Tension between scalar/pseudoscalar new physics contribution to $B_s \to \mu^+\mu^-$ and $B \to K\mu^+\mu^-$

Ashutosh Kumar Alok,[1] Amol Dighe,[1] and S. Uma Sankar[2]

[1]*Tata Institute of Fundamental Research,*
Homi Bhabha Road, Mumbai 400005, India
[2]*Indian Institute of Technology Bombay, Mumbai-400076, India*

New physics in the form of scalar/pseudoscalar operators cannot lower the semileptonic branching ratio $B(B \to K\mu^+\mu^-)$ below its standard model value. In addition, we show that the upper bound on the leptonic branching ratio $B(B_s \to \mu^+\mu^-)$ sets a strong constraint on the maximum value of $B(B \to K\mu^+\mu^-)$ in models with multiple Higgs doublets: with the current bound, $B(B \to K\mu^+\mu^-)$ cannot exceed the standard model prediction by more than 2.5%. The conclusions hold true even if the new physics couplings are complex. However these constraints can be used to restrict new physics couplings only if the theoretical and experimental errors in $B(B \to K\mu^+\mu^-)$ are reduced to a few per cent. The constraints become relaxed in a general class of models with scalar/pesudoscalar operators.

One of the major aims of the large hadron collider (LHC), about to start operating soon, is to look for Higgs particles within and beyond the standard model (SM). Even a direct observation of a Higgs particle will not suffice to tell us whether it is the SM Higgs or not. An understanding of possible scalar/pseudoscalar new physics (SPNP) interactions through indirect means is therefore extremely crucial.

The flavor changing neutral interaction $b \to s\mu^+\mu^-$ serves as an important probe to test higher order corrections to the SM as well as to constrain many new physics models. This four-fermion interaction is responsible for the purely leptonic decay $B_s \to \mu^+\mu^-$, for the semileptonic decays $B \to (K, K^*)\mu^+\mu^-$ and also for the radiative leptonic decay $B_s \to \mu^+\mu^-\gamma$. The semileptonic decays have been experimentally observed at BaBar and Belle [1, 2, 3, 4]. The pseudoscalar semileptonic decay has the branching ratio

$$B(B \to K\mu^+\mu^-) = (4.2^{+0.9}_{-0.8}) \times 10^{-7} \, , \tag{1}$$

which has been obtained with ~ 350 fb^{-1} of data. These values are consistent with the SM predictions [6, 7, 8, 9], and the experimental errors are expected to reduce to $\sim 2\%$ at the forthcoming Super-B factories [10]. At the moment there is about 20% uncertainty in these SM predictions

due to the error in the quark mixing matrix element V_{ts} and the uncertainties related to strong interactions. Improvements in the lattice calculations and the measurement of V_{ts} are likely to bring this error down to a few per cent within the next decade.

The purely leptonic decay $B_s \to \mu^+ \mu^-$ is highly suppressed in the SM, the prediction for its branching ratio being $(3.35 \pm 0.32) \times 10^{-9}$ [11]. The uncertainty in the SM prediction is mainly due to the uncertainty in the decay constant f_{B_s} and V_{ts}. This decay is yet to be observed in experiments. Recently the upper bound on its branching ratio has been improved to [12]

$$B(B_s \to \mu^+ \mu^-) < 0.58 \times 10^{-7} \quad (95\% \text{ C.L.}) , \tag{2}$$

which is still more than an order of magnitude away from its SM prediction. The decay $B_s \to \mu^+ \mu^-$ will be one of the important rare B decay channels to be studied at the LHC and we expect that the sensitivity of about 10^{-9} can be reached in a few years [13].

In the context of these decays, one needs to focus only on new physics from scalar/pseudoscalar interactions, since (i) new physics in the form of vector/axial-vector operators is highly constrained by the data on $B \to (K, K^*) \mu^+ \mu^-$ as shown in [14], and (ii) new physics in the form of tensor and magnetic dipole operators does not contribute to $B(B_s \to \mu^+ \mu^-)$. A measured value of $B(B_s \to \mu^+ \mu^-) \gtrsim 10^{-8}$ indicates that the new physics must be in the form of scalar/pseudoscalar operators.

