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# The propagation and backscattering of soliton-like pulses in a chain of quartz beads and related problems. (II). Backscattering

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## Abstract

We demonstrate that the propagation of solitons, soliton-like excitations and acoustic pulses discussed in the preceding article (M. Manciu, S. Sen and A.J. Hurd, Physica A, preceding article) can be used to detect buried impurities in a chain of elastic grains with Hertzian contacts. We also present preliminary data for 3D granular beds, where soliton-like objects can form and can be used to probe for buried impurities, thus suggesting that *soliton-pulse spectroscopy* has the potential to become a valuable tool for probing the structural properties of granular assemblies. The effects of restitution are briefly discussed. We refer to available experiments which support our contention. © 1999 Elsevier Science B.V. All rights reserved.

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## 1. Introduction

In the preceding article [1], hereafter referred to as I, we have discussed the properties of solitons and soliton-like objects that can be initiated in monodisperse chains of elastic grains, where the grains are in mutual contact. Let us now consider the problem in which the chain possesses an impurity mass at some position. The impurity mass can be defined by distinct coupling constants characterized by different  $a(R_{\text{imp}}, \text{imp})$ 's in Eq. (1) of I [1], where  $R_{\text{imp}}$  and  $\text{imp}$  denote the radius and the Young's modulus of the impurity grain or by a different mass compared to the mass of the host grains or both.

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One can now ask the question, what happens to a propagating soliton or soliton-like pulse as it passes through the impurity

This is precisely the question that we address in the present article. We discuss the equations of motion in Section 2. The results on backscattering of solitons and soliton-like objects from an impurity in 1D systems are presented in Section 3. We briefly discuss the corresponding problem in 3D beds in Section 4. Our results are summarized in Section 5.

## 2. Relevant equations

We solve the dynamical problem for the following system of equations:

$$m_i \ddot{r}_i = n[a_{i,i-1}\{\Delta_0 - (r_i - r_{i-1})\}^{n-1} - a_{i,i+1}\{\Delta_0 - (r_{i+1} - r_i)\}^{n-1}], \quad (1)$$

where the equation describes the grain displacements and accelerations in vector form. The quantity  $a_{ij}$  is described in Eq. (1) of I [1]. We define  $\Delta_0 \equiv R_i + R_{i+1} - l_0$ , where  $R_i$  and  $R_{i+1}$  are used generically to describe the distance between adjacent grains in a polydisperse chain and  $l_0$  is a parameter that describes external loading on the chain that could decrease the distance between the centers of the grains.

We now set  $m_i = m$  for all masses except the impurity mass. For most of the work to be discussed in this article, we do not consider gravitationally loaded systems. In the few instances in which we use gravitational loading, we carry out the determination of the initial coordinates of the grains in the manner described in Section 2.2 of I [1] (see Eq. (4) in I). The calculations have been carried out using the velocity Verlet algorithm [2] as described in I.

The numerical constants used in the study can be summarized as follows. We have used Young's Modulus  $E = 7.87 \times 10^{10} \text{ N m}^{-2}$  for quartz,  $E = 2.0 \times 10^{11} \text{ N m}^{-2}$  for steel and  $E = 1.0 \times 10^{10} \text{ N m}^{-2}$  for plastic, and Poisson's ratios of  $\sigma = 0.144$  for quartz,  $\sigma = 0.300$  for steel and  $\sigma = 0.120$  for plastic. With these data, the constant  $a$  in the Hertz potential (see Eq. (1)) is  $3.39 \times 10^8 \text{ N m}^{-3/2}$  for quartz–quartz interaction,  $4.96 \times 10^8 \text{ N m}^{-3/2}$  for quartz–steel interaction and is  $7.60 \times 10^7 \text{ N m}^{-3/2}$  for quartz–plastic interaction. The grain mass for all except the impurity grain is  $m = 1.41 \times 10^{-6} \text{ kg}$ . The impurity mass will be specified with respect to the mass of the host grains.

We have already discussed the problem of impulse propagation in 1D systems in I. In the following section we consider the problem of backscattering of impulses and of acoustic pulses from impurities.

## 3. Backscattering of impulses from impurities horizontal and vertical columns

We address the dynamical process in which a pulse travels down a granular chain and encounters a light or a heavy mass impurity located at the mid-point of the chain. We first address the case in which  $g = 0$ . This discussion is followed by the study of the case in which  $g > 0$ .

