

# Possibility of conversion of neutron star to quark star in presence of high magnetic field

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## ABSTRACT

Recent results and data suggests that high magnetic field in neutron stars (NS) strongly affects the characteristic (radius, mass) of the star. They are even separated as a class known as magnetars, for whom the surface magnetic field are greater than  $10^{14}$  G. In this work we discuss the effect of such high magnetic field on the phase transition of NS to quark star (QS). We study the effect of magnetic field on the transition from NS to QS including the magnetic field effect in equation of state (EoS). The inclusion of the magnetic field increases the range of baryon number density, for which the flow velocities of the matter in the respective phase are finite. The magnetic field helps in initiation of the conversion process. The velocity of the conversion front however decreases due to the presence of magnetic field, as the presence of magnetic field reduces the effective pressure (P). The magnetic field of the star gets decreased by the conversion process, and the resultant QS has lower magnetic field than that of the initial NS.

stars: neutron, equation of state, gravitation, hydrodynamics, stars: magnetic fields, shock waves

## 1 INTRODUCTION

The discovery of radio pulsar (Hewish et al. 1968), brought about the theoretical proposition of NS (Baade & Zwicky 1934) to much attention. Inside a NS the matter, composed of neutrons, protons, electrons, and sometimes muons, is in a highly dense state, whose density may be as high as 3 – 10 times normal nuclear matter (NM) saturation density ( $n_0$ ). Naturally, the constituent particles therein interact via strong forces, forming a non-ideal nuclear fluid. However, the nature of the strong interaction at this high density is not well understood yet. There are many theoretical models of nuclear matter at high density describing different EoSs for the matter. Based on different EoSs, different mass-radius relations of NS are obtained, which can only be justified by comparing the theoretical results with observed properties of NS.

The possibility of emergence and existence of QS, containing deconfined quarks in their free states has been intensively discussed in the literature (Alcock et al. 1986; Olinto 1987,1991). If the QS consists only of almost equal number of free up (u), down (d) and strange (s) quarks it is termed as strange star (SS) (Witten 1984; Alcock et al. 1986), otherwise a more general star consisting of all kinds of quarks in their free states is called a QS. In

this work our final EoS is for that of a SS. The measured periods and spin down rates of soft-gamma repeaters (SGR) and of anomalous X-ray pulsars (AXP), and the observed X-ray luminosities of AXP, indicate that some NSs have extremely high surface magnetic fields, as large as  $10^{14} - 10^{15}$  G (Thompson & Duncan 1993; Mereghetti & Stella 1995) which are known as magnetars. Furthermore, if these sources are the central engine of gamma-ray bursts (GRBs), as suggested in (Usov 1992; Kluzniak & Ruderman 1998), their surface magnetic field might even be larger. The discovery of magnetars has triggered a growing interest in the study of the structure, dynamics and the evolution of NS with large magnetic fields, which has raised a number of interesting issues.

It is known that magnetic field plays important role in the astrophysical phenomena, such as supernovae, GRBs, galaxy jet, and so on. Recently, there is a growing consensus in explaining SGRs via the magnetar model (Thompson & Duncan 1993). Magnetars are believed to be NS with strong magnetic field which is responsible for the observed flare activity. Three giant flares, SGR 0526-66, SGR 1900+14 and SGR 1806-20, have been detected so far. This huge amount of energy can be explained by the presence of a strong magnetic field whose strength is estimated to be larger than  $4 \times 10^{14}$  G. In the late part of the flares, a careful analysis revealed the existence of characteristics quasi-periodic oscillations (QPO) (Stohmayer & Watts 2006). It is not clear whether they are associated to crustal modes, or to modes of the magnetic

field or both; if the spacing between the observed frequencies would be explained, one may gain information on the internal structure of the star (Sotani et al. 2007).

