TIFR/TH/08-15

Probing extended Higgs sector through rare $b \to s\mu^+\mu^-$ transitions

Ashutosh Kumar Alok,[1,*] Amol Dighe,[1,†] and S. Uma Sankar[2,‡]

[1] *Tata Institute of Fundamental Research, Homi Bhabha Road, Mumbai 400005, India*

[2] *Indian Institute of Technology Bombay, Mumbai 400076, India*

We study the constraints on the contribution of new physics in the form of scalar/pseudoscalar operators to the average forward backward asymmetry $\langle A_{FB}\rangle$ of muons in $B \to K\mu^+\mu^-$ and the longitudinal polarization asymmetry A_{LP} of muons in $B_s \to \mu^+\mu^-$. We find that the maximum possible value of $\langle A_{FB}\rangle$ allowed by the present upper bound on $B(B_s \to \mu^+\mu^-)$ is about 1% at 95% C.L. and hence will be very difficult to measure. On the other hand, the present bound on $B(B_s \to \mu^+\mu^-)$ fails to put any constraints on A_{LP}, which can be as high as 100% even if $B(B_s \to \mu^+\,\mu^-)$ is close to its standard model prediction. The measurement of A_{LP} will be a direct evidence for an extended Higgs sector, and combined with the branching ratio $B(B_s \to \mu^+\mu^-)$ it can even separate the new physics scalar and pseudoscalar contributions.

PACS numbers: 13.20.He, 12.60.-i

*Electronic address: alok@theory.tifr.res.in

†Electronic address: amol@theory.tifr.res.in

‡Electronic address: uma@phy.iitb.ac.in

I. INTRODUCTION

The quark level flavor changing neutral interaction $b \to s\mu^+\mu^-$ is forbidden at the tree level in the standard model (SM) and can occur only at the one-loop level. Therefore it can serve as an important probe to test SM at loop level and also constrain many new physics models beyond the SM. This quark level interaction is responsible for the purely leptonic decay $B_s \to \mu^+\mu^-$ and also the semi-leptonic decays $B \to (K, K^*)\mu^+\mu^-$. The semi-leptonic decays have been observed by BaBar and Belle [1, 2, 3] with the following branching ratios:

$$
\begin{aligned}
B(B \to K\mu^+\mu^-) &= (5.7^{+2.2}_{-1.8}) \times 10^{-7}, \\
B(B \to K^*\mu^+\mu^-) &= (11.0^{+2.99}_{-2.6}) \times 10^{-7}.
\end{aligned} \tag{1}
$$

These values are close to the SM predictions [4, 5, 6]. However there is about 20% uncertainty in these predictions mainly due to the errors in the determination of the hadronic form factors and the CKM matrix element $|V_{ts}|$.

The decay $B_s \to \mu^+\mu^-$ is highly suppressed in SM. Its branching ratio is predicted to be $(3.35 \pm 0.32) \times 10^{-9}$ [7, 8, 9]. This decay is yet to be observed experimentally. Recently the upper bound on its branching ratio has been improved to [10]

$$
B(B_s \to \mu^+\mu^-) < 5.8 \times 10^{-8} \quad (95\%\ \text{C.L.})\ , \tag{2}
$$

which is still more than an order of magnitude above its SM prediction. $B_s \to \mu^+\mu^-$ will be one of the important rare B decays to be studied at the upcoming Large Hadron Collider (LHC) and we expect that the sensitivity of the level of the SM prediction can be reached with $\sim 1\ \text{fb}^{-1}$ of data. [11, 12].

Many new physics models predict an order of magnitude enhancement or more in $B(B_s \to \mu^+\mu^-)$. These include theories with Z' mediated vector bosons [13], as well as multi-Higgs doublet models that violate [13] or obey [14] natural flavor conservation. In [15], it was shown that the new physics mediated by vector bosons is highly constrained by the measured values of the branching ratio of $B \to (K, K^*)\mu^+\mu^-$. As a result, an order of magnitude enhancement in $B(B_s \to \mu^+\, \mu^-)$ from new physics vector or axial vector operators is ruled out. On the other hand, such an enhancement from the scalar/pseudoscalar new physics (SPNP) operators is still allowed, since the most stringent bound on the SPNP operators comes from $B(B_s \to \mu^+ \mu^-)$ itself. In particular, multi-Higgs doublet models or supersymmetric (SUSY) models with large $\tan\beta$ can give rise to such an enhancement.

Apart from the branching ratios of the purely leptonic and semi-leptonic decays, there are other observables which are sensitive to the SPNP contribution to $b \to s$ transitions. These are forward-backward (FB) asymmetry A_{FB} of muons [16] in $B \to K\mu^+ \mu^-$ and longitudinal polarization (LP) asymmetry A_{LP} of muons in $B_s \to \mu^+ \mu^-$ [17]. Both these are predicted to be zero in the SM. Therefore, any nonzero measurement of one of these asymmetries is a signal for new physics. In addition, these asymmetries are almost independent of form factors and CKM matrix element uncertainties, which makes them attractive candidates in searches for new physics. In this paper we investigate what constraints the recently improved upper bound on $B(B_s \to \mu^+\mu^-)$ puts on the possible SPNP contribution to A_{FB} and A_{LP}. Do SPNP operators enhance these observables to sufficiently large values to be measurable in future experiments?

The paper is organized as follows. In section II, we study the effect of possible SPNP contribution to A_{FB}. In section III, we calculate the possible A_{LP} enhancement due to SPNP, and point out some interesting experimental possibilities. In section IV, we present our conclusions.

II. FORWARD-BACKWARD ASYMMETRY IN $B \to K\mu^+ \mu^-$

There are numerous studies in literature of the FB asymmetry of leptons in the SM and its possible extensions [18, 19, 20, 21, 22, 23, 24]. In the SM, the FB asymmetry of muons in $B \to K\mu^+\mu^-$ vanishes (or to be more precise, is negligibly small) because the hadronic current for $B \to K$ transition does not have any axial vector contribution. However this asymmetry can be nonzero in multi-Higgs doublet models and supersymmetric models with large $\tan\beta$, due to the contributions from Higgs bosons. Therefore FB asymmetry in $B \to K\mu^+\mu^-$ is expected to serve as an important probe to test the existence and importance of an extended Higgs sector [21, 24]. Any nonzero measurement of this asymmetry will be a clear signal of new physics.

The average (or integrated) FB asymmetry of muons in $B \to K\mu^+\mu^-$, which is denoted by $\langle A_{FB}\rangle$, has been measured by BaBar [2] and Belle [25, 26] to be

$$\langle A_{FB}\rangle = (0.15^{+0.21}_{-0.23} \pm 0.08) \quad \text{(BaBar)}\,, \tag{3}$$

$$\langle A_{FB}\rangle = (0.10 \pm 0.14 \pm 0.01) \;\; \text{(Belle)}. \tag{4}$$

These measurements are consistent with zero. But on the other hand, they can be as high as $\sim 40\%$ within 2σ error bars.