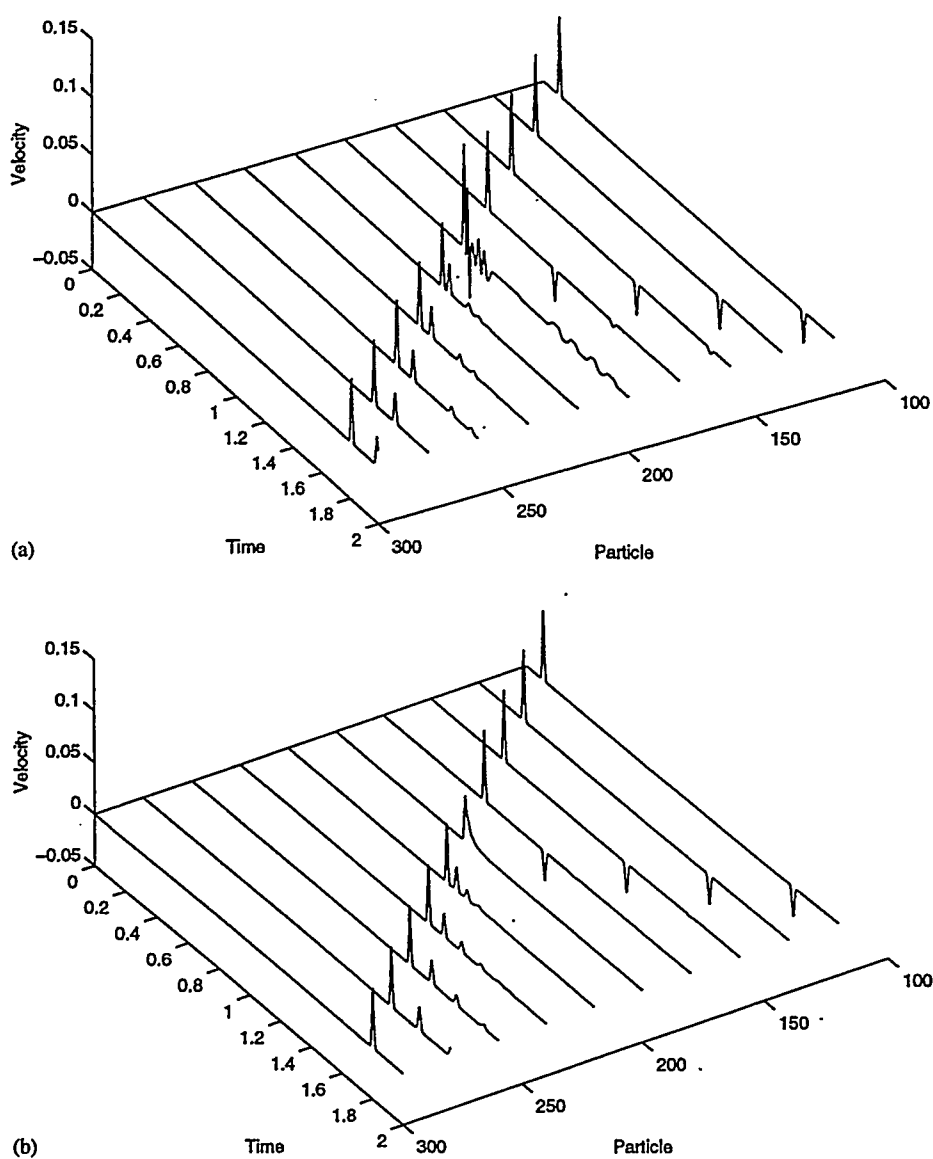


Fig. 1. Plot of velocity (in m/s), time (in s) and particle number (which measures depth) for an incoming soliton from the right that backscatters off (a) a light mass (see text) and (b) a heavy mass. Observe that the light mass in (a) produces secondary solitons while the heavier mass essentially yields a pure backscattered pulse.

*Case 1:  $g=0$ .* When  $g=0$ , there is no loading on the grains in the chain. Therefore, any impulse initiated in the chain will develop into a soliton [1].

Fig. 1(a) shows the velocity (in m/s) of a group of grains along the chain as functions of time (in s) and space (measured in number of grains or particles). We do

not show all the grains in the chain in the graph. The soliton takes a certain distance to form and hence we show the chain from grain number 100 from the surface. The 1st grain defines the surface of the chain. We use  $\eta$  to denote the displacement suffered by the first grain with respect to its initial equilibrium position when the impulse is initiated. For  $\eta \gg l_0$ , the pulse behaves like a robust soliton as is evident by taking note of the magnitudes of the maximum velocities suffered by the grains in the pulse. The soliton collides with a plastic sphere, i.e., a light impurity that is placed in a dense medium in the mid-point of the chain, and fragments into a propagating pulse with slightly lower energy and a reflected or backscattered pulse which eventually reaches the surface grain. The soliton pulse moves with uniform velocity in the Hertzian chain, and as indicated earlier, this velocity is dependent upon the magnitude of  $\eta/l_0$  and the system parameters.

