

Probing CP violation with the three years ultra-high energy neutrinos from IceCube

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The IceCube collaboration has recently announced the discovery of ultra-high energy neutrino events. These neutrinos can be used to probe their production source, as well as leptonic mixing parameters. In this work, we have used the first IceCube data to constrain the leptonic CP violating phase δ_{CP} . For this, we have analyzed the data in the form of flux ratios. We find that the fit to δ_{CP} depends on the assumptions made on the production mechanism of these astrophysical neutrinos. Consequently, we also use this data to impose constraints on the sources of the neutrinos.

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Introduction: The study of cosmic particles, and through them the study of astrophysical phenomena has gradually moved up the energy scale over the last few decades. GeV scale cosmic rays have already been observed in atmospheric neutrino experiments for many years. With the recent detection of ultra-high energy (UHE) neutrinos at the IceCube detector [1, 2], we have emphatically entered the TeV regime. In fact, the PeV energy events seen by IceCube underscore the prospects of neutrino astrophysics with large telescopes.

The first data set announced by the IceCube collaboration consists of 28 events above 25 TeV, detected over a period of 662 days of live time (May 2011 – May 2012 with 79 strings, and May 2012 – May 2013 with 86 strings). 7 out of these 28 events are tracks signifying $(\nu_\mu + \bar{\nu}_\mu)$ charged-current (CC) events; while the other 21 are showers indicating either $(\nu_e + \bar{\nu}_e)$ or $(\nu_\tau + \bar{\nu}_\tau)$, or $(\nu_\mu + \bar{\nu}_\mu)$ neutral-current (NC) events [3]. This 4σ detection marked the first discovery of UHE neutrinos. Further data was collected for next one year. For the full 988 days IceCube collected 37 events, adding 1 track, 7 shower events and 1 was produced by a coincident pair of background muons from unrelated air showers that cannot be reconstructed with a single direction and energy, to the previously detected 28 events including the highest neutrino of energy 2000 TeV, ever detected [2].

UHE cosmic rays secondaries like photons and neutrinos carry information about their production (source) and propagation. UHE neutrinos can be produced by a wide array of astrophysical and cosmological processes. It may be possible to probe the mechanism of their production by observing them at neutrino telescopes. Moreover, as these neutrinos travel from their source to the earth, they oscillate. Therefore, one can use this information to constrain mixing in the leptonic sector. In this paper, we have used the first data set from IceCube to address questions about astrophysical neutrino production and neutrino oscillations.

Data from various neutrino oscillation experiments have constrained the mixing angles and mass-squared differences at the $\leq 10\%$ level [4]. However, the value of the CP violating phase δ_{CP} is not constrained by the data. The measurement of δ_{CP} is one of the outstanding

problems in particle physics today, since CP violation in the leptonic sector can be linked to leptogenesis and the matter-antimatter asymmetry of the Universe [5]. Measuring the value of δ_{CP} can also provide valuable insights into new physics beyond the Standard Model, since CP violation can arise in various models of neutrino mass generation through complex couplings of neutrinos to other particles, or through complex vacuum expectation values [6].

The measurement of δ_{CP} through oscillations of atmospheric/artificially-produced neutrinos is difficult using existing technology, and therefore new strategies have to be devised. Many interesting proposals exist in the literature for getting an evidence of CP violation and/or measuring δ_{CP} (for a non-exhaustive list, see Refs. [7–9]). In this paper, for the first time, we analyze actual UHE neutrino data to measure δ_{CP} and determine the source of astrophysical neutrinos. In Ref. [7], the author discussed in detail the complementary nature of astrophysical and terrestrial neutrino experiments in CP studies. In that study (and more recently in Ref. [8]), data in the form of flavour ratios of observed neutrinos was used. In this work, we have analysed data from IceCube using a similar approach to get a hint about the value of δ_{CP} .

Astrophysical sources: The data recorded by the IceCube telescope is the first evidence of extra-terrestrial events in the UHE range. These neutrinos can have their origin in extragalactic astrophysical sources like low power Gamma-Ray Burst (GRB) jets in stars [10] or Active Galactic Nuclei (AGN) cores [11]. (Note however, that based on the data collected by the photon detectors Fermi, MAGIC, HESS, etc. in the 100 GeV – TeV range, one can predict galactic sources of TeV neutrinos [12].) The energy of the 28 detected neutrino events are in the range 25 – 2000 TeV. By tracing the hadronic origin [13] of these events, one can estimate the proton energies at their sources to be within 0.5 – 40 PeV. Supernova Remnants (SNRs), AGNs, GRBs and other astrophysical sources can accelerate protons to such energies (and above) by the Fermi acceleration mechanism. The interactions of these protons with soft photons or matter from the source can give UHE neutrinos