A. Calculation of A_{FB}

We consider new physics in the form of scalar/pseudoscalar operators. The effective Lagrangian for the quark level transition $b \to s\mu^+\mu^-$ can be written as

$$L(b \to s\mu^+\mu^-) = L_{SM} + L_{SP}\,, \tag{5}$$

where

$$\begin{aligned} L_{SM} = & \frac{\alpha G_F}{\sqrt{2}\pi} V_{tb}V_{ts}^{\star}\Bigg\{ C_9^{\rm eff}\,(\bar{s}\gamma_\mu P_L b)\,\bar{\mu}\gamma_\mu\mu + C_{10}(\bar{s}\gamma_\mu P_L b)\,\bar{\mu}\gamma_\mu\gamma_5\mu \\ & - 2\frac{C_7^{\rm eff}}{q^2} m_b\,(\bar{s}i\sigma_{\mu\nu}q^\nu P_R b)\,\bar{\mu}\gamma_\mu\mu \Bigg\}\,, \end{aligned} \tag{6}$$

$$L_{SP} = \frac{\alpha G_F}{\sqrt{2}\pi} V_{tb}V_{ts}^{\star}\Bigg\{ R_S\,(\bar{s}\,P_R\,b)\,\bar{\mu}\,\mu + R_P\,(\bar{s}\,P_R\,b)\,\bar{\mu}\gamma_5\mu \Bigg\}\,. \tag{7}$$

Here $P_{L,R} = (1 \mp \gamma_5)/2$ and q_μ is the sum of 4-momenta of μ^+ and μ^-. R_S and R_P are the new physics scalar and pseudoscalar couplings respectively. In our analysis we assume that there are no additional CP phases apart from the single CKM phase. Under this assumption, R_S and R_P are real. Within SM, the Wilson coefficients in eq. (6) have the following values:

$$C_7^{\rm eff} = -0.310\;,\; C_9^{\rm eff} = +4.138 + Y(q^2)\;,\; C_{10} = -4.221\;, \tag{8}$$

where the function $Y(q^2)$ is given in [27, 28].

The normalized FB asymmetry is defined as

$$A_{FB}(z) = \frac{\int_0^1 dcos\theta \frac{d^2\Gamma}{dzdcos\theta} - \int_{-1}^0 dcos\theta \frac{d^2\Gamma}{dzdcos\theta}}{\int_0^1 dcos\theta \frac{d^2\Gamma}{dzdcos\theta} + \int_{-1}^0 dcos\theta \frac{d^2\Gamma}{dzd\cos\theta}}\;. \tag{9}$$

In order to calculate the FB asymmetry, we first need to calculate the differential decay width. The decay amplitude for $B(p) \to K(p')\mu^+(p_+)\mu^-(p_-)$ is given by

$$
\begin{aligned}
M\left(B \to K\mu^+\mu^-\right) \;=\; & \frac{\alpha G_F}{2\sqrt{2}\pi} V_{tb}V_{ts}^\star \\
\times & \Bigg[\langle K(p')\,|\bar{s}\gamma_\mu b|\, B(p)\rangle \left\{ C_9^{\rm eff}\bar{u}(p_+)\gamma_\mu v(p_-) + C_{10}\bar{u}(p_+)\gamma_\mu\gamma_5 v(p_-) \right\} \\
& -2\frac{C_7^{\rm eff}}{q^2} m_b \langle K(p')\,|\bar{s}i\sigma_{\mu\nu}q^\nu b|\, B(p)\rangle\, \bar{u}(p_+)\gamma_\mu v(p_-) \\
& + \langle K(p')\,|\bar{s}b|\, B(p)\rangle \left\{ R_S\bar{u}(p_+)v(p_-) + R_P\bar{u}(p_+)\gamma_5 v(p_-) \right\} \Bigg] \,, \qquad (10)
\end{aligned}
$$

where $q_\mu = (p - p')_\mu = (p_+ + p_-)_\mu$. The relevant matrix elements are

$$
\langle K(p')\,|\bar{s}\gamma_\mu b|\, B(p)\rangle \;=\; (2p - q)_\mu f_+(z) + (\frac{1-k^2}{z})\, q_\mu [f_0(z) - f_+(z)]\,, \qquad (11)
$$

$$
\langle K(p')\,|\bar{s}i\sigma_{\mu\nu}q^\nu b|\, B(p)\rangle = -\Big[(2p-q)_\mu q^2 - (m_B^2 - m_K^2)q_\mu\Big]\, \frac{f_T(z)}{m_B + m_K}\,, \qquad (12)
$$

$$
\langle K(p')\,|\bar{s}b|\, B(p)\rangle = \frac{m_B(1-k^2)}{\hat{m}_b}\, f_0(z)\,. \qquad (13)
$$

Here, $k \equiv m_K/m_B$, $z \equiv q^2/m_B^2$ and $\hat{m}_b \equiv m_b/m_B$. In this paper, we approximate $\hat{m}_b$ by 1.

Using the above matrix elements, the double differential decay width can be calculated as

$$
\begin{aligned}
\frac{d^2\Gamma}{dzdcos\theta} \;=\; & \frac{G_F^2\alpha^2}{2^9\pi^5}\, |V_{tb}V_{ts}^*|^2\, m_B^5\, \phi^{1/2}(1,k^2,z)\, \beta_\mu \\
\times & \Bigg[\Big(|A|^2\,\beta_\mu^2 + |B|^2 \Big) z + \frac{1}{4}\phi(1,k^2,z)\Big(|C|^2 + |D|^2 \Big)(1 - \beta_\mu^2\cos^2\theta) \\
& + 2\hat{m}_\mu(1 - k^2 + z)Re(BC^*) + 4\hat{m}_\mu{}^2\,|C|^2 \\
& + 2\hat{m}_\mu\, \phi^{\frac{1}{2}}(1,k^2,z)\, \beta_\mu\, Re(AD^*)\, \cos\theta \Bigg]\,, \qquad (14)
\end{aligned}
$$

where

$$\begin{aligned}
A &\equiv \frac{1}{2}(1-k^2) f_0(z) R_S \ , \\
B &\equiv -\hat{m}_\mu C_{10} \left\{ f_+(z) - \frac{1-k^2}{z}(f_0(z) - f_+(z)) \right\} + \frac{1}{2}(1-k^2) f_0(z) R_P \ , \\
C &\equiv C_{10}\, f_+(z) \ , \\
D &\equiv C_9^{eff}\, f_+(z) + 2\, C_7^{eff}\, \frac{f_T(z)}{1+k} \ , \\
\phi(1,k^2,z) &\equiv 1 + k^4 + z^2 - 2(k^2 + k^2 z + z) \ , \\
\beta_\mu &\equiv (1 - \frac{4\hat{m}_\mu{}^2}{z}) \ . \qquad (15)
\end{aligned}$$

Also, $\hat{m}_\mu = m_\mu / m_B$ and θ is the angle between the momenta of K meson and μ^- in the dilepton centre of mass frame. The kinematical variables are bounded as

$$\begin{aligned}
-1 \leq \cos\theta \leq 1 \ , \\
4\hat{m}_\mu^2 \leq z \leq (1-k)^2 \ .
\end{aligned}$$

The form factors $f_{+,0,T}$ can be calculated in the light cone QCD approach. Their q^2 dependence is given by [18]

$$f(z) = f(0)\ \exp(c_1 z + c_2 z^2 + c_3 z^3) \ , \qquad (16)$$

where the parameters $f(0), c_1$, c_2 and c_3 for each form factor are given in Table I. The FB asymmetry arises from the $\cos\theta$ term in the last line of eq. (14).

	$f(0)$	c_1	c_2	c_3
f_+	$0.319^{+0.052}_{-0.041}$	1.465	0.372	0.782
f_0	$0.319^{+0.052}_{-0.041}$	0.633	− 0.095	0.591
f_T	$0.355^{+0.016}_{-0.055}$	1.478	0.373	0.700

TABLE I: Form factors for the $B \to K$ transition [18].