We take the effective Lagrangian for the four-fermion transition $b \to s \mu^+ \mu^-$ to be [6]

$$L(b \to s \mu^+ \mu^-) = L_{SM} + L_{SP} , \tag{3}$$

where

$$L_{SM} = \frac{\alpha G_F}{\sqrt{2}\pi} V_{tb} V_{ts}^\star \left\{ C_9^{\text{eff}} (\bar{s} \gamma_\mu P_L b) \, \bar{\mu} \gamma_\mu \mu + C_{10} (\bar{s} \gamma_\mu P_L b) \, \bar{\mu} \gamma_\mu \gamma_5 \mu + 2 \frac{C_7^{\text{eff}}}{q^2} m_b \, (\bar{s} i \sigma_{\mu\nu} q^\nu P_R b) \, \bar{\mu} \gamma_\mu \mu \right\} , \tag{4}$$

$$L_{SP} = \frac{\alpha G_F}{\sqrt{2}\pi} V_{tb} V_{ts}^\star \left\{ \tilde{R}_S \, (\bar{s} \, P_R \, b) \, \bar{\mu} \, \mu + \tilde{R}_P \, (\bar{s} \, P_R \, b) \, \bar{\mu} \gamma_5 \mu \right\} . \tag{5}$$

Here $P_{L,R} = (1 \mp \gamma_5)/2$ and q is the sum of the μ^+ and μ^- momenta. $\tilde{R}_S$ and $\tilde{R}_P$ are the scalar and pseudoscalar new physics couplings respectively, which in general can be complex. We use the notation $\tilde{R}_S \equiv R_S e^{i\delta_S}, \tilde{R}_P \equiv R_P e^{i\delta_P}$. Here the phases are restricted to be $0 \leq (\delta_S, \delta_P) < \pi$, whereas R_S and R_P can take positive as well as negative values. Within SM, the Wilson coefficients in Eq. (4) have the following values [6]:

$$C_7^{\text{eff}} = -0.310 , \; C_9^{\text{eff}} = +4.138 + Y(q^2) , \; C_{10} = -4.221, \tag{6}$$

where the function $Y(q^2)$ is given in [15]. These coefficients have an uncertainty of about 5%, which arises mainly due to their scale dependence.

In Eq. (5), we have taken only P_R in the quark bilinear, while the most general Lagrangian must have a linear combination of P_L and P_R. Here we start by considering the simpler case because SPNP operators mostly arise due to multiple Higgs doublets. In such models, the coefficient of P_R in the Lagrangian is much larger than that of P_L [6]. In two Higgs doublet model, for instance, the coefficient of P_L is smaller by a factor of m_s/m_b [16]. We shall examine the consequences of considering the most general quark bilinear in the latter part of this Letter.

In the following, we consider the interrelations between the contributions of L_{SP} to the branching ratios of the decays $B_s \to \mu^+\mu^-$ and $B \to K\mu^+\mu^-$. The effect of SPNP couplings on additional observables related to these decays, viz. forward-backward asymmetry in the semileptonic decay and the polarization asymmetry in the leptonic decay, has been studied in [5]. The contribution of L_{SP} to $B \to K^*\mu^+\mu^-$ is so small [6] that no worthwhile correlation can be established between it and other decays. Also, L_{SP} does not contribute to the radiative leptonic decay $B_s \to \mu^+\mu^-\gamma$ [17, 18].