It should be noted that there is a small soliton-like pulse (barely visible in Fig. 1(a)) that trails the first big pulse on all occasions in our simulations. We find that the energy imparted to the surface grain of the chain by the initial perturbation leads to the formation of a single large soliton and the residual energy, if any, is used by the system to form one or more smaller soliton pulses. This “quantization” of the perturbation energy into solitons at different energies is an interesting feature that we have encountered in all of our analyses and is poorly understood at the present time.

Fig. 1(b) depicts the same calculations as above except that they are for the case where the light impurity is replaced by a heavy impurity. Soliton signals, which are shock-like pulses, are too dominant to respond with characteristic differences when they encounter light or heavy impurities (compare with Fig. 1(a)). As we shall see, purely acoustic pulses, when backscattered, on the other hand are too weak to reveal significant information about what they hit.

It is of interest to probe the amount of acoustic energy that is reflected or perhaps backscattered by an impurity placed at the middle of the chain. Fig. 2 (upper panel) describes the ratio of the transmitted and reflected kinetic energies for a chain in which all the masses are kept equal but the material dependent coupling  $a$  (see Eqs. (1)–(2) in I [1]) is made to differ for the impurity mass which is named  $a$ . Clearly, for  $a = a$ , we do not expect any reflected energy and this is confirmed by the data. When  $a \rightarrow \infty$ , the period of oscillation of the impurity grain becomes vanishingly small and thus high-frequency oscillations are to be expected. Such oscillations are largely overwhelmed by the traveling impulse and hence there is only modest reflection that is possible. The data reveal that the ratio of the reflected to transmitted energies level off to about 0.2. When  $a/a$  becomes very small, the period of oscillation of the impurity is large compared to that of the rest of the masses. The energy of the soliton gets “temporarily trapped” in the impurity. The rattling of the impurity spawns secondary solitons, which result in large reflection from the impurity. Further studies need to be carried out to better understand soliton reflection in this regime. Such analyses is difficult to perform numerically because long time dynamical calculations become a necessity. Given that  $a$  is controlled by  $E$ ,  $\sigma$  and grain size, it may be possible

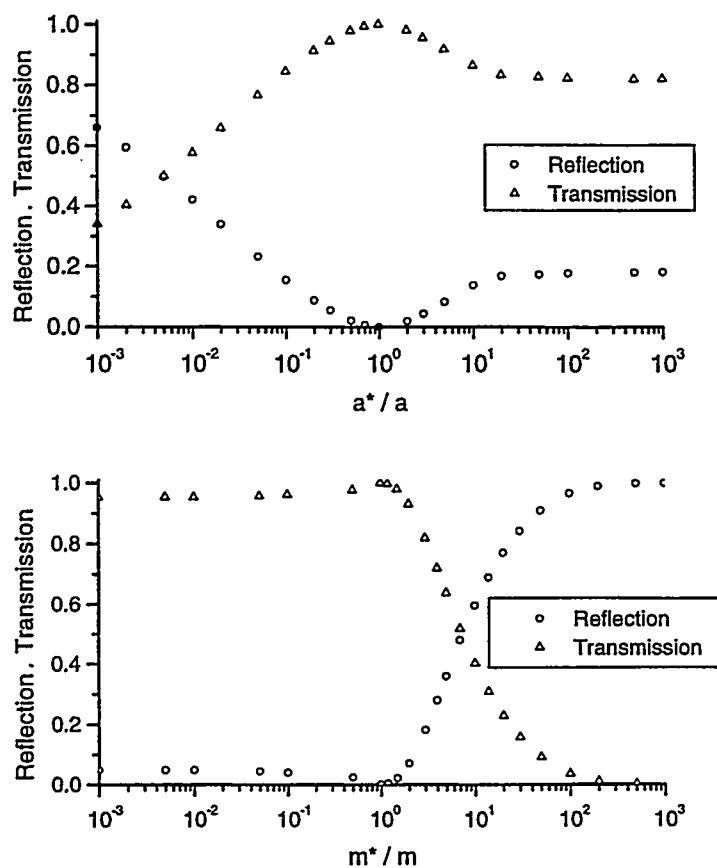
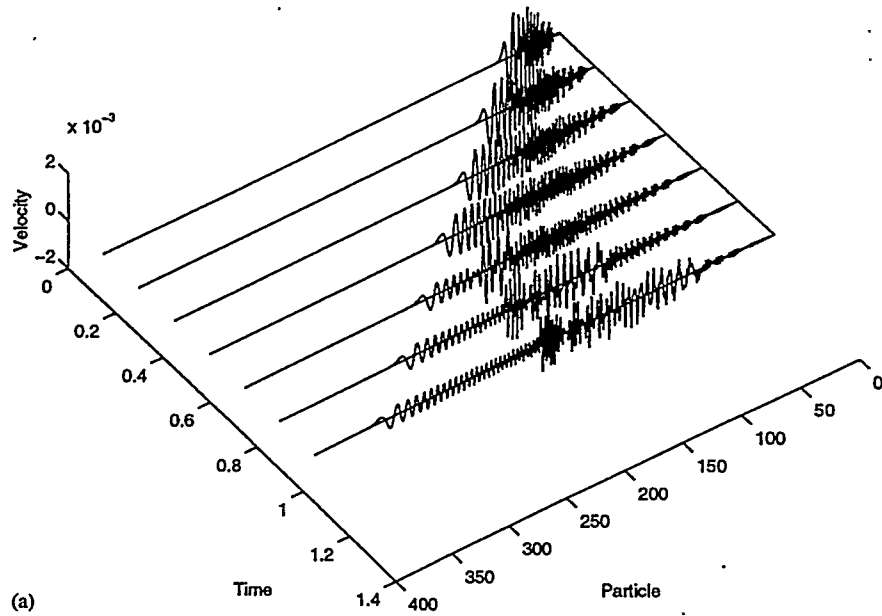