The calculation of FB asymmetry gives

$$A_{FB}(z) = \frac{2\Gamma_0\, \hat{m}_\mu\, a_1(z)\, \phi(1,k^2,z)\, \beta_\mu^2\, R_S}{d\Gamma/dz} \ , \qquad (17)$$

where

$$\Gamma_0 = \frac{G_F^2 \alpha^2}{2^9 \pi^5}\, |V_{tb} V_{ts}^*|^2\, m_B^5 \ , \qquad (18)$$

$$a_1(z) = \frac{1}{2}(1-k^2)C_9 f_0(z) f_+(z) + (1-k)C_7 f_0(z) f_T(z) \;, \tag{19}$$

$$\begin{aligned}
\frac{1}{\Gamma_0}\frac{d\Gamma}{dz} \;=\;& \frac{1}{2}(1-k^2)\,\beta_\mu\,\phi^{\frac{1}{2}}\,z\,f_0^2(z)\,(R_P^2+\beta_\mu^2 R_S^2) \\
&+\; 2(1-k^2)\,\hat{m}_\mu\,C_{10}\,f_0(z)f_+(z)\,\beta_\mu\,\phi^{\frac{1}{2}}(z)\,(1-k^2+z)R_P \\
&-\; 2(1-k^2)\,\hat{m}_\mu\,C_{10}\,\beta_\mu\,z\,\phi^{\frac{1}{2}}f_0(z)\left\{f_+(z)-\frac{1-k^2}{z}(f_0(z)-f_+(z))\right\}R_P \\
&+\; 2\hat{m}_\mu^2\,C_{10}^2\,\beta_\mu\,\phi^{\frac{1}{2}}(z)\left\{f_+(z)-\frac{1-k^2}{z}(f_0(z)-f_+(z))\right\}^2 \\
&+\; 8\hat{m}_\mu^2\,C_{10}^2\,\beta_\mu\,\phi^{\frac{1}{2}}(z)f_+^2(z) \\
&+\; \frac{1}{3}(1+\frac{2\hat{m}_\mu^2}{z})\beta_\mu\phi^{\frac{3}{2}}(z)\times \\
&\left\{(C_{10}^2+C_9^{eff2})f_+^2(z)+\frac{4C_7^{eff2}}{(1+k)^2}f_T^2(z)+\frac{4C_9^{eff}C_7^{eff}}{(1+k)}f_+(z)f_T(z)\right\} \\
&-\; 4\hat{m}_\mu{}^2C_{10}^2 f_+(z)\,\beta_\mu\,(1-k^2+z)\,\phi^{\frac{1}{2}}(z)\times \\
&\left\{f_+(z)-\frac{1-k^2}{z}(f_0(z)-f_+(z))\right\}\;.
\end{aligned} \tag{20}$$

From eq. (17), it is clear that $A_{FB}(z)$ is proportional to $\hat{m}_\mu(\approx 0.02)$, and to the scalar new physics coupling R_S. In the minimal supersymmetric standard model (MSSM) and two Higgs doublet models, R_S itself is proportional to $\hat{m}_\mu$ and $\tan^2\beta$. Hence a large FB asymmetry is possible only for exceptionally large values of $\tan\beta$.

The average FB asymmetry is obtained by integrating the numerator and denominator of eq. (17) separately over dilepton invariant mass, which leads to

$$\langle A_{FB}\rangle = \frac{2\Gamma_0\,\hat{m}_\mu\,\beta_\mu^2\,R_S\,\int dz\,a_1(z)\,\phi(1,k^2,z)}{\Gamma(B\to K\mu^+\mu^-)} = \frac{2\tau_B\Gamma_0\,\hat{m}_\mu\,\beta_\mu^2\,R_S\,\int dz\,a_1(z)\,\phi(1,k^2,z)}{B(B\to K\mu^+\mu^-)}\;. \tag{21}$$

where $B(B\to K\mu^+\mu^-)$ is the total branching ratio of $B\to K\mu^+\mu^-$. The numerator in eq. (21) can be calculated to be

$$2\tau_B\Gamma_0\,\hat{m}_\mu\,\beta_\mu^2\,R_S\int dz\,a_1(z)\,\phi(1,k^2,z) = (5.25\times 10^{-9})(1\pm 0.20)R_S\;, \tag{22}$$

whereas the total branching ratio, including the contribution of SPNP operators, is given by [20]

$$B(B\to K\mu^+\mu^-) = \left[5.25 + 0.18(R_S^2+R_P^2) - 0.13R_P\right]\,(1\pm 0.20)\times 10^{-7}\;. \tag{23}$$

In the SM calculation of $B(B\to K\mu^+\,\mu^-)$, two vector form factors, f_0 and f_+, as well as the tensor form factor f_T appear. The SPNP contribution, on the other hand,

$G_F = 1.166 \times 10^{-5}$ GeV^{-2}	$m_{B_s} = 5.366$ GeV
$\alpha = 7.297 \times 10^{-3}$	$m_B = 5.279$ GeV
$\tau_{B_s} = (1.437^{+0.031}_{-0.030}) \times 10^{-12} s$	$V_{tb} = 1.0$
$\tau_{B_d} = 1.53 \times 10^{-12} s$	$V_{ts} = (40.6 \pm 2.7) \times 10^{-3}$
$m_\mu = 0.105$ GeV	$f_{B_s} = (0.259 \pm 0.027)$ GeV [29]
$m_K = 0.497$ GeV	

TABLE II: Numerical inputs used in our analysis. Unless explicitly specified, they are taken from the Review of Particle Physics [30].

is only through f_0. We have made the assumption that the fractional uncertainties in all the form factors are the same. The $|V_{ts}|$ dependence in the numerator and denominator of eq. (21) cancels completely, whereas the errors due to the form factors uncertainties cancel partially. We conservatively take the net error in $\langle A_{FB}\rangle$ to be 30%, leading to

$$\langle A_{FB}\rangle = \frac{5.25 \times 10^{-9}\, R_S}{[5.25 + 0.18(R_S^2 + R_P^2) - 0.13 R_P] \times 10^{-7}}\,(1 \pm 0.3)\,. \qquad (24)$$

B. Constraints on $\langle A_{FB}\rangle$ from $B(B_s \to \mu^+\,\mu^-)$

We now want to see what constraints the present upper bound on $B(B_s \to \mu^+\,\mu^-)$ puts on the maximum possible value of $\langle A_{FB}\rangle$. The present experimental upper limit on $B(B_s \to \mu^+\mu^-)$ is an order of magnitude larger than the SM prediction. In such a situation, the SM amplitude for this decay will be much smaller than the new physics amplitude and hence can be neglected in determining the constraints on new physics couplings, R_S and R_P. In other words, we will assume that SPNP operators saturate the present upper limit. Therefore we need to consider only the contribution of L_{SP} to the decay rate of $B_s \to \mu^+\mu^-$.