through the following process: $p\gamma, pp \rightarrow \pi^\pm X$, $\pi^\pm \rightarrow \mu^\pm \nu_\mu (\bar{\nu}_\mu)$, $\mu^\pm \rightarrow e^\pm \bar{\nu}_\mu (\nu_\mu) \nu_e (\bar{\nu}_e)$ [14] with a flux ratio of $\phi_{\nu_e} : \phi_{\nu_\mu} : \phi_{\nu_\tau} = 1 : 2 : 0$ (known as πS process). Some of the muons, due to their light mass, can get cooled in the magnetic field quickly resulting in a neutrino flux ratio of $0 : 1 : 0$ (μDS process). K -mesons, produced from $p\gamma$ interactions with a cross-section two orders of magnitude less than pions, will cool in the magnetic field of the source at higher energies compared to the pions. $K^+ \rightarrow \mu^+ \bar{\nu}_\mu$ is the dominant channel of neutrino production from cooled pions, with a branching fraction of 63%, and with the same flux ratio as the pion decay [15]. The $p\gamma$ interaction also produces high energy neutrons which would decay as $n \rightarrow p + e^- + \bar{\nu}_e$ to anti-neutrinos [16] with the flux ratio of $1 : 0 : 0$ (nS process). The relative contribution of each channel depends on different parameters of the astrophysical source like the magnetic field, the strength of the shock wave and density of photon background [17]. Apart from neutrinos these processes also produce high energy photons inside the source. Correlation of high energy photons with the UHE neutrinos can be considered as a signature of hadronic production inside the source. For example, a TeV neutrino can have an accompanying TeV photon at the source. However due to attenuation in the background radiation during propagation, PeV photons will have typical mean free path ~ 10 kpc [18]. Thus, the associated photons of TeV neutrinos from extragalactic sources cannot reach earth.

Analysis: The main sources of astrophysical neutrinos in the energy range 10 TeV to 1200 TeV are the πS , μDS and nS channels. However, the exact fraction of events in the detector from each of these sources is not known. Therefore, we have introduced relative fractions k_1 , k_2 and k_3 for these three sources respectively, which are treated as free parameters in the problem subject to the normalization constraint $\sum k_i = 1$. In this study, we have not considered any other sub-dominant mode of neutrino production.

Neutrinos oscillate during propagation, and our aim is to observe these oscillations. Given that the value of E/L for such neutrinos is very large compared to the mass-squared differences between the neutrino mass states, we can only observe the average oscillation probability. Therefore, the probabilities take the simple form:

$$P(\nu_\alpha \rightarrow \nu_\beta) \equiv P_{\alpha\beta} = \sum_i |U_{\alpha i}|^2 |U_{\beta i}|^2. \quad (1)$$

It is worth emphasizing that this oscillation probability depends only on the mixing angles and CP phase, but not on the mass-squared differences. Therefore, unlike in beam-based experiments where knowledge of the mass hierarchy is essential for CP sensitivity [19], in this case we can (at least in principle) detect CP violation without suffering from the hierarchy degeneracy. Also note that $P_{\alpha\beta} = P_{\beta\alpha}$, therefore the probability can only be an even function of δ_{CP} . As a consequence, we can treat neutrino and antineutrino oscillations on an equal footing. Another consequence of this is that every value of δ_{CP} allowed by the data will be accompanied by a de-

generate solution $-\delta_{CP}$.