Fig. 2. (Upper panel) Plot of normalized reflected and normalized transmitted kinetic energies (see legend) for solitons as function of (upper panel)  $a^*/a$  and of (lower panel)  $m^*/m$ , where  $a^*$  and  $m^*$  are the quantities that allow calculation of the effective spring constant and the mass of the impurity, respectively.

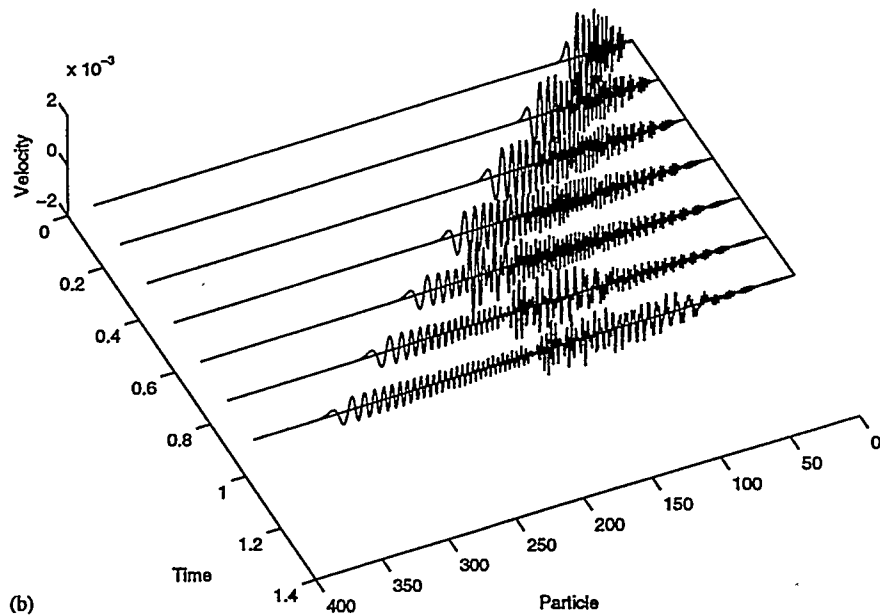
to engineer materials with very low  $a$ , which may turn out to be excellent shock reflectors.

When  $m/m \ll 1$ , one would expect that most of the energy in a strong impulse would easily pass through the light impurity. Indeed, we find that transmitted energy is rather large for small  $m/m$  as shown in the lower panel of Fig. 2. When  $m/m \gg 1$ , as expected, one obtains a high reflection. Because the details of soliton properties are dependent upon  $n$  and hence on contact geometry of the grains, one would expect that the behavior of  $a/a$  would be affected as one changes  $n$ . These studies will be published separately [3].