We first consider the contribution of L_{SP} to the decay rate of $B_s \to \mu^+\mu^-$. The branching ratio is given by

$$B_{SP}(B_s \to \mu^+\mu^-) \; = \; \frac{G_F^2 \alpha^2 m_{B_s}^3 \tau_{B_s}}{64\pi^3} |V_{tb}V_{ts}^*|^2 f_{B_s}^2 (R_S^2 + R_P^2) \; . \tag{7}$$

Taking $f_{B_s} = (0.259 \pm 0.027)$ GeV [19], we get

$$B_{SP}(B_s \to \mu^+\mu^-) \; = \; (1.43 \pm 0.30) \times 10^{-7} \, (R_S^2 + R_P^2) \; . \tag{8}$$

Note that the present experimental upper limit on $B(B_s \to \mu^+\mu^-)$ is an order of magnitude larger than the SM prediction. In the following, we will assume that the SPNP will provide an order of magnitude increase of $B(B_s \to \mu^+\mu^-)$. In such a situation, the SM amplitude can be neglected in the calculation of the branching ratio. Equating the expression in Eq. (8) to the present 95% C.L. upper limit in Eq. (2), we get the inequality

$$(R_S^2 + R_P^2) \leq 0.70 \; , \tag{9}$$

where we have taken the 2σ lower bound for the coefficient in Eq. (8). Thus, the allowed region in the R_S–R_P parameter space is the interior of a "leptonic" circle of radius $r_\ell \approx 0.84$ centered at the origin, as indicated in both the panels of Fig. 1. As the upper bound on $B(B_s \to \mu^+\mu^-)$ goes down, the radius of the circle will shrink.
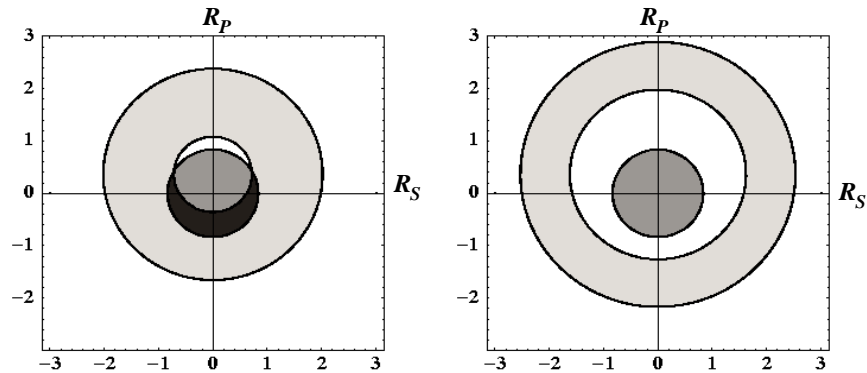
FIG. 1: The allowed ranges of R_S and R_P, when the new physics couplings are real. In both figures, the dark grey circles centered at origin represent the regions allowed by the current 2σ upper bound on $B(B_s \to \mu^+\mu^-)$. The light grey annulus in each figure represents the parameter space allowed by $B(B \to K\mu^+\mu^-)$ at 2σ. The width of the annulus corresponds to the sum of the theoretical and experimental errors, both of which are taken to be 2%. In the left panel, we take $B(B \to K\mu^+\mu^-) = (5.64 \pm 0.11) \cdot 10^{-7}$. The overlap between the allowed regions is represented by the black crescent. In the right panel we take $B(B \to K\mu^+\mu^-) = (6.04 \pm 0.12) \cdot 10^{-7}$, where the allowed parameter spaces do not overlap.

We now turn to the semileptonic decay $B \to K\mu^+\mu^-$. The measured branching ratio is consistent with the SM prediction, though there is a 25% error in the measurement and about 20% error in the theoretical prediction due to uncertainties in V_{ts}, form factors and Wilson coefficients (which in turn depend on V_{ts}). With the addition of the SPNP contribution, the theoretical prediction for the net branching ratio becomes [6]

$$B(B \to K\mu^+\mu^-) \;=\; \left[5.25 + 0.18(R_S^2 + R_P^2) - 0.13 R_P \cos\delta_P\right](1 \pm 0.20) \times 10^{-7}\,, \qquad (10)$$

In Eq. (10), the first term is purely due to the SM, the second term is purely due to SPNP and the third term is due to the interference of the two. The theoretical errors arise from one tensor and two vector form factors in the SM, and a scalar form factor in SPNP (which is related to one of the SM vector form factors). We have made the simplifying assumption that the fractional uncertainties in all the form factors are the same.