The distinction between tracks (which we assume to be ν_μ CC events) and showers (which we assume to be ν_e or ν_τ , or ν_μ NC events) is quite clear in the IceCube detector. We have folded the relative initial fluxes with the oscillation probabilities to get the relative number of events at the detector. Separation of muon events into CC and NC has been done using the ratio of the cross-sections at the relevant energy [20]. We have done a simple analysis using the total events, instead of binning the data in energy and angle. Since the probability is almost independent of energy, this simplification is not expected to affect the analysis. This also allows us to neglect the effect of energy resolution. In Ref. [1], the number of background events in the IceCube data set is estimated to be $10.6^{+5.0}_{-3.6}$. Of these, 6.0 ± 3.4 are expected to be veto penetrating atmospheric muons and $4.6^{+3.7}_{-1.2}$ are from the atmospheric neutrino background above energy 10 TeV. The background assumed by IceCube could be an overestimation [21], since (a) it has been estimated by extrapolating data, and (b) for atmospheric neutrinos the background has been calculated from 10 TeV while the events have been detected with lowest energy nearly 28 TeV. Therefore, we have used an estimate of 3 background atmospheric muon tracks and 3.4 (the lower limit) background atmospheric neutrinos. IceCube have predicted a total of 8.4 ± 4.2 muon events and $6.6^{+5.9}_{-1.6}$ atmospheric neutrinos [2] including the next set of neutrino events detected for the period of 988 days. Using the same analysis method we have taken the lowest limit of the backgrounds for our calculation. We have separated the background atmospheric neutrinos into tracks and showers using the same cross-sections as mentioned earlier. These background events are subtracted from the data set in our analysis. The neutrino background flux ratio has been included as 0:1:0, (which is close to 0.05:1:0 estimated in Ref. [22]) since TeV range muons will penetrate all the way through the atmosphere.

In Refs. [7, 8, 23], the authors have proposed the use of the variable $R = N_\mu / (N_e + N_\tau)$ for the study of CP violation with astrophysical sources, where N_α is the flux of $\nu_\alpha + \bar{\nu}_\alpha$ at the detector. This variable helps by eliminating the overall source and detector-dependent normalization. Moreover, as studies of the up/down ratio as well as data/MC ratio in atmospheric neutrinos have shown, taking ratios of event rates can also reduce the effect of systematics [24]. For our study, we have constructed a similar quantity $\rho = N_{track} / N_{shower}$, with the flavour compositions of the track and shower events as mentioned before.

We have constructed the quantity ρ^{data} using the IceCube data, and calculated ρ^{theory} for a certain value of δ_{CP} as described above. Background events are subtracted from the data, as mentioned above. The statistical χ^2 is then computed using the Gaussian definition

$$\chi^2(\delta_{CP}) = \left(\frac{\rho^{data} - \rho^{theory}}{\sigma_\rho} \right)^2, \quad (2)$$

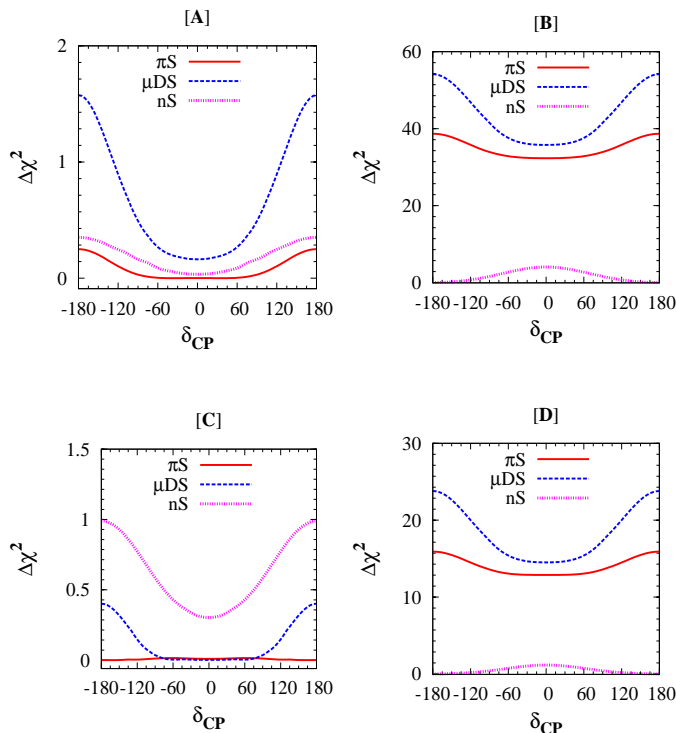


FIG. 1. Fit to δ_{CP} considering a single source i.e., πS ($k_1 = 1, k_2 = 0, k_3 = 0$, solid red), μDS ($k_1 = 0, k_2 = 1, k_3 = 0$, dashed blue), nS ($k_1 = 0, k_2 = 0, k_3 = 1$, dotted magenta), considering all the events being from astrophysical environment. Panel [A]: Three-year data, without background; Panel [B]: Three-year data, with background; Panel [C]: Two-year data, without background; Panel [D]: Two-year data, with background.

where $\sigma_\rho = \sqrt{\rho^{data}(1-\rho^{data})/N}$, N being the number of data points [25]. We have incorporated systematic effects using the method of pulls, with a systematic error of 5%. Note that, we have marginalized the $\Delta\chi^2$ over the mixing angles ($\theta_{23}, \theta_{13}, \theta_{12}$) within the ranges $\theta_{23} = 35^\circ$ to 55° , $\sin^2 2\theta_{13} = 0.085$ to 0.115 and $\theta_{12} = 30^\circ$ to 36° respectively. The priors added are $\sigma(\sin^2 2\theta_{13}) = 0.01$, $\sigma(\sin^2 2\theta_{23}) = 0.1$ and $\sigma(\sin^2 \theta_{12}) = 0.0155$.