The existence of a magnetar motivates to study the effects of strong magnetic field on NS properties. A strong magnetic field affects, the structure of a NS through its influence on the underlying metric (Bocquet et al. 1995; Cardall et al. 2001) and EoS through the Landau quantization of charged particles and then the interaction of magnetic moments of charged particles with the magnetic field. For the NM with a  $n$ - $p$ - $e$  system, the effect of magnetic field was studied by several authors (Chakrabarty et al. 1997; Yuan & Zhang 1999; Broderick et al. 2001; Chen et al. 2005; Wei et al. 2006). However, as discussed earlier the composition of the core of a NS is very uncertain, and different EoSs have been proposed to describe the matter at such extreme condition. The matter in the star may contain only deconfined quarks, which are known as SS, or the hyperons may appear, making hyperonic matter. The effect of magnetic field on quark matter using the MIT bag model has been studied earlier (Chakrabarty 1996; Ghosh & Chakrabarty 2001; Felipe et al. 2008). There are other models of quark matter with phenomenological density dependent quark masses (Fowler et al. 1981; Chakrabarty 1991; Dey et al. 1998; Li et al. 2010). Broderick et al. (Broderick et al. 2002) studied the effect of strong magnetic field on hyperonic matter, where the field strength does not depend on density. However, in reality, we expect the field strength should be higher at core than at surface of a NS. Therefore, the field strength should vary with radius, and hence with density.

So the study of magnetic field in NS may provide with new different results and will help in understanding the basic properties of NS in a much better way. The magnetic field will also play an important role in the conversion of NS to SS. The NS may convert to a SS by several different ways. A few possible mechanisms for the production of SQM in a NS have been discussed by Alcock et al (Alcock et al. 1986). The conversion from hadron matter to quark matter is expected to start as the star comes in contact with a seed of external strange quark nugget. Another mechanism for the initiation of the conversion process was given by Glendenning (Glendenning 1982,1992,1992). It was suggested there that a sudden spin down of the star may increase the density at its core thereby triggering the conversion process spontaneously.

Conversion of NM to SQM has been studied by several authors, which are discussed in detail by Bhattacharyya et al (Bhattacharyya et al. 2006) and for brevity we do not discuss them here.

In this paper we plan to study the effect of the density dependent magnetic field EoS on the conversion front. We will write down the Rankine-Hugoniot condition for the matter velocities and solve them with the EoS derived in presence of magnetic field. In presence of magnetic field, both pressure ( $P$ ) and energy density ( $\varepsilon$ ) of the system are affected. This will indirectly give the effect of magnetic field on the conversion front. The paper is arranged in the following way: In the next section we construct our model for the magnetic field dependent EoS. In section 3 we will discuss the kinematics of the phase transition for the magnetic field

dependent EoS. In section 4 we will discuss about the propagation of the front along the star and in the final section we will summarize our results.

## 2 MODEL

First we construct the magnetic field induced EoS. We use nonlinear Walecka model (Ellis et al. 1991), which has been successful in describing the nuclear ground state properties and elastic scattering (Walecka 1974; Chin 1977; Serot 1979; Serot & Walecka 1986). In addition to the model, here we consider the possibility of appearance of hyperons ( $\Lambda, \Sigma^-, \Sigma^0, \Sigma^+, \Xi^-, \Xi^0$ ) and muons ( $\mu^-$ ) at higher density.

The detail of the calculation is similar to that of Sinha et al (Sinha & Mukhopadhyay 2010) and for brevity we only mention the important results of the model here. For the magnetic field inclusion we choose the gauge to be,  $A^\mu \equiv (0, -y\mathcal{B}, 0, 0)$ ,  $\mathcal{B}$  being the magnitude of magnetic field and  $eQ$  the charge of the particle with  $e$  the positive unit of charge. For this particular gauge choice  $\vec{B} = \mathcal{B}\hat{z}$ . In the presence of magnetic field, the motion of the charged particles is Landau quantized in the perpendicular direction to the magnetic field. The momentum in the  $x$ - $y$  plane is quantized and hence the energy in the  $n$ th Landau level is given by

$$E_n = \sqrt{p_z^2 + m^2 + 2ne|Q|\mathcal{B}}. \quad (1)$$

With the above consideration we can write down the total energy density of matter as