Figs. 3(a) and (b) describe the propagation and backscattering of purely acoustic signals from light and heavy impurities, respectively. Acoustic signals disperse quickly in granular chains (presumably also in granular beds) and hence it is inconvenient to decipher backscattered acoustic data. While our calculations show that the reflected



(a)



(b)

Fig. 3. Plot of velocity (in m/s), time (in s) and particle number (which measures depth) for an incoming acoustic impulse from the right that backscatters off (a) a light mass (see text) and (b) a heavy mass. The backscattered acoustic signals are typically too weak and noisy to allow easy detection of the properties of the backscatterer.



signals carry information about the density and position of the impurity, the data in this regime are highly noisy and we believe it will be cumbersome to obtain information about the impurity using purely acoustic signals.

*Case 2:  $g > 0$ .* The effect of gravitational loading is profound in Hertzian systems. Grains that are buried at progressively larger depths in such chains are more compressed compared to those at shallower depths. Hence the properties of the propagating impulse evolves in space and time.

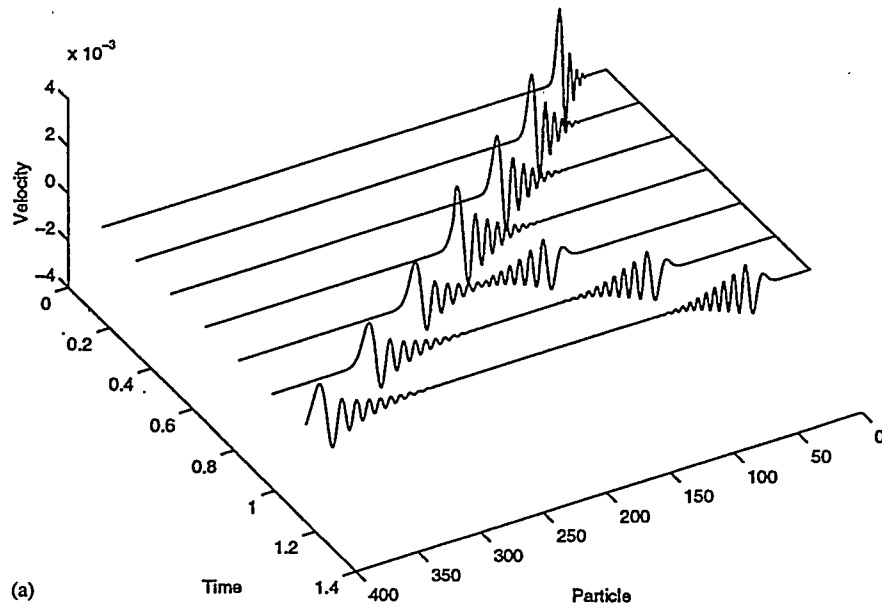
Figs. 4(a) and (b) describe the propagation and backscattering of weak (acoustic) impulses in a chain of Hertzian grains in the presence of gravitational loading. Observe that the magnitudes of the velocities are identical in Figs. 3(a), (b) and 4(a), (b). For strong impulses, one generates granular compressions that are significantly larger than the same due to gravitational loading. Therefore, the solitons that are generated by strong impulses are unaffected by gravity and are indistinguishable from the results shown in Figs. 1(a) and (b).

For intermediate-range impulses, the soliton-like pulses that are generated possess oscillatory tails as seen in Figs. 4(a) and (b) [4]. In all instances, we find that the propagating pulse interacts with the impurity, which is placed at the center of the granular chain, and splits up into a part that continues to move down the chain and another which is backscattered and which eventually reaches the surface. The backscattered pulse contains information about the impurity, which acts as the backscatterer. Thus, the backscattered pulses differ from one another depending upon whether the backscattering impurity is a lighter or a heavier mass with respect to the granular beads that make up the chain. The ratio of the amplitudes (of velocities) of the leading peaks of the backscattered and ingoing pulses is positive if the impurity mass is less than that of the grains in the chain and is negative if the impurity mass is larger as is evident from Figs. 3(a) and (b). Our preliminary analyses indicate that the ratio of the amplitudes of the leading peaks of the backscattered and ingoing pulses decay approximately linearly with  $m/m_0$ , where  $m$  is the impurity mass and  $m_0$  is the mass of the quartz grains in the chain, for  $0 < m/m_0 < 10$  [5]. The study of backscattering in the gravitationally loaded chain remains analytically intractable at the present time and more work must be done to acquire a deeper understanding of the physics of impulse propagation in these systems (see Ref. [6] for related work).