The decay amplitude for $B_s \to \mu^+ \mu^-$ is given by

$$M\left(B_s \to \mu^+\mu^-\right) \;=\; \frac{\alpha G_F}{2\sqrt{2}\pi} V_{tb} V_{ts}^{\star} \langle 0 \left| \bar{s}\gamma_5 b \right| B_s \rangle \left[R_S \bar{u}(p_\mu) v(p_{\bar{\mu}}) + R_P \bar{u}(p_\mu)\gamma_5 v(p_{\bar{\mu}}) \right] \; . \tag{25}$$

On substituting

$$\langle 0 \left| \bar{s}\gamma_5 b \right| B_s \rangle \;=\; -i \frac{f_{B_s} m_{B_s}^2}{m_b + m_s} \; , \tag{26}$$

we get

$$M\left(B_s \to \mu^+\mu^-\right) \;=\; -i \, \frac{\alpha G_F}{2\sqrt{2}\pi} V_{tb} V_{ts}^{\star} \frac{f_{B_s} m_{B_s}^2}{m_b + m_s} \left[R_S \bar{u}(p_\mu) v(p_{\bar{\mu}}) + R_P \bar{u}(p_\mu)\gamma_5 v(p_{\bar{\mu}}) \right] \; , \tag{27}$$

where m_b and m_s are the masses of bottom and strange quark, respectively. The calculation of the branching ratio $B(B_s \to \mu^+ \mu^-)$ gives

$$B(B_s \to \mu^+\mu^-) \;=\; \frac{G_F^2 \alpha^2 m_{B_s}^3 \tau_{B_s}}{64\pi^3} |V_{tb} V_{ts}^{*}|^2 \, f_{B_s}^2 \, (R_S^2 + R_P^2) \; . \tag{28}$$

Here we have neglected terms of order m_s/m_b and approximated m_{B_s}/m_b by 1. Taking $f_{B_s} = (0.259 \pm 0.027)\,\mathrm{GeV}$, we get

$$B(B_s \to \mu^+\mu^-) \;=\; (1.43 \pm 0.30) \times 10^{-7} \, (R_S^2 + R_P^2) \, . \tag{29}$$

Equating the expression in eq. (29) to the present 95% C.L. upper limit in eq. (2), we get the inequality

$$(R_S^2 + R_P^2) \leq 0.70 \; , \tag{30}$$

where we have taken the 2σ lower bound for the coefficient in eq. (29). Thus, the allowed region in the R_S–R_P parameter space is the interior of a circle of radius ≈ 0.84 centered at the origin.

In [31], it was shown that the SPNP operators cannot lower $B(B \to K\mu^+\mu^-)$ below its SM prediction. Therefore from eq. (24), the maximum value of $\langle A_{FB} \rangle$ with the current upper bound on $B(B \to K\mu^+ \mu^-)$ is 1.34% at 2σ. If $B(B_s \to \mu^+ \mu^-)$ is bounded to 10^{-8}, the 2σ maximum value of $\langle A_{FB} \rangle$ will be 0.56%.

A naive estimation suggests that the measurement of an asymmetry $\langle A_{FB} \rangle$ of a decay with the branching ratio $\mathcal{B}$ at $n\sigma$ C.L. with only statistical errors require

$$N \sim \left(\frac{n}{\mathcal{B} \, \langle A_{FB} \rangle} \right)^2 \tag{31}$$

number of events. For $B \to K\mu^+\mu^-$, if $\langle A_{FB}\rangle$ is 1% at 2σ C.L., then the required number of events will be as high as 10^{18} ! Therefore it is very difficult to observe such a low value of FB asymmetry in experiments. Hence FB asymmetry of muons in $B \to K\mu^+\mu^-$ will play no role in testing SPNP.

III. LONGITUDINAL POLARIZATION ASYMMETRY IN $B_s \to \mu^+\,\mu^-$

The longitudinal polarization asymmetry of muons in $B_s \to \mu^+\mu^-$ is a clean observable that depends only on SPNP operators. It vanishes in the SM, whereas its value is nonzero if and only if the new physics contribution is in the form of scalar operator. Therefore any nonzero measurement of this observable A_{LP} will confirm the existence of an extended Higgs sector. The observable A_{LP} was introduced in ref. [17], though the corresponding analysis in the context of $K_L \to \mu^+\mu^-$ had been carried out earlier [32, 33, 34, 35]. In this section, we will determine the allowed values of A_{LP} consistent with the present upper bound on $B(B_s \to \mu^+\mu^-)$, and explore the correlation between these two quantities.

The most general model independent form of the effective Lagrangian for the quark level transition $b \to s\mu^+\mu^-$ that contributes to the decay $B_s \to \mu^+\mu^-$ has the form [36, 37]

$$\begin{aligned} L \;=\; & \frac{G_F\alpha}{2\sqrt{2}\pi}\,(V_{ts}^{*}V_{tb})\;\{R_A(\bar{s}\,\gamma_\mu\gamma_5\,b)(\bar{\mu}\,\gamma^\mu\gamma_5\,\mu) \\ & +R_{\rm S}(\bar{s}\,\gamma_5\,b)(\bar{\mu}\,\mu) + R_{\rm P}(\bar{s}\,\gamma_5\,b)(\bar{\mu}\,\gamma_5\,\mu)\}\;, \end{aligned} \tag{32}$$

where R_P, R_S and R_A are the strengths of the scalar, pseudoscalar and axial vector operators respectively. Note that the effective Lagrangian in eq. (32) is essentially the same as the effective Lagrangian given in eq. (5). Here we have dropped C_7 and C_9 terms which do not contribute to $B_s \to \mu^+\,\mu^-$. In addition, the R_A in eq. (32) is the sum of SM and new physics contributions.

In SM, the scalar and pseudoscalar couplings $R_{\rm S}^{\rm SM}$ and $R_{\rm P}^{\rm SM}$ receive contributions from the penguin diagrams with physical and unphysical neutral scalar exchange and are highly suppressed:

$$R_{\rm S}^{\rm SM} = R_{\rm P}^{\rm SM} \propto \frac{(m_\mu m_b)}{m_W^2} \sim 10^{-5}\;. \tag{33}$$

Also, $R_{\rm A}^{\rm SM} = Y(x)/\sin^2\theta_W$, where $Y(x)$ is the Inami-Lim function [38]

$$Y(x) = \frac{x}{8}\left[\frac{x-8}{x-1} + \frac{3x}{(x-1)^2}\ln x\right] , \qquad (34)$$

with $x = (m_t/M_W)^2$. Thus, $R_{\rm A}^{\rm SM} \simeq 4.3$.

The calculation of the branching ratio gives [17, 36]

$$B(B_s \to \mu^+\,\mu^-) = a_s\left[\left|2m_\mu R_{\rm A} - \frac{m_{B_s}^2}{m_b+m_s}R_{\rm P}\right|^2 + \left(1-\frac{4m_\mu^2}{m_{B_s}^2}\right)\left|\frac{m_{B_s}^2}{m_b+m_s}R_{\rm S}\right|^2\right] , \qquad (35)$$

where

$$a_s \equiv \frac{G_F^2\alpha^2}{64\pi^3}\,\left|V_{ts}^*V_{tb}\right|^2\tau_{B_s}f_{B_s}^2 m_{B_s}\sqrt{1-\frac{4m_\mu^2}{m_{B_s}^2}} . \qquad (36)$$

Here τ_{B_s} is the lifetime of B_s. Eq. (35) represents the most general expression for the branching ratio of $B_s \to \mu^+\mu^-$.

We now derive an expression for the lepton polarization. In the rest frame of μ^+, we can define only one direction $\overrightarrow{p}_-$, the three momentum of μ^-. The unit longitudinal polarization 4-vectors along that direction are

$$\bar{s}^\mu_{\mu^\pm} = (0,\ \hat{e}_L^\pm) = \left(0,\ \pm\frac{\overrightarrow{p}_-}{|\overrightarrow{p}_-|}\right) . \qquad (37)$$

Transformation of unit vectors from the rest frame of μ^+ to the center of mass frame of leptons (which is also the rest frame of B_s meson) can be accomplished by the Lorentz boost. After the boost, we get

$$s^\mu_{\mu^\pm} = \left(\frac{|\overrightarrow{p}_-|}{m_\mu},\ \pm\frac{E_\mu\overrightarrow{p}_-}{m_\mu|\overrightarrow{p}_-|}\right) , \qquad (38)$$

where E_μ is the muon energy.

