. Conclusions

[32] In this study, the effects of sediment suspension on turbulent momentum transfer have been investigated using a new mixing length model. A new local damping factor ψ arose from the modified mixing length model for sediment-laden flows. This damping factor is given by a function of the distance to the wall that depends on the velocity and concentration profiles and their gradients. The damping factor allows one to define a von Kármán parameter as a local variable function of the distance to the wall. The velocity profile obtained is based on theoretical considerations, and it is the major difference as compared to the log-wake model [*Coleman*, 1981; *Valiani*, 1988] or the log-law model [*Montes*, 1973; *Gust*, 1984], where the name "von Kármán constant" is given to a free adjustable parameter determined by fitting to observations.

[33] The prediction of velocity from the proposed model agrees well with experimental observations. Thus, the defined von Kármán parameter as a function of the distance to the wall describes the turbulent momentum transfer with suspended sediment. The proposed model is further compared with a more general model using CFD simulation of turbulent sediment-laden flows, again resulting in good agreement. The model proposed is a good estimator of the velocity profile, as confirmed by experimental data. This requires integration of a first order differential equation, which can be achieved by using standard and well-known techniques. The model application is more complex than the use of a simple log-wake model, but, in contrast, the latter requires the calibration of the κ_{fit} parameter whereas our model simulates density stratification by theoretical means, which is an advantage. The numerical solution of our first order differential equation is simpler than programming the solution of the two-dimensional RANS equations using the k-ε closure, making the proposed model suitable for a broader audience. The improved velocity profile proposed can also be used to obtain more accurate estimations of the suspended load.

[34] The theoretical results obtained with the proposed model have been applied to analyze log-wake data fitting methods. It was found that if near-wall data are used, the conclusions of *Coleman* [1981, 1984] are generally supported, resulting in $\kappa \approx 0.41$ and a wake strength Π_s dependent on $\overline{\mathsf{R}}_i$. However, it is shown that this result does not imply that suspended sediment affects the turbulent structure of the outer layer. Further, it has been observed that when the whole wall-layer is considered, the fitted von Kármán parameter in a log-wake model abruptly drops below 0.41. This is a measure of density stratification effects, also reflected by the minimum values of the depth-distribution curve of $\kappa_m = \kappa_m(\eta)$ of the numerical model.

[35] The new density stratification theoretical results presented back up the experimental findings of *Valiani* [1988] and *Guo and Julien* [2001] that $\kappa_{fit} < 0.41$. Their results are an average measure in statistical terms of the density stratification of the flow, whereas this model provides a physical interpretation of their results. The density stratification effect causes a damping of the mixing length which mainly affects the momentum transfer in the wall-layer. This effect is theoretically approximated by the new von Kármán parameter distribution across the flow depth. The empirically fitted reduced κ_{fit} found by other researchers can be mechanistically interpreted in the light of our new von Kármán parameter: κ_{fit} is below 0.41 when the log-wake law is fitted to

experimental data because of the damping of turbulence near the wall, that is, $\psi < 1$ or $\kappa_s = (0.41\psi) < 0.41$. This indicates that the log-wake model might be considered reliable for sediment-laden flows if it is carefully calibrated by using test data in the whole stratified water depth.

[36] The present results overcome the conceptual problem regarding the von Kármán parameter and its relation with suspended sediment: Does the suspended sediment affect κ, or not? From our model results we highlight that a von Kármán parameter can be defined as a turbulence variable in a mixing length differential model of the velocity profile. The effect of $\kappa_m = \kappa_m(\eta)$ on log-wake law matching to test data is indirectly accounted for by setting $\kappa_{fit} = 0.41$ and $\Pi_s(\overline{\mathsf{R}}_i)$, following *Coleman* [1986], or $\kappa_{fit}(\overline{\mathsf{R}}_i)$ and $\Pi_s(\overline{\mathsf{R}}_i)$, according to *Valiani* [1988] and *Guo and Julien* [2001]. The *Coleman* [1986] approximation is not a coherent solution for the velocity profile, and it is, therefore, not recommended for practical applications on sediment transport. Our model results show that a physically coherent log-wake fit should have $\kappa_{fit} < 0.41$, and, in short, the suspended sediment affects κ.

[37] **Acknowledgments.** The good improvements and suggestions offered by the Chief Editor A. Densmore and the Associate Editor J.M. Buffington are gratefully acknowledged. Four anonymous reviewers and Junke Guo provided useful critiques and comments to improve the manuscript. The critical review and kind support received from Neville Robinson, Flinders University, Australia, is greatly appreciated. This research was partially funded by the project P08-AGR-03925 (Junta de Andalucía).

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