Eq. (10) can be rewritten as

$$B(B \to K\mu^+\mu^-) = (1 + \epsilon)B_{\rm SM}\,, \qquad (11)$$

where ϵ is the fractional change in the branching ratio due to SPNP. The maximum negative value that ϵ can take is -0.005, thus implying that the SPNP new physics cannot lower the branching ratio $B(B \to K\mu^+\mu^-)$ by more than 0.5% below its standard model value. Indeed, if the theoretical

and experimental errors in this quantity were improved to 5%, with the central values unchanged, the discrepancy cannot be accounted for by SPNP at 2σ.

Let us first consider the case where the new couplings R_S and R_P are real, which is typical for the class of models where the only charge-parity violation comes from the CKM matrix elements. Using Eqs. (1) and (10), we get

$$R_S^2 + (R_P - 0.36)^2 = \frac{B_{\text{exp}}}{(0.18 \pm 0.036) \times 10^{-7}} - 29.04 \ , \tag{12}$$

where B_{exp} is the measured value of $B(B_s \to K\mu^+\mu^-)$. The region in the R_S–R_P plane allowed by the measurement of $B(B_s \to K\mu^+\mu^-)$ is then an "semileptonic" annulus centered at $(0, 0.36)$, as shown in both the panels of Fig. 1. The inner and outer boundaries of this region correspond to the lower and upper bounds of the right hand side of Eq. (12). The right hand side turns out to be negative if B_{exp} is below the SM prediction by more than 0.5%. Then the radius of the circle becomes imaginary, which implies that the discrepancy of the measurement with the SM cannot be explained by SPNP.

To illustrate the tension between the quantities $B(B_s \to \mu^+\mu^-)$ and $B(B \to K\mu^+\mu^-)$, we consider the scenario where the errors in both B_{SM} and B_{exp} have been reduced to 2%, while keeping the upper limit on $B(B_s \to \mu^+\mu^-)$ at its current value. The allowed R_S–R_P parameter space is shown in Fig. 1. If the lower limit on B_{exp} is small enough, the semileptonic annulus will overlap with leptonic circle, as shown in the left panel. However, if the lower limit on B_{exp} is larger than a critical value (determined by the bound on the leptonic branching ratio), then there is no region of overlap as shown in the right panel. In such a situation, the difference between B_{exp} and B_{SM} cannot be accounted for by SPNP because of the constraint coming from the leptonic mode.

We represent the radius of the leptonic circle by r_ℓ and the inner (outer) radius of the semileptonic annulus by r_{in} (r_{out}). There is tension between the two measurements if

$$r_{in} - r_\ell > 0.36 \ , \tag{13}$$

in which case the regions allowed by the two branching ratios do not overlap. Given the current value of $r_\ell = 0.84$, we require $0 < r_{in} < 1.2$ for an overlap. This implies that the 2σ lower limit on B_{exp} should be between 4.93×10^{-7} and 5.67×10^{-7}. (We have added the theoretical and experimental errors in quadrature.) If the upper bound on $B(B_s \to \mu^+\mu^-)$ is improved by a factor of 5, the 2σ range for the lower limit on B_{exp} would be $(4.93 - 5.57) \times 10^{-7}$. For the tension to be manifest in future experiments, the reduction of errors in B_{exp} and B_{SM} is the most crucial.

When $\tilde{R}_S$ and $\tilde{R}_P$ are complex, the constraint Eq. (12) becomes

$$R_S^2 + (R_P - 0.36\cos\delta_P)^2 = \frac{B_{\text{exp}}}{(0.18\pm0.036)\times10^{-7}} - 29.17 + (0.36\cos\delta_P)^2 . \tag{14}$$

For nonzero δ_P, the center of the semileptonic annulus shifts along the R_P axis, while the radius of the annuli are almost unchanged. If the allowed regions do not overlap for $\delta_P = 0$ (as illustrated in the right panel of Fig. 1), then they will not overlap for any value of δ_P. Hence the tension between $B(B_s \to \mu^+\mu^-)$ and $B(B \to K\mu^+\mu^-)$ persists, and gives rise to the same constraints on the semileptonic branching ratio even if the SPNP couplings are complex.