$$\begin{aligned} \varepsilon = & \frac{1}{2}m_\sigma^2\sigma^2 + U(\sigma) + \frac{1}{2}m_\omega^2\omega^2 + \frac{1}{2}m_\rho^2\rho_3^2 \\ & + \sum_N \frac{1}{8\pi^2} \left( 2p_F^{(N)}\mu_N^{*3} - p_F^{(N)}m_N^{*2}\mu_N^* - m_N^{*4} \ln \left[ \frac{p_F^{(N)} + \mu_N^*}{m_N^*} \right] \right) \\ & + \frac{e|Q|\mathcal{B}}{(2\pi)^2} \sum_C \sum_{n=0}^{n_{max}} (2 - \delta_{n,0}) \\ & \times \left( p_C(n)\mu_C^* + (m_C^{*2} + 2ne|Q|\mathcal{B}) \ln \left[ \frac{p_C(n) + \mu_C^*}{\sqrt{(m_C^{*2} + 2ne|Q|\mathcal{B})}} \right] \right) \\ & + \frac{e|Q|\mathcal{B}}{(2\pi)^2} \sum_{l=e,\mu} \sum_{n=0}^{n_{max}} (2 - \delta_{n,0}) \\ & \times \left( p_l(n)\mu_l + (m_l^2 + 2ne|Q|\mathcal{B}) \ln \left[ \frac{p_l(n) + \mu_l}{\sqrt{(m_l^2 + 2ne|Q|\mathcal{B})}} \right] \right) \\ & + \frac{\mathcal{B}^2}{8\pi}, \end{aligned} \quad (2)$$

where  $N$  denotes charge neutral baryons,  $C$  charged baryons and  $l$  leptons.  $\psi_B, \psi_l, \sigma, \omega$  and  $\rho$  are fields of baryons, leptons,  $\sigma$ -mesons,  $\omega$ -mesons and  $\rho$ -mesons, with masses  $m_B, m_l, m_\sigma, m_\omega$  and  $m_\rho$  respectively,  $g_{\sigma B}, g_{\omega B}$  and  $g_{\rho B}$  are coupling constants for interactions of  $\sigma, \omega$  and  $\rho$  mesons respectively with the baryon  $B$ .  $U(\sigma)$  is the scalar self interaction term (Glendenning 1982,1985,1987; Boguta & Bodmer 1977). We define  $p(n) = \sqrt{p_F^2 - 2ne|Q|\mathcal{B}}$ , where  $p_F$  is the Fermi momentum.

The total  $P$  is then

$$P = \sum_B \mu_B n_B + \sum_l \mu_l n_l - \varepsilon. \quad (3)$$

For SQM we employ one of the realistic EoS with density dependent quark masses given by Dey et al. (Dey et al. 1998). In this model quarks interact among themselves via Richardson potential (Richardson 1979). The matter is composed of  $u$ ,  $d$  and  $s$  quarks and electrons. In the presence of magnetic field the energy density is then given by

$$\varepsilon = \varepsilon_k + \varepsilon_p + \varepsilon_e + \frac{\mathcal{B}^2}{8\pi}, \quad (4)$$

where

$$\varepsilon_k = \frac{3e\mathcal{B}}{(2\pi)^2} \sum_i |Q|_i \sum_{n=0}^{n_{max}} (2 - \delta_{n,0}) \times \left( p_i(n)\mu_i^* + (m_i^2 + 2ne|Q|_i\mathcal{B}) \ln \left[ \frac{p_i(n) + \mu_i^*}{\sqrt{(m_i^2 + 2ne|Q|_i\mathcal{B})}} \right] \right) \quad (5)$$

is the kinetic energy density of quarks with  $\mu_i^* = \sqrt{m_i^2 + p_F^{(i)2}}$ ,  $i = u, d, s$ ,

$$\varepsilon_p = -\frac{1}{(2\pi)^4} \sum_{i,j} (e\mathcal{B})^2 |Q|_i |Q|_j \int_0^{p_i(n_i)} \int_0^{p_j(n_j)} \int_0^{2\pi} V(q) \times F(p_i, p_j, m_i, m_j, \phi) dp_z^{(i)} dp_z^{(j)} d\phi \quad (6)$$

is the potential energy density with

$$F(p_i, p_j, m_i, m_j) = \frac{(E_i + m_i)(E_j + m_j)}{4E_i E_j} \times \left\{ 1 + \frac{p_i^2 p_j^2}{(E_i + m_i)^2 (E_j + m_j)^2} + \frac{2\mathbf{p}_i \cdot \mathbf{p}_j}{(E_i + m_i)(E_j + m_j)} \right\} \quad (7)$$

and

$$\varepsilon_e = \frac{e|Q|_e\mathcal{B}}{(2\pi)^2} \sum_{n=0}^{n_{max}} (2 - \delta_{n,0}) \times \left( p_e(n)\mu_e + (m_e^2 + 2ne|Q|_e\mathcal{B}) \ln \left[ \frac{p_e(n) + \mu_e}{\sqrt{(m_e^2 + 2ne|Q|_e\mathcal{B})}} \right] \right) \quad (8)$$

is the energy density of electrons. The total pressure is then

$$P = \sum_i \mu_i n_i + \mu_e n_e - \varepsilon. \quad (9)$$

The relation between energy density and pressure describes the EoS of the matter.