Results:

To demonstrate the impact of the origin of these astrophysical neutrinos on the precision of δ_{CP} , we start with various possibilities, like, single, double or a combination of three sources as the origin. First we show the fit to the data as a function of δ_{CP} for the single source assumption, in Fig. 1. The upper row shows the results of our analysis of the full three-year data set. We have also included the results from analysing data from only the first two years (lower row) to show the improvement in results from additional data.

In the left panels we assume that all the events seen at IceCube are purely of astrophysical origin whereas in the right panels we include the effect of backgrounds. The latter is the realistic assumption. From these figures we

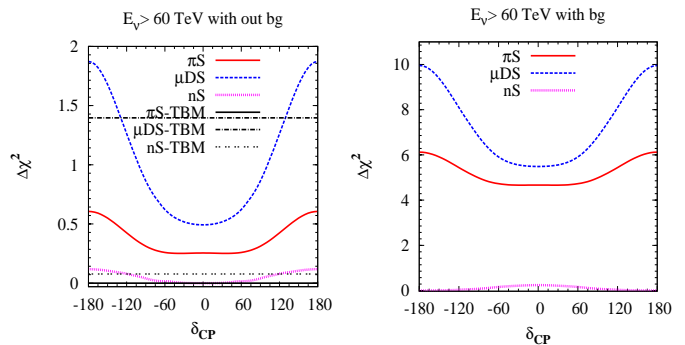


FIG. 2. Analysis for neutrinos within energy 60 TeV to 3 PeV.

can see that, in case of no background the πS source is favoured by the data as compared to the nS and μDS source (though the sensitivity is quite small, as $\Delta\chi^2$ is always < 1.5). However, when we include the background, the scenario changes completely. Pure πS and pure μDS sources are ruled out by the data at $> 3\sigma$, while the pure nS source is favoured by data, though it is not sufficient to put any significant constraint on the value of δ_{CP} . This has also been pointed out recently in Ref. [21]. This result can be understood qualitatively in the following way. In the 2nd column of Table I we have listed the theoretically calculated values for track by shower ratio for all the three sources keeping the oscillation parameters fixed at their tri-bimaximal (TBM) values ($\theta_{23} = 45^\circ$, $\theta_{13} = 0^\circ$, $\sin^2 \theta_{12} = \frac{1}{3}$)¹ whereas the third column contains the experimental values of the track by shower ratio without and with backgrounds. We can clearly see that for a pure signal, the track to shower ratio for πS is closest to the data. But the difference becomes quite high when backgrounds are taken under consideration, resulting in a very high $\Delta\chi^2$. A comparison of the upper and lower panels shows a marked increase in $\Delta\chi^2$. This shows the importance of additional data in both, excluding certain combinations of sources as well as constraining the value of δ_{CP} .

We have also done an analysis of the events in the energy range $60 \text{ TeV} < E < 3 \text{ PeV}$ considering the 3 years of IceCube data. This is motivated by the fact that, this energy interval contains the atmospheric muon background less than one. In this energy range there are 4 track events and 16 shower events with an atmospheric muon background of 0.435 and atmospheric neutrino background of 2.365 [2]. The result is plotted in Fig. 2. In the left panel there is no background and in the right panel background has been considered. From the right panel we can see that we are still getting nS

¹ Due to the present non zero value of θ_{13} , there will be deviations from the TBM values but as shown in Ref. [8], this deviation is quite small.

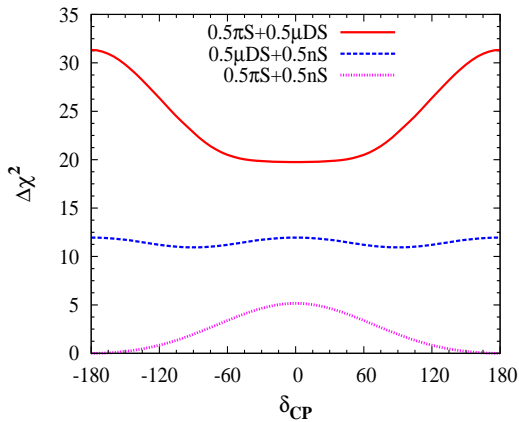


FIG. 3. Fit to δ_{CP} considering contribution from two sources at a time, in equal proportion i.e., $k_1 = k_2 = 0.5, k_3 = 0$ (solid, red), $k_1 = 0, k_2 = k_3 = 0.5$ (dashed, blue), $k_1 = k_3 = 0.5, k_2 = 0$ (dotted, magenta) for both old **a** and new **B**.