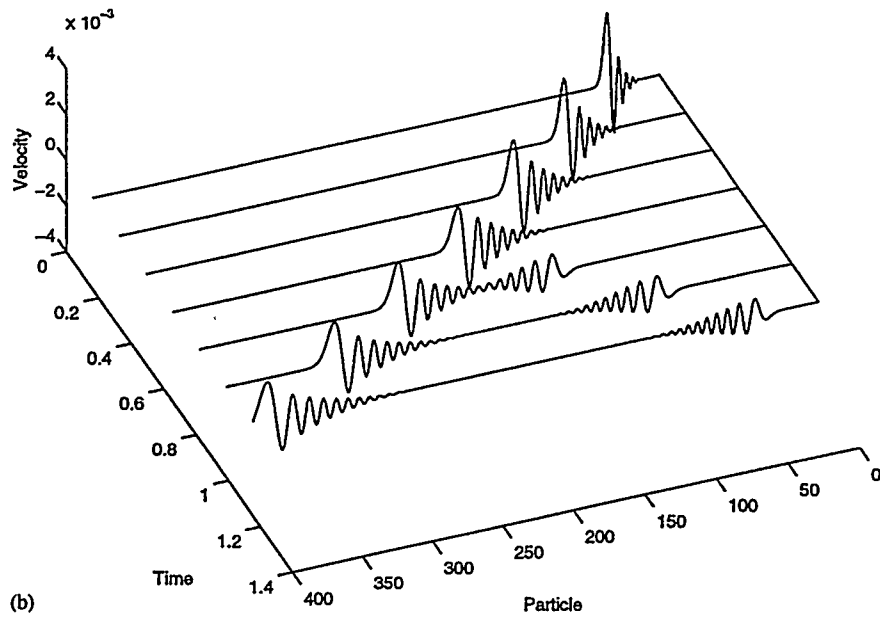
### 3.1. Backscattering of pulses from an inclusion in ideal 3D beds

In the recent past, Rogers and Don [7] reported experiments in which they sent acoustic impulses into soil and recorded backscattered impulses from known buried objects which contained specific signatures associated with the buried objects. Detailed theoretical analyses of the Rogers and Don experiments [7] remain to be carried out. In this section, we report the first studies in which we demonstrate that an acoustic impulse can be successfully backscattered by a buried object in a 3D granular bed.

Our simulations have been carried out in 3D granular beds of dimensions  $20 \times 20 \times 40$  grains with walls along the  $y-z$ ,  $x-z$  and  $x-y$  planes and an open surface. The



(a)



(b)

Fig. 4. Plot of velocity (in m/s), time (in s) and particle number (which measures depth) for an incoming acoustic impulse (same as in Fig. 8 of I) from the right in a gravitationally loaded chain that backscatters off (a) a light mass (see text) and (b) a heavy mass.

surface grains are constrained to move only along the direction of the impulse, which is compressive, and along the  $c$ -axis of the box. We ignored the effects of restitution and friction in our calculations. We also ignored the effects of gravitational compaction in our studies. The rationale for neglecting gravity is that we are interested in pulses of significant amplitude that propagate relatively short distances. In our studies, we introduced an impulse in the entire surface layer. The square inclusions of dimensions  $10 \times 10$  were placed at a depth of 20 granular layers and were taken to be one granular layer thick. The elastic constants and hence  $a$  for the inclusions were adjusted to be that of plastic and of steel, respectively, for the light and heavy inclusions that were probed. The results for the light and heavy inclusion cases are shown in Figs. 5(a) and (b), respectively.

We have used a variety of conditions to specify the reflection of energy from the walls. Our preliminary analyses indicate that the details of reflection from the wall play an insignificant role when studying the propagation of large amplitude perturbations through the bed.

Our results can be summarized as follows. (i) We find that distinct backscattering occurs depending upon whether the inclusion is lighter or heavier than the medium. (ii) The signal attenuates in space and time as it travels down the bed. The attenuation process is multifaceted and cannot be described via a simple functional fit (e.g., the kinetic energy of the pulse does not simply decay exponentially or algebraically in space and time). Recall that in the nonlinear acoustic regime, the velocity of the signal is amplitude dependent. Typically, the initial signal suffers backscattering from the layers as it travels down. Such backscattering eventually leads to the formation of secondary pulses that start to propagate downward in time-delayed fashion. The most dominant of these secondary signals is visible in Figs. 5(a) and (b). Energy localization is significant in the light inclusion as evidenced by the two-peaked structure of kinetic energy in the center of Fig. 5(a). This feature indicates that backscattering from an inclusion is a rich process which involves the “residence times” of the signal on the inclusion and the formation of leading and secondary pulses that are emitted by the signal.