As the configuration of the magnetic field inside a star is not known, following previous work (Bandyopadhyay et al. 1998) we adopt a magnetic field profile

$$\mathcal{B}(n_b/n_0) = \mathcal{B}_S + \mathcal{B}_C \left\{ 1 - e^{-\beta \left( \frac{n_b}{n_0} \right)^\gamma} \right\}, \quad (10)$$

where  $\beta$  and  $\gamma$  are two parameters determining the magnetic field profile with given  $\mathcal{B}_S$  and  $\mathcal{B}_C$ , and  $n_b$  is the total baryon number density. We assume that the magnetic field at the center is higher than at the surface by a few order of magnitude. As magnetars are observed to have  $\mathcal{B}_S$  as large as  $10^{15}$  G, in our model we keep  $\mathcal{B}_S$  fixed at  $10^{15}$  G and vary the  $\mathcal{B}_C$ . It is found that the effect of magnetic field is important for  $\mathcal{B}_C \geq 10^{17}$  G. However, for  $\mathcal{B}_C < 10^{18}$  G,

[t]

Threshold densities	
$\mu^-$	0.9
$\Lambda$	2.6
$\Xi^-$	3.1
$\Xi^0$	5.7

**Table 1.** Threshold densities for muons and hyperons to appear in units of  $n_0$ .

the effect is significant only when the field reaches  $\mathcal{B}_C$  at a very low density, away from the center, and remains almost constant up to center. Therefore, we restrict our study with  $\mathcal{B}_C \sim 10^{18}$  G.

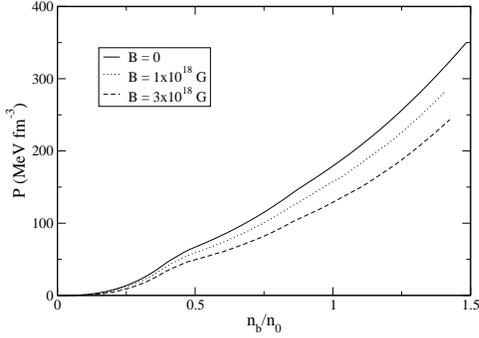
At lower densities, the matter is composed of only neutrons, protons and electrons. Hence, at the low density regime, the particles which are affected by the magnetic field are electrons and protons. Since the electrons are highly relativistic, electron Fermi momentum is very large compared to electron mass. Therefore, the number of occupied Landau levels by electrons is very large, even though the field strength under consideration is larger than the critical field strength of electron by several orders. On the other hand, the field strength under consideration is very less than the critical field strength of protons. Consequently, the number of occupied Landau levels by protons is also large. As density increases, the heavier particles appear gradually. In addition, the magnetic field increases with the increase of density. As a result, the number of occupied Landau levels gradually decreases for every species. The threshold densities for muons and other hyperons to appear for  $\mathcal{B}_C = 10^{18}$  G are given in Table 1. The threshold densities for various species to appear do not differ from their respective values when the magnetic field is absent.

In fig. 1 we show the EoSs with and without magnetic field when the magnetic field profile corresponds to  $\beta = 0.1$  and  $\gamma = 1$  [see Eq. (10)]. We observe that the EoS becomes softer with the increase of  $\mathcal{B}_C$ . Here we should mention that, when the field strength is high enough, the field energy and the field pressure are not negligible. In calculating the EoS, we thus add this contribution too which is necessary to construct the structure of NS.

### 3 RANKINE-HUGONIOT CONDITION

We heuristically assume the existence of a combustive phase transition front. Using the macroscopic conservation conditions, we examine the range of densities for which such a combustion front exists. We next study the outward propagation of this front through the model star by using the hydrodynamic (*i.e.* Euler) equation of motion and the equation of continuity for the energy density flux (Folomeev et al. 2005). Let us now consider the physical situation where a combustion front has been generated in the core of the NS. This front propagates outwards through the NS with a certain hydrodynamic velocity. In the following,

[t]



**Figure 1.** Variation of  $P$  as a function of normalized baryon number density. The solid curve is for without magnetic field. The dotted and dashed curves correspond to  $\mathcal{B}_C = 1 \times 10^{18}$  G, and  $3 \times 10^{18}$  G respectively.

we denote all the physical quantities in the hadronic sector by subscript 1 and those in the quark sector by subscript 2.