In writing the effective SPNP Lagrangian in Eq. (5), we considered only the quark bilinear $\bar{s}P_R b$. Lorentz Invariance of the Lagrangian also allows the bilinear $\bar{s}P_L b$ in general. We can take this generalization into account by replacing $\bar{s}P_R b$ by $\bar{s}(\alpha P_L + P_R)b$, where α is the strength of the $\bar{s}P_L b$ bilinear relative to that of $\bar{s}P_R b$. With this modification, $B(B \to K\mu^+\mu^-)$ is driven by the sum of the two quark bilinears with different chiralities, whereas $B(B_s \to \mu^+\mu^-)$ depends on their difference [20]. The expressions for the branching ratios of the two processes considered here are:

$$B(B_s \to \mu^+\mu^-) = (1-\alpha)^2(R_S^2 + R_P^2)\,(1.43\pm0.30)\times10^{-7}, \tag{15}$$

$$B(B \to K\mu^+\mu^-) = \left[5.25 + 0.18\,(1+\alpha)^2\,(R_S^2 + R_P^2) - 0.13\,(1+\alpha)R_P\right]\,(1\pm0.20)\times10^{-7}. \tag{16}$$

Here we have taken R_S, R_P and α to be real for simplicity. For $\alpha = 0$, Eqs. (15) and (16) reduce to Eqs. (8) and (10) respectively. For the special case $\alpha = 1$, the new physics has no contribution to $B_s \to \mu^+\mu^-$ because the quark bilinear is pure scalar and the corresponding pseudoscalar meson to vacuum transition matrix element is zero. In such cases, $B(B_s \to \mu^+\mu^-)$ is entirely due to the SM, and provides no constraints on $B(B \to K\mu^+\mu^-)$.