The longitudinal polarization asymmetry of muons in $B_s \to \mu^+\mu^-$ is defined as

$$A_{LP}^\pm \ = \ \frac{\Gamma(\hat{e}_L^\pm)\ -\ \Gamma(-\hat{e}_L^\pm)}{\Gamma(\hat{e}_L^\pm)\ +\ \Gamma(-\hat{e}_L^\pm)} . \qquad (39)$$

Thus we get [17]

$$A_{LP} \ = \ \frac{2\sqrt{1-\frac{4m_\mu^2}{m_{B_s}^2}}\left[\frac{m_{B_s}^2}{m_b+m_s}R_{\rm S}\left(2m_\mu R_{\rm A} - \frac{m_{B_s}^2}{m_b+m_s}\ R_{\rm P}\right)\right]}{\left|2m_\mu R_{\rm A} - \frac{m_{B_s}^2}{m_b+m_s}R_{\rm P}\right|^2 + (1-\frac{4m_\mu^2}{m_{B_s}^2})\left|\frac{m_{B_s}^2}{m_b+m_s}R_{\rm S}\right|^2} , \qquad (40)$$

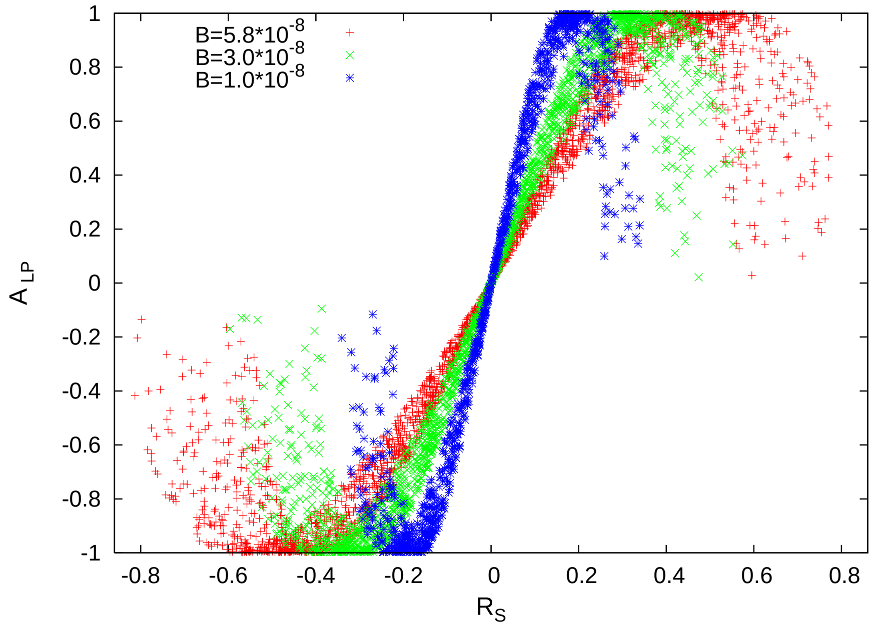

FIG. 1: A_{LP} vs R_s plot for $B(B_s \to \mu^+\mu^-) = (5.8, 3.0, 1.0) \times 10^{-8}$

with $A^+_{LP} = A^-_{LP} \equiv A_{LP}$. It is clear from eq. (40) that A_{LP} can be nonzero if and only if $R_{\rm S} \neq 0$, i.e. for A_{LP} to be nonzero, we must have contribution from SPNP operators. Within the SM, $R_{\rm S} \simeq 0$ and hence $A_{LP} \simeq 0$.

Using eq. (35), we can eliminate R_A and R_P from eq. (40) in favour of the physical observables $B(B_s \to \mu^+\,\mu^-)$ and a_s. We get [17]

$$\begin{aligned} A_{LP} \;&=\; \pm\frac{2a_s}{B(B_s \to \mu^+\,\mu^-)}\sqrt{1-\frac{4m_\mu^2}{m_{B_s}^2}}\times \\ &\frac{m_{B_s}^2\,R_S}{m_b+m_s}\sqrt{\frac{B(B_s\to\mu^+\,\mu^-)}{a_s}-\left(1-\frac{4m_\mu^2}{m_{B_s}^2}\right)\left|\frac{m_{B_s}^2\,R_S}{m_b+m_s}\right|^2}\;. \end{aligned} \tag{41}$$

Eq. (41) represents a general relation between the longitudinal polarization asymmetry A_{LP} and the branching ratio of $B_s \to \mu^+\mu^-$.

We now explore the correlation between A_{LP} and $B(B_s \to \mu^+\,\mu^-)$. It is quite obvious that when $B(B_s \to \mu^+\,\mu^-) \gtrsim 10^{-8}$, we can neglect the SM contribution in obtaining the bounds on R_S and R_P. However if $B(B_s \to \mu^+\,\mu^-)$ is of the order of the SM prediction, then we will have to take into account the SM contribution as well. Therefore it is reasonable to consider both the cases separately.

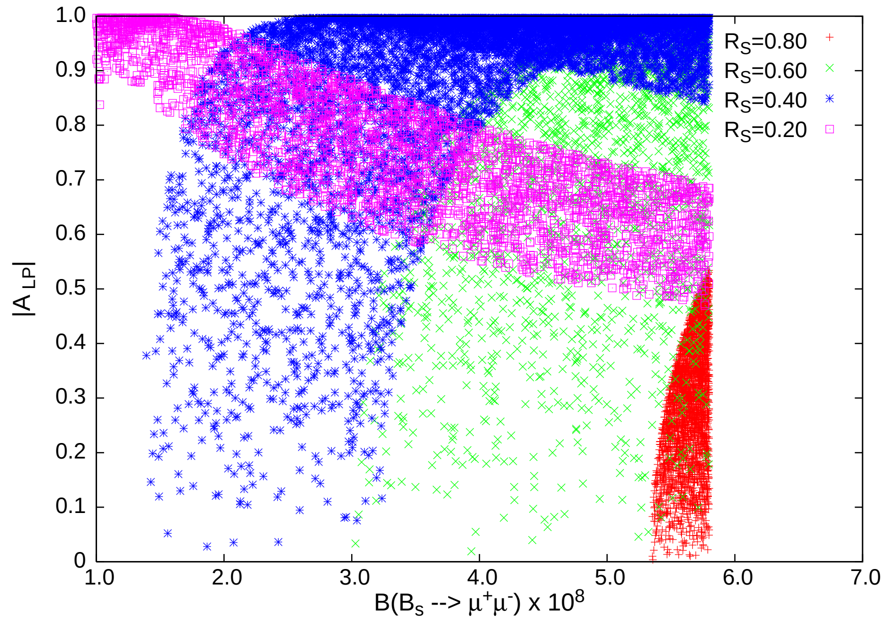

FIG. 2: Plot between $|A_{LP}|$ and $B(B_s \to \mu^+\mu^-)$ for different R_S values, when $B(B_s \to \mu^+\mu^-) \gtrsim 10^{-8}$. The region $B(B_s \to \mu^+\mu^-) > 5.8 \times 10^{-8}$ is ruled out by experiments to 95% C.L..

A. $B(B_s \to \mu^+\mu^-) \gtrsim 10^{-8}$

We first consider the constraints on A_{LP} coming from the present upper bound on $B(B_s \to \mu^+\mu^-)$. Fig. 1 shows the plot between A_{LP} and R_S for three different values of $B(B_s \to \mu^+\mu^-) \gtrsim 10^{-8}$. Fig. 2 is a plot between $|A_{LP}|$ and $B(B_s \to \mu^+\mu^-)$ for various allowed values of R_S. The bands in Figs. 1 and 2 are mainly due to the uncertainties in CKM matrix element $|V_{ts}|$ and decay constant f_{B_s}.