### 3.2. *role of restitution*

Restitution plays an extremely important role in granular media. When two spherical grains are progressively compressed against one another and are subsequently released, they never regain the original compressions at fixed loadings. This “hysteretic” behavior leads to a coefficient of restitution which, at the simplest level, can be used to estimate the energy lost to internal degrees of freedom of the individual grains during the loading and unloading process.

There appears to be no universally acceptable way of quantifying restitution in granular systems at this stage. We define restitution in a simplistic manner that closely follows a well-accepted treatment of restitution due to Walton and Braun [8]. We assume that the force associated with compressing two adjacent grains, which we call

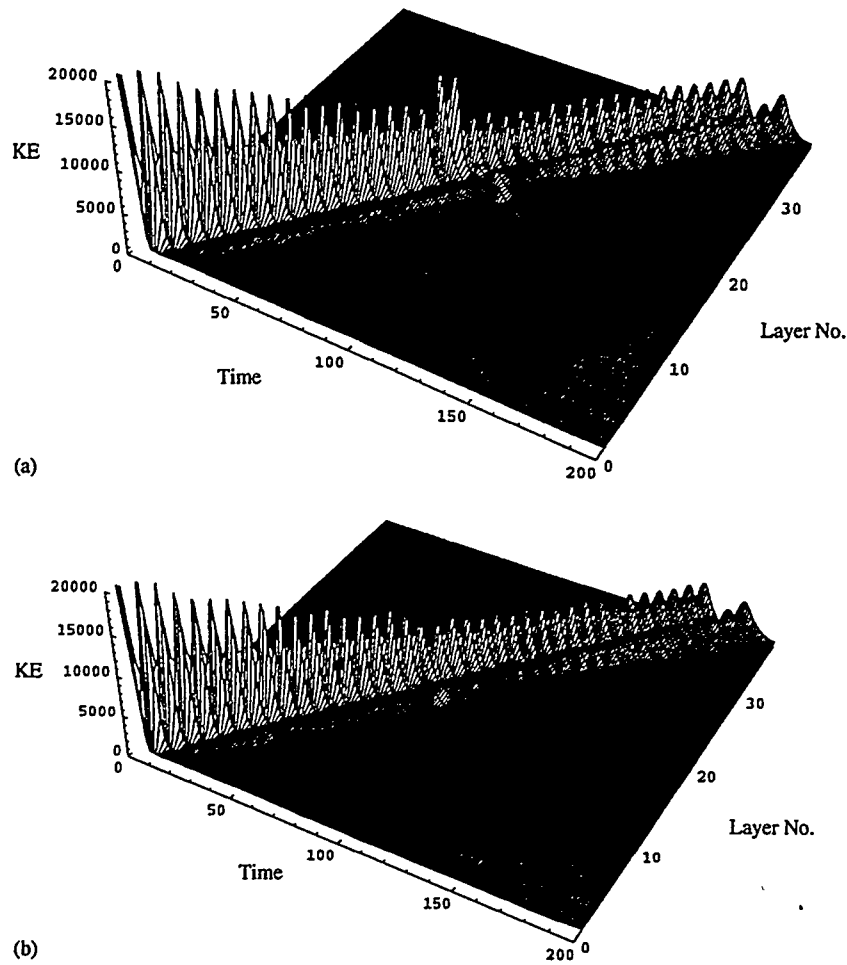


Fig. 5. Plots show the propagation and backscattering of the kinetic energy (in arbitrary units) associated with an impulse in a 3D box of grains of dimension  $20 \times 20 \times 40$  grains as functions of time (in arbitrary units). The impulse is given to all the surface grains in the box in both the cases. The buried objects are of dimension  $10 \times 10 \times 1$  grains and are placed half-way down the height of the box (see text). In (a) we study backscattering from a lighter object (plastic) and in (b) the same from a heavier object (steel). The localization of energy on the site of the lighter object is evident.