In Fig. 2, we show ϵ_{max}, the maximum fractional deviation of $B(B \to K\mu^+\mu^-)$ from its SM prediction as defined in Eq. (11), as a function of the 2σ upper bound on $B(B_s \to \mu^+\mu^-)$. The minimum allowed value of ϵ is almost independent of the value of α and the leptonic upper bound, and is approximately -0.005. For the class of models with multiple Higgs doublets, $\alpha = 0$, and the maximum value of ϵ is restricted to $+0.025$, as seen in earlier discussions. With the additional freedom generated by the extra parameter α, this severe constraint is relaxed. For example, for the models with $\alpha \approx 1.5$, the value of ϵ may be as large as $+0.7$, as can be seen in the figure. In general for positive α values, ϵ_{max} increases with α for $\alpha < 1.0$, and decreases thereafter. When $\alpha < 0$, Eq. (15) indicates that the constraints on R_S and R_P should become more restrictive. As a result, ϵ is constrained to be even smaller. From the figure, ϵ_{max} for negative α are seen to be very close to zero, and the corresponding ϵ_{max} curves are almost overlapping. This implies that for negative α, any significant deviation of $B(B \to K\mu^+\mu^-)$ from SM is impossible with SPNP.
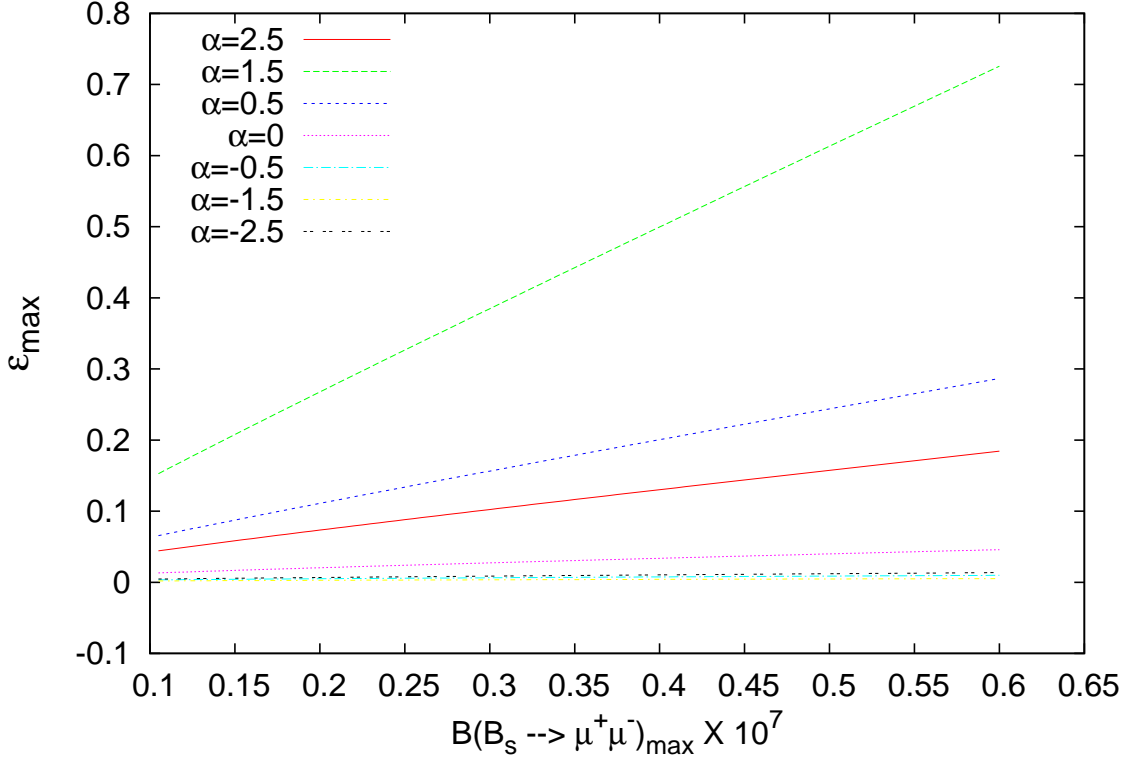
FIG. 2: $\epsilon_{\max}$ as a function of the 2σ upper bound on $B(B_s \to \mu^+\mu^-)$ for different values of α. We have taken the theoretical error on $B(B_s \to \mu^+\mu^-)$ to be 20%; decreasing it would further constrain $\epsilon_{\max}$.

For the measurements of $B(B_s \to \mu^+\mu^-)$ and $B(B \to K\mu^+\mu^-)$ to be compatible with SPNP, the lower bound on $B(B \to K\mu^+\mu^-)$ should be less than $(1 + \epsilon_{\max})B_{\mathrm{SM}}$. Thus, the upper bound on $B(B_s \to \mu^+\mu^-)$ and the lower bound on $B(B \to K\mu^+\mu^-)$ allow us to constrain the value of α in a class of models that involve new physics scalar/pseudoscalar couplings.

In this letter, we have parameterized scalar/pseudoscalar new physics in terms of the effective operators given in Eq. (5). In general, the introduction of new scalar/pseudoscalar fields into a model leads to not only new effective operators but also modification of the coefficients of the SM operators, e.g. the Wilson coefficients C_7, C_9 and C_{10} shown in Eq. (4). However, it has been shown that these modifications due to new scalar/pseudoscalar fields are very small [16, 21]. We have computed these changes in the two Higgs doublet model and found them to be at most 1%. Thus, our assumption of retaining the SM values for the Wilson coefficients, even in the presence of new scalar/pseudoscalar fields, is valid.

In summary, we have shown that in a class of models with new scalar/pseudoscalar operators, which includes models with multiple Higgs doublets, the SPNP couplings are strongly constrained

by the upper bound on $B(B_s \to \mu^+\mu^-)$, and in turn restrict the allowed values of $B(B \to K\mu^+\mu^-)$ to within a narrow range around its SM prediction. Future precise measurements of these two branching ratios have the potential not only to give an evidence for new physics, but also to reveal the nature of its Lorentz structure. However in order to achieve this, the theoretical as well as experimental errors on $B(B \to K\mu^+\mu^-)$ need to be reduced to a few per cent.

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