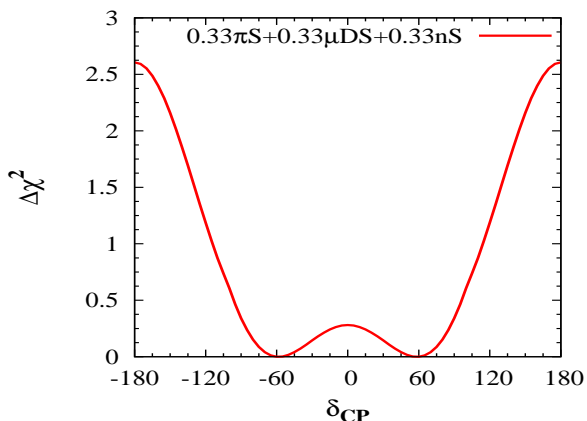


FIG. 4. Fit to δ_{CP} considering equal contribution from all the sources, i.e., $k_1 = k_2 = k_3 = 0.33$ for both old **a** and new **B**.

as the favoured source whereas πS and μDS sources are excluded at more than 2σ . This is due to the fact that though the atmospheric muon background is less than one in this energy range but due to the presence of atmospheric neutrino background nS is getting preferred over πS source. This can be seen from the left panel where no background is considered. There we can note that the data agrees with the final flavor ratio 1:1:1 i.e. it favours the πS source over nS source marginally when TBM mixing is assumed. But when we vary the oscillation parameters in their allowed 3σ range then due to the deviation from TBM, nS is getting slightly preferred over πS .

In Fig. 3 and Fig. 4 we show the fit to the data when neutrinos are coming from two/all the three sources respectively, with equal contributions. These results are for the full data set, and backgrounds have been included in generating these plots. In Fig. 3, we find only the combination of πS and nS neutrinos are allowed at 3σ level. We

Source	$\frac{N_{track}}{N_{shower}}$ (Calculated)	$\frac{N_{track}}{N_{shower}}$ (Data)
πS	0.30	8/28=0.287(Without background)
μDS	0.38	
nS	0.18	0.06(With background)

TABLE I. Theoretical values of track by shower ratio for all the three sources along with experimental values with and without background.

also see that the CP dependence is maximum if the neutrinos come from the combination of πS and μDS modes. The data may also rule out one-third of δ_{CP} values (approximately -60° to 60°) at $\sim 2\sigma$. The poor sensitivity from nS neutrinos is the reason why the combination of $\pi S + \mu DS$ in Fig. 3 has a higher χ^2 than the combinations involving nS . When we consider equal contributions from all these channels (Fig. 4), we find that the data favours the first and fourth quadrants of δ_{CP} at 1σ .

We have then performed a check to constrain the astrophysical parameters k_i vs δ_{CP} using the IceCube data, by plotting the allowed contours in the $k_i - \delta_{CP}$ plane. In Fig. 5, we have showed the 2σ (light) and 3σ (dark) contours in the $k_1 - \delta_{CP}$ plane for three fixed values of k_2 . The best-fit point indicated by the data has been marked with a red dot. We see that the data favours a smaller value of k_1 and larger values of k_2 and k_3 . Similarly, Fig. 6 shows that for a given value of k_1 , the data disfavours the μDS process (small value of k_2) but favours the nS process (large value of k_3). Likewise, Fig. 7 shows the data favouring the largest possible value of k_3 allowed by the normalization condition. These features can be understood from Fig. 1, where we see that the data prefers the nS source. From these contours, we may draw certain constraints on the astrophysical sources most favoured. In particular if we obtain a good prior on δ_{CP} from other experiments, then the most favoured ratio of k_1, k_2 and k_3 may be obtained. Alternately, if we obtain a better picture of the sources of the IceCube events, a refined and constrained range on δ_{CP} would be predicted.