We see from Fig. 1 that the maximum possible value of A_{LP} consistent with the present upper bound on $B(B_s \to \mu^+\mu^-)$ is 100%, i.e. the present upper bound of $B(B_s \to \mu^+\mu^-)$ does not put any constraint on A_{LP}. Indeed, $B(B_s \to \mu^+\mu^-)$ will be unable to put any constraint on A_{LP} even if it is as low as 10^{-8}.

Thus we see that the recently improved upper bound on the branching ratio of $B_s \to \mu^+\mu^-$, which provides the most stringent bound on SPNP couplings, fails to put any bound on A_{LP}. *Therefore A_{LP} is more sensitive to SPNP operators as compared to $B(B_s \to \mu^+\mu^-)$.* Any nonzero measurement of A_{LP} will be evidence for an extended Higgs sector.

We would like to emphasize another important point: The measurement of $B(B_s \to \mu^+\mu^-)$ will only give the allowed range for the values of the SPNP couplings R_S and R_P. However the simultaneous determination of $B(B_s \to \mu^+\mu^-)$ and A_{LP} will allow the determination of new physics scalar coupling R_S (see Fig. 2) and this in turn will enable us to determine the new physics pseudoscalar coupling R_P.

B. $B(B_s \to \mu^+\,\mu^-) \lesssim 10^{-8}$

LHC is expected to reach the SM sensitivity in $B_s \to \mu^+\,\mu^-$. In fact, it may even go 5σ below the SM prediction [11]. Therefore it is worth considering the case when $B(B_s \to \mu^+\,\mu^-)$ is of the order of the SM prediction. In this section we study the correlation between A_{LP} and $B(B_s \to \mu^+\mu^-)$ under the assumption that $B(B_s \to \mu^+\,\mu^-)$ is close to its SM prediction.

Taking $R_A = R_A^{SM}$, eq. (35) gives

$$B(B_s \to \mu^+\,\mu^-) = a_s\Big[28.8(R_S^2\,+\,R_P^2)\,-\,9.7R_P\,+\,0.81\Big]\,, \tag{42}$$

which leads to

$$R_S^2\,+\,(R_P\,-\,0.165)^2\;=\;\frac{0.035\,B(B_s \to \mu^+\,\mu^-)}{a_s}\,. \tag{43}$$

This corresponds to a circle in $R_S - R_P$ plane with centre at $(R_S = 0,\ R_P = 0.165)$ and radius given by $r = \sqrt{0.035\,B(B_s \to \mu^+\,\mu^-)/a_s}$.

Fig. 3 shows the plot between A_{LP} and R_S for three different values of $B(B_s \to \mu^+\,\mu^-) \lesssim 10^{-8}$. Fig. 4 is a plot between $|A_{LP}|$ and $B(B_s \to \mu^+\mu^-)$ for various allowed values of R_S. It is obvious from fig. 3 that A_{LP} can be 100% even if $B(B_s \to \mu^+\mu^-)$ is close to its SM prediction.

We now consider three exciting experimental possibilities, all of which can be accounted for with SPNP.

1. $B(B_s \to \mu^+\,\mu^-)$ *is consistent with SM but* $A_{LP} \neq 0$

It is possible to have a non-zero value of A_{LP} even if $B(B_s \to \mu^+\mu^-)$ is equal to its SM prediction. We can re-write eq. (35) in the following form:

$$B(B_s \to \mu^+\,\mu^-) = a_s[(b_{SM} - b_P)^2\,+\,b_S^2]\,, \tag{44}$$

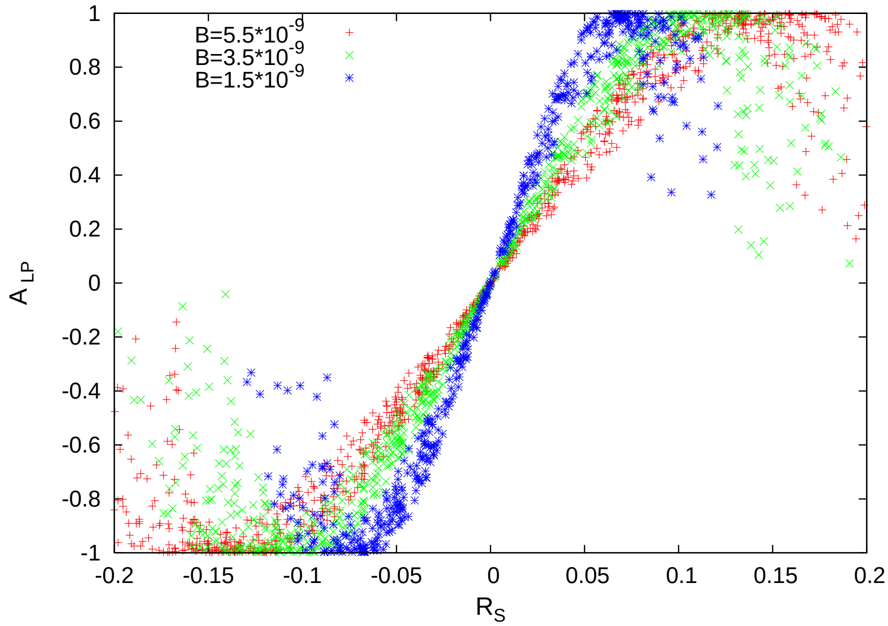

FIG. 3: A_{LP} vs R_s plot for $B(B_s \to \mu^+\mu^-) = (5.5, 3.5, 1.5) \times 10^{-9}$

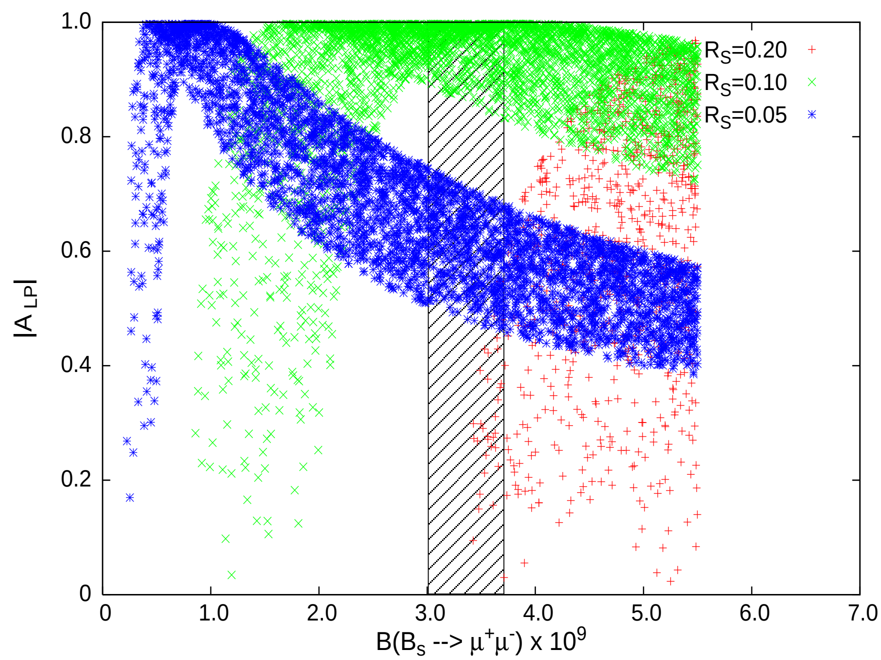

FIG. 4: Plot between $|A_{LP}|$ and $B(B_s \to \mu^+\mu^-)$ for different R_S values, when $B(B_s \to \mu^+\,\mu^-) \lesssim 10^{-8}$. The vertical shaded band corresponds to 1σ theoretical prediction within the SM. The region $B(B_s \to \mu^+\,\mu^-) \gtrsim 5 \times 10^{-9}$ is not ruled out; here we just concentrate on the region near the SM prediction.