Quantities on opposite sides of a front are related through the energy density, the momentum density and the baryon number density flux conservation. In the rest frame of the combustion front, these conservation conditions can be written as (Folomeev et al. 2005; Landau & Lifshitz; Anile):

$$\omega_1 v_1^2 \gamma_1^2 + P_1 = \omega_2 v_2^2 \gamma_2^2 + P_2, \quad (11)$$

$$\omega_1 v_1 \gamma_1^2 = \omega_2 v_2 \gamma_2^2, \quad (12)$$

and

$$n_1 v_1 \gamma_1 = n_2 v_2 \gamma_2. \quad (13)$$

In the above three Rankine-Hugoniot conditions  $v_i$  ( $i = 1, 2$ ) is the velocity,  $\gamma_i = \frac{1}{\sqrt{1-v_i^2}}$  is the Lorentz factor,  $\omega_i = \varepsilon_i + P_i$  is the specific enthalpy of the respective phases.

The velocities of the matter in the two phases, given by eqns. (11-13), are written as (Landau & Lifshitz):

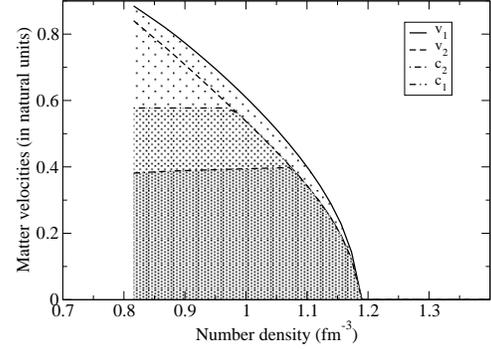
$$v_1^2 = \frac{(P_2 - P_1)(\varepsilon_2 + P_1)}{(\varepsilon_2 - \varepsilon_1)(\varepsilon_1 + P_2)}, \quad (14)$$

and

$$v_2^2 = \frac{(P_2 - P_1)(\varepsilon_1 + P_2)}{(\varepsilon_2 - \varepsilon_1)(\varepsilon_2 + P_1)}. \quad (15)$$

It is possible to classify the various conversion mechanism by comparing the velocities of the respective phases with the corresponding sound speed, denoted by  $c_{si}$ , in these phases. For the conversion to be physically possible, velocities should satisfy an additional condition, namely,  $0 \leq v_i^2 \leq 1$ .

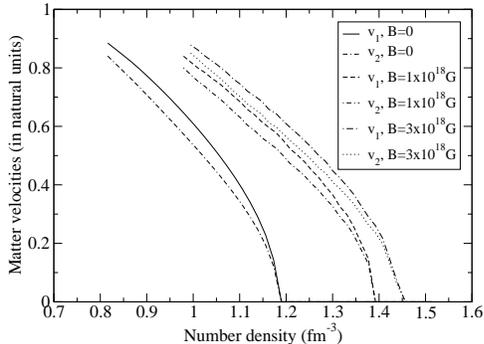
For the above constructed EoS with magnetic field we first plot (fig. 2) the variation of different matter flow velocities as a function of baryon number density. We shade the different portion of the graph which represents different modes of conversion mechanism. For this EoS, beyond a certain density (on the lower side) the curve does not comes



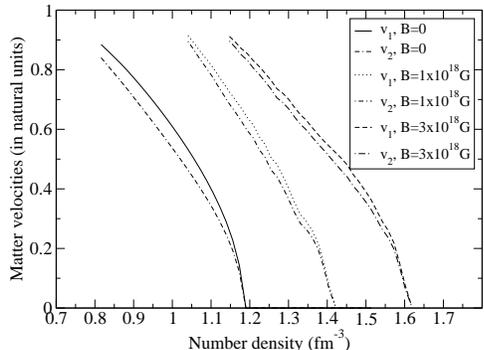
**Figure 2.** Variation of different flow velocities with baryon number density. The most densely shaded region correspond to deflagration, moderate shaded region correspond to detonation and the most lightly shaded region correspond to supersonic conversion processes.

down. This is because, below that density the energy and velocity criterion are not satisfied. If we now construct a star with this hyperon matter EoS then the minimum central density has to be that high where the curve end in the lower baryon density region. At that density there is a high chance of detonation wave formation as the difference in  $v_1$  and  $v_2$  is high. Whereas at very high density a star may be formed but the conversion process would not start. This is because at that very high density the star becomes so dense that whatever may be the perturbation, it fails to grow to give rise to a wave which would start the conversion process.