To show the statistical improvement of the 3 year data over 2 year data, in Fig. 5 we have also plotted the 2σ and 3σ contours for the latter for $k_2 = 0$. Here we can clearly see that for $\delta_{CP} = 0$, 3 year data can exclude 73%(91.5%) of k_1 values at $3\sigma(2\sigma)$ where as the 2 year data can only rule out 13%(61%) of K_1 values at $3\sigma(2\sigma)$. For $\delta_{cp} = \pi$ the exclusion percentages are 58%(73%) at $3\sigma(2\sigma)$ for 3 years and 20%(48%) at $3\sigma(2\sigma)$ for 2 years. One can understand this qualitatively from the πS curve of Fig. 1 showing a significant improvement in the $\Delta\chi^2$ with 3 years of data compared to 2 years.

Conclusion: In this work, we have analyzed the first IceCube data on TeV-PeV scale neutrinos. We have used the flux ratios of the three neutrino flavours to put constraints on δ_{CP} . We find that the results depend strongly on the source of the neutrinos. After taking into account

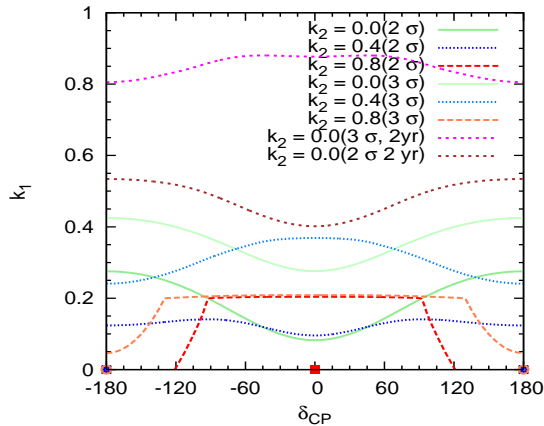


FIG. 5. Contour plots for allowed region in the $k_1 - \delta_{CP}$ plane, for three representative values of k_2 . The points marked in the respective colours indicate the best-fit point with new IceCube data.

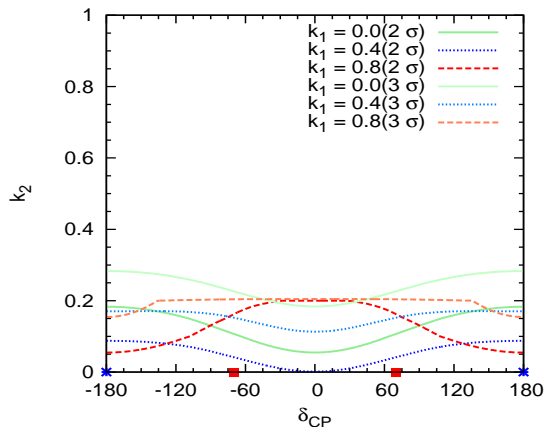


FIG. 6. Contour plots for allowed region in the $k_2 - \delta_{CP}$ plane, for three representative values of k_1 . The points marked in the respective colours indicate the best-fit point.

the effect of backgrounds, we find that the nS source of neutrinos is favoured by the data. Depending on the particular combination of sources for these neutrinos, current data can only hint at the allowed region of the δ_{CP} range. However, we have shown that additional data gives a remarkable improvement in results, which underlines the importance of future data from IceCube. We have also put constraints on the astrophysical parameters k_1 , k_2 and k_3 that determine which of the modes of neutrino production is more close to the data. In fact if δ_{CP} is measured by other experiments, then IceCube data can be used to determine the production mechanism of these neutrinos. Similar analyses can also be carried out for other parameters related to neutrino physics and astrophysics.

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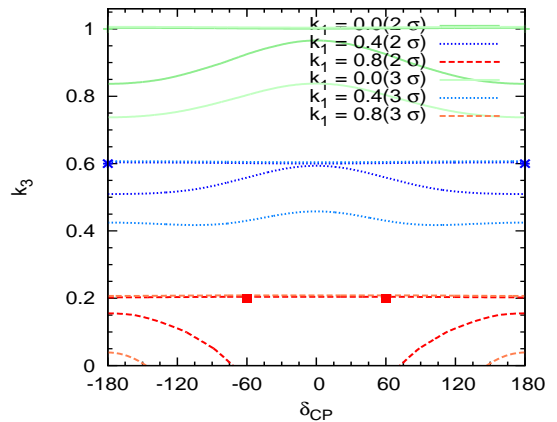


FIG. 7. Contour plots for allowed region in the $k_3 - \delta_{CP}$ plane, for three representative values of k_1 . The points marked in the respective colours indicate the best-fit point.

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