where

$$b_{SM} = 2m_\mu R_A^{SM} \;, \quad b_P = \frac{m_{B_s}^2}{m_b + m_s} R_P \;, \quad b_S = \sqrt{1 - \frac{4m_\mu^2}{m_{B_s}^2}} \frac{m_{B_s}^2}{m_b + m_s} R_S \;. \tag{45}$$

Here we have taken $R_A = R_A^{SM}$, i.e. we have considered new physics only through the SPNP operators. Now if $B(B_s \to \mu^+\mu^-)$ is equal to its SM prediction, then

$$a_s[(b_{SM} - b_P)^2 \,+\, b_S^2] = a_s\, b_{SM}^2 \;, \tag{46}$$

which leads to

$$(b_P - b_{SM})^2 \,+\, b_S^2 = b_{SM}^2 \;, \tag{47}$$

or

$$R_S^2 \,+\, \left[R_P - \frac{(m_b + m_s)}{m_{B_s}^2} b_{SM}\right]^2 = \left(\frac{m_b + m_s}{m_{B_s}^2} b_{SM}\right)^2 \;. \tag{48}$$

Eq. (48) represents a circle in $R_S - R_P$ plane with center at $\big(0, (m_b + m_s) b_{SM}/m_{B_s}^2\big)$.

The circle representing eq. (48) passes through the origin ($R_S = R_P = 0$), which corresponds to the SM. However, in general the points on the circle have nonzero R_S, and hence imply nonvanishing A_{LP}. Therefore it is possible to have a nonzero value of A_{LP} even if $B(B_s \to \mu^+\mu^-)$ is equal to its SM prediction. Thus A_{LP} can still serve as an important observable to search for SPNP even if $B(B_s \to \mu^+\mu^-)$ is observed to be very close to its SM prediction.

2. LHCb fails to find $B_s \to \mu^+\,\mu^-$

If LHCb fails to find $B_s \to \mu^+\,\mu^-$ or puts an upper bound on its branching ratio which is smaller than 2×10^{-9} (5σ *below SM prediction*), this scenario can still be accomodated within the SPNP.

The interference between the SPNP and SM operators can decrease the branching ratio $B(B_s \to \mu^+\mu^-)$ far below its SM prediction. In fact it can be seen from eq. (35), $B(B_s \to \mu^+\mu^-)$ can even vanish, provided the following conditions are satisfied simultaneously:

$$R_S = 0, \quad R_P = \frac{2m_\mu m_b}{m_{B_s}^2} R_A = 0.04\, R_A \;. \tag{49}$$

From Fig. 4, it can be seen that for low R_S values, it is indeed possible to suppress $B(B_s \to \mu^+\,\mu^-)$ much below its SM value.

3. Both $B(B_s \to \mu^+ \mu^-)$ and A_{LP} are consistent with the SM

The lepton polarization asymmetry is a result of the interference of the scalar term with pseudoscalar / axial vector, as can be seen from eq. (40). Therefore it vanishes when either b_S or $(b_P - b_{SM})$, defined in eq. (45), vanishes. Thus there exists the interesting possibility of nontrivial SPNP even when both $B(B_s \to \mu^+ \mu^-)$ and A_{LP} are consistent with the SM. This occurs when

$$b_S = 0 \; , \quad b_P = 2\, b_{SM} \; , \tag{50}$$

$$\text{or} \qquad b_S = b_{SM} \; , \quad b_P = b_{SM} \; , \tag{51}$$

as can be confirmed from eq. (47). Therefore, the absence of SPNP is not guaranteed simply by the consistency of these observables with the SM; more channels need to be examined to rule out this possibility completely.

IV. CONCLUSIONS

An order of magnitude enhancement in $B(B_s \to \mu^+\mu^-)$ is possible only due to SPNP operators. Apart from $B(B_s \to \mu^+\mu^-)$, observables such as FB asymmetry of muons in $B \to K\mu^+ \mu^-$ and LP asymmetry of muons in $B_s \to \mu^+\mu^-$ are also sensitive to SPNP operators. In this paper we consider the constraints on possible SPNP contribution to these observables coming from the present upper bound on $B(B_s \to \mu^+\mu^-)$.

We find that $B(B_s \to \mu^+\mu^-)$ puts very stringent constraint on SPNP contribution to $\langle A_{FB} \rangle$ and restricts its value to be less than $\sim 1\%$. Such a small FB asymmetry is almost impossible to be measured in experiments. In the literature, $\langle A_{FB} \rangle$ of muons in $B \to K\mu^+ \mu^-$ has been considered a promising measurement for probing SPNP operators. Our results show that the present upper bound on $B(B_s \to \mu^+ \mu^-)$ makes searching for SPNP through $\langle A_{FB} \rangle$ a futile exercise.

On the other hand, the present upper bound on $B(B_s \to \mu^+\mu^-)$ does not put any constraint on A_{LP}. Indeed, A_{LP} can be 100% even if $B(B_s \to \mu^+\mu^-)$ is close to its SM prediction. A_{LP} is sensitive only to SPNP operators and hence its nonzero value will give direct evidence for a non-standard Higgs sector.

A simultaneous determination of $B(B_s \to \mu^+\mu^-)$ and A_{LP} will enable us to separate the new physics scalar and pseudoscalar contributions. Therefore it is worth considering this observable in experiments to probe the extended Higgs sector.

Acknowledgments

One of the authors, A. K. Alok would like to acknowledge Y. Grossman for his valuable suggestions.

---

[1] B. Aubert *et al.* [BABAR Collaboration], *"Evidence for the rare decay $B \to K^*\ell^+\ell^-$ and measurement of the $B \to K\ell^+\ell^-$ branching fraction"*, Phys. Rev. Lett. **91**, 221802 (2003) [arXiv:hep-ex/0308042].

[2] B. Aubert *et al.* [BABAR Collaboration], *"Measurements of branching fractions, rate asymmetries, and angular distributions in the rare decays $B \to K\ell^+\ell^-$ and $B \to K^*\ell^+\ell^-$"*, Phys. Rev. D **73**, 092001 (2006) [arXiv:hep-ex/0604007].

[3] A. Ishikawa *et al.* [Belle Collaboration], *"Observation of the electroweak penguin decay $B \to K^*\ell^+\ell^-$"*, Phys. Rev. Lett. **91**, 261601 (2003) [arXiv:hep-ex/0308044].

[4] A. Ali, E. Lunghi, C. Greub and G. Hiller, *"Improved model-independent analysis of semileptonic and radiative rare B decays"*, Phys. Rev. D **66**, 034002 (2002) [arXiv:hep-ph/0112300].

[5] E. Lunghi, *"Improved model-independent analysis of semileptonic and radiative rare B decays"*, arXiv:hep-ph/0210379.

[6] F. Kruger and E. Lunghi, *"Looking for novel CP violating effects in $\bar{B} \to K^*\ell^+\ell^-$"*, Phys. Rev. D **63**, 014013 (2001) [arXiv:hep-ph/0008210].

[7] M. Blanke, A. J. Buras, D. Guadagnoli and C. Tarantino, *"Minimal Flavor Violation Waiting for Precise Measurements of ΔM_s, $S_{\psi\phi}$, A^s_{SL}, $|V_{ub}|$, γ and $B^0_{s,d} \to \mu^+\mu^-$"*, JHEP **0610**, 003 (2006) [arXiv:hep-ph/0604057].

[8] G. Buchalla and A. J. Buras, *"QCD corrections to rare K and B decays for arbitrary top quark mass"*, Nucl. Phys. B **400**, 225 (1993).