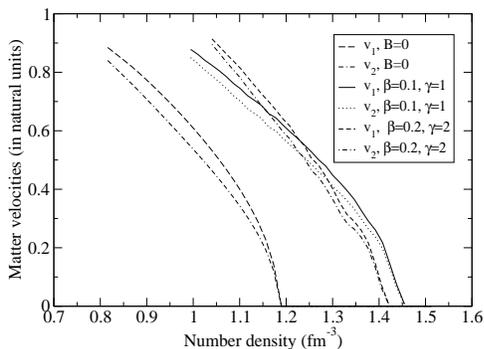
In fig. 3 we plot  $v_1$  and  $v_2$  for different  $\mathcal{B}_C$  corresponding to  $\beta = 0.1$  and  $\gamma = 1$ . For comparison we also plot for the non magnetic case. We find that the nature of the curve remains same, that is  $v_1$  is always greater than  $v_2$ , which means the shock front propagates outward of the star. Due to introduction of the magnetic field the range of baryon density, for which the flow velocities are physical, increases. The magnetic field decreases the effective  $P$  for the same baryon number density, rendering the matter to be more compressible. That is for the same density now the matter is less rigid or more compressible now. For much higher value of the baryon density where the matter was very dense previously, there was little chance of shock formation, but now due to the introduction of the magnetic field there is a finite chance of shock formation. Therefore the range of baryon density gets much wider. In fig. 4 we plot the same, but for the case of the EoS with magnetic field corresponding to  $\beta = 0.2$  and  $\gamma = 2$ . The nature of the graph remains the same. From eqn. 10, we can see that  $\beta$  and  $\gamma$  describes how the magnetic field varies from the core to the surface for fixed  $\mathcal{B}_C$  and  $\mathcal{B}_S$  values. It determines the slope of the magnetic field, that is higher the  $\beta, \gamma$  value larger the slope. Therefore for the same value of the magnetic field the range of density, for which the flow velocities are finite increases with the increase of  $\beta, \gamma$ . This becomes much more clear from fig. 5, where we have plotted for two sets of  $\beta, \gamma$  value for same  $\mathcal{B}_C$ . The slope of  $v_1$  and  $v_2$  changes with the change in slope of the magnetic field configuration.



**Figure 3.** Variation of  $v_1$  and  $v_2$  with baryon number density. The curves are plotted for the EoS with magnetic field profile corresponding to  $\beta = 0.1$  and  $\gamma = 1$ . The variation is plotted for three cases, one without magnetic field and the other two with  $\mathcal{B}_C = 1 \times 10^{18}$  G and  $3 \times 10^{18}$  G.



**Figure 4.** Variation of  $v_1$  and  $v_2$  with baryon number density. The curves are plotted for the EoS with magnetic field profile corresponding to  $\beta = 0.2$  and  $\gamma = 2$ . The variation is plotted for three cases, one without magnetic field and the other two with  $\mathcal{B}_C = 1 \times 10^{18}$  G and  $3 \times 10^{18}$  G.



**Figure 5.** Variation of  $v_1$  and  $v_2$  with baryon number density. The curves are plotted for the EoSs with magnetic field profiles corresponding to  $\beta = 0.1$ ,  $\gamma = 1$  and  $\beta = 0.2$ ,  $\gamma = 2$  with  $\mathcal{B}_C = 3 \times 10^{18}$  G along with the non magnetic case.

#### 4 PROPAGATION OF THE FRONT

Next we come to the dynamic picture of the front propagation, that is the evolution of the hydrodynamic combustion front along the radius of the star. To examine such an evolution, we move to a reference frame in which the NM is at rest. The speed of the combustion front in such a frame is given by  $v = -v_1$ . The velocity of matter in the combustion frame is in natural units, with  $c = \hbar = k_B = 1$ . All the other physical quantities discussed below are also in natural units, that is they are converted to  $eV$ .

Using special relativistic formalism to study the evolution of combustion front we derive the relevant Eulers and continuity equation, given by (Folomeev et al. 2005):

$$\frac{1}{\omega} \left( \frac{\partial \varepsilon}{\partial \tau} + v \frac{\partial