$\epsilon_{\text{loading}}$ , differs from the force associated with restoring these grains to their original states, which we call  $\epsilon_{\text{unloading}}$ . We simplify the computation by ignoring the change in the size of the grains. Since some energy is lost in the process of loading and unloading, we assume that  $\epsilon_{\text{unloading}} < \epsilon_{\text{loading}}$ . The ratio,  $\epsilon_{\text{unloading}} / \epsilon_{\text{loading}} \equiv \epsilon$ . We regard  $\epsilon$  as the coefficient of restitution in our studies. The quantity of utmost interest turns out to be  $1 - \epsilon$ , rather than  $\epsilon$  when comparing the effects of restitution with experiments. Thus,  $\epsilon = 1$  defines the absence of restitution.

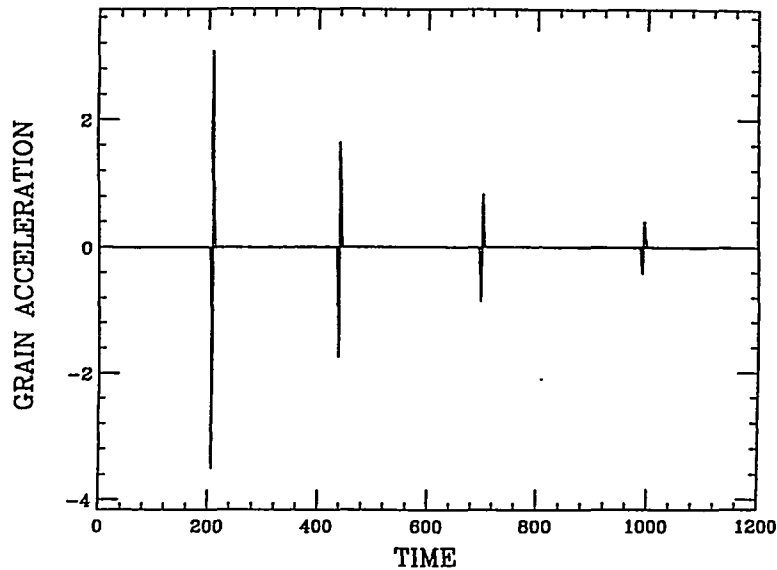


Fig. 6. Plot showing the acceleration (in arbitrary units) of particle number 350 (first peak) where the soliton resides at time  $t = 200$  (time is given in arbitrary units), the acceleration of particle number 300, which the soliton reaches at  $t = 440$ , the acceleration of particle number 250, which the soliton reaches at  $t = 650$  and the acceleration of particle number 200, which the soliton reaches at  $t = 1000$ . The first particle in the chain is numbered 400. Restitution is defined in the Hertzian chain via the condition  $\text{unloading}/\text{loading} = 0.95$ . The attenuation of the soliton amplitude is exponential (see text). Observe that the soliton slows down in space.

Fig. 6 presents the decay of the amplitude of a soliton pulse for  $\mu = 0.95$  as it travels through a 1D chain of grains in the absence of gravity and loading. The reader may note that as the soliton propagates in space (from grain 350 to grain 200 in intervals of 50 grains), it progressively loses its amplitude and hence its energy while maintaining its shape. Subsequently, the soliton slows down in space and time. At sufficiently large depths, we find that the soliton possesses sufficiently low energy to become dispersive and hence behaves as an acoustic pulse that travels with constant speed (for the acoustic waves  $\delta \ll l_0$ , the speed is dictated only by the initial loading).

We have tracked the decay of the kinetic energy as a function of depth  $z$  of a traveling pulse in the presence of various restitution coefficients. We find that our studies can be described by an exponential decay of the kinetic energy of a soliton pulse which can be written as  $\exp(-0.38(1 - \mu)z)$ , where  $z$  is the distance through which the soliton has traveled.

#### 4. Discussion

We have presented a detailed study of the backscattering of mechanical pulses of various amplitudes through a chain of spherical quartz beads and through a close-packed 3D bed of Hertzian spheres.

We have considered the study of horizontal and vertical quartz chains with a single impurity placed at its center. The impure grain was assumed to possess a density that is either less than (plastic) or more than (steel) that of quartz. In either case, our simulations showed that traveling soliton or acoustic pulses were effectively backscattered by the impurity. The reflected pulse carried signatures that revealed the density contrast of the impurity with respect to that of quartz.

We have extended our work to report that backscattering of nonlinear acoustic impulses is found in simulations of 3D beds with buried inclusions.

Finally, we have explored the effects of restitution on the amplitude of the mechanical pulses in our analyses. We report that restitution typically leads to an exponential decay of the amplitude of the mechanical pulse.

The present study suggests that acoustic or shock pulses may be useful in probing for buried impurities in granular systems. Restitution plays a key role in determining the maximum distances across which acoustic sensing may be feasible.

#### Acknowledgements

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