[9] A. J. Buras, *"Relations between $\Delta M_{s,d}$ and $B_{s,d} \to \mu\bar{\mu}$ in models with minimal flavor violation"*, Phys. Lett. B **566**, 115 (2003) [arXiv:hep-ph/0303060].

[10] T. Aaltonen *et al.* [CDF Collaboration], *"Search for $B_s \to \mu^+\mu^-$ and $B_d \to \mu^+\mu^-$ Decays with $2fb - 1$ of ppbar Collisions"*, Phys. Rev. Lett. **100**, 101802 (2008) [arXiv:0712.1708 [hep-ex]].

[11] O. Schneider [for LHCb Collaboration], *"Expected LHCb physics with first 2 years of data"*, Talk given at CERN SPC meeting, Dec 2007. `http://lhcb-doc.web.cern.ch/lhcb-doc/progress/Source/CERNSPC/OSchneider_SPC_Dec2007.ppt`

[12] M. Smizanska [ATLAS, CMS, LHCb Collaborations], *"LHC preparations for measurements of very rare B decays"*, Nucl. Phys. Proc. Suppl. **170**, 210 (2007).

[13] M. Gronau and D. London, *"New physics in CP asymmetries and rare B decays"*, Phys. Rev. D **55**, 2845 (1997) [arXiv:hep-ph/9608430].

[14] J. L. Hewett, S. Nandi and T. G. Rizzo, *"$B \to \mu^+\mu^-$ in the two Higgs doublet model"*, Phys. Rev. D **39**, 250 (1989).

[15] A. K. Alok and S. U. Sankar, *"New physics upper bound on the branching ratio of $B_s \to l^+l^-$"*, Phys. Lett. B **620**, 61 (2005) [arXiv:hep-ph/0502120].

[16] A. Ali, T. Mannel and T. Morozumi, *"Forward backward asymmetry of dilepton angular distribution in the decay $b \to sl^+l^-$"*, Phys. Lett. B **273**, 505 (1991).

[17] L. T. Handoko, C. S. Kim and T. Yoshikawa, *"Longitudinal lepton polarization asymmetry in pure leptonic B decays"*, Phys. Rev. D **65**, 077506 (2002) [arXiv:hep-ph/0112149].

[18] A. Ali, P. Ball, L. T. Handoko and G. Hiller, *"A comparative study of the decays $B \to (K, K^*)l^+l^-$ in standard model and supersymmetric theories"*, Phys. Rev. D **61**, 074024 (2000) [arXiv:hep-ph/9910221].

[19] Q. S. Yan, C. S. Huang, W. Liao and S. H. Zhu, *"Exclusive semileptonic rare decays $B \to (K, K^*)l^+l^-$ in supersymmetric theories"*, Phys. Rev. D **62**, 094023 (2000) [arXiv:hep-ph/0004262].

[20] C. Bobeth, T. Ewerth, F. Kruger and J. Urban, *"Analysis of neutral Higgs-boson contributions to the decays $\bar{B}_s \to l^+l^-$ and $\bar{B} \to Kl^+l^-$"*, Phys. Rev. D **64**, 074014 (2001) [arXiv:hep-ph/0104284].

[21] G. Erkol and G. Turan, *"The exclusive $B \to (K, K^*)l^+l^-$ decays in a CP spontaneously broken two Higgs doublet model"*, Nucl. Phys. B **635**, 286 (2002) [arXiv:hep-ph/0204219].

[22] D. A. Demir, K. A. Olive and M. B. Voloshin, *"The forward-backward asymmetry of $B \to (\pi, K)l^+l^-$: Supersymmetry at work"*, Phys. Rev. D **66**, 034015 (2002) [arXiv:hep-ph/0204119].

[23] W. J. Li, Y. B. Dai and C. S. Huang, *"Exclusive semileptonic rare decays $B \to K^*l^+l^-$ in a SUSY $SO(10)$ GUT"*, Eur. Phys. J. C **40**, 565 (2005) [arXiv:hep-ph/0410317].

[24] C. H. Chen, C. Q. Geng and A. K. Giri, *"Looking for forward backward asymmetries in $B \to K\mu^+\mu^-$ and $K^+ \to \pi^+\mu^+\mu^-$"*, Phys. Lett. B **621**, 253 (2005) [arXiv:hep-ph/0507022].

[25] A. Ishikawa *et al.*, *"Measurement of forward-backward asymmetry and Wilson coefficients in $B \to K^*l^+l^-$"*, Phys. Rev. Lett. **96**, 251801 (2006) [arXiv:hep-ex/0603018].

[26] K. Ikado [Belle Collaboration], *"Measurements of forward-backward asymmetry in $B \to K^*l^+l^-$ and evidence of $B^- \to \tau^-\bar{\nu}$"*, arXiv:hep-ex/0605067.

[27] A. J. Buras and M. Munz, *"Effective Hamiltonian for $B \to X_s e^+e^-$ beyond leading logarithms in the NDR and HV schemes"*, Phys. Rev. D **52**, 186 (1995) [arXiv:hep-ph/9501281].

[28] M. Misiak, *"The $b \to se^+e^-$ and $b \to s\gamma$ decays with next-to-leading logarithmic QCD corrections"*, Nucl. Phys. B **393**, 23 (1993) [Erratum-ibid. B **439**, 461 (1995)].

[29] P. B. Mackenzie, *"CKM physics from lattice QCD", In the Proceedings of 4th Flavor Physics and CP Violation Conference (FPCP 2006) Vancouver, British Columbia, Canada, 9-12 Apr 2006, pp 022* [arXiv:hep-ph/0606034].

[30] W. M. Yao *et al.* [Particle Data Group], *"Review of particle physics"*, J. Phys. G **33**, 1 (2006).

[31] A. K. Alok, A. Dighe and S. U. Sankar, *"Tension between scalar/pseudoscalar new physics contribution to $B_s \to \mu^+\mu^-$ and $B \to K\mu^+\mu^-$"*, arXiv:0803.3511 [hep-ph].

[32] F. J. Botella and C. S. Lim, *"Flavor changing Yukawa coupling in the standard model and muon polarization in $K_L \to \mu\bar{\mu}$"*, Phys. Rev. Lett. **56**, 1651 (1986).

[33] P. Herczeg, *"Muon polarization in $K_L \to \mu^+\mu^-$"*, Phys. Rev. D **27**, 1512 (1983).

[34] C. Q. Geng and J. N. Ng, *"Relating fermion electric dipole moments and muon polar-*

ization in $\eta \to \mu\bar{\mu}$, $K_L^0 \to \mu\bar{\mu}$ decays with the scalar-pseudoscalar mixing mechanism", Phys. Rev. Lett. **62**, 2645 (1989).

[35] G. Ecker and A. Pich, *"The Longitudinal Muon Polarization In $K_L \to \mu^+\mu^-$"*, Nucl. Phys. B **366**, 189 (1991).

[36] Y. Grossman, Z. Ligeti and E. Nardi, *"$B \to \tau^+\tau^- X$ decays: First constraints and phenomenological implications"*, Phys. Rev. D **55**, 2768 (1997) [arXiv:hep-ph/9607473].

[37] D. Guetta and E. Nardi, *"Searching for new physics in rare $B \to \tau$ decays"*, Phys. Rev. D **58**, 012001 (1998) [arXiv:hep-ph/9707371].

[38] T. Inami and C. S. Lim, *"Effects Of Superheavy Quarks And Leptons In Low-Energy Weak Processes $K_L \to \mu\bar{\mu}$, $K^+ \to \pi^+\nu\bar{\nu}$ and $K^0 \leftrightarrow \bar{K^0}$"*, Prog. Theor. Phys. **65**, 297 (1981) [Erratum-ibid. **65**, 1772 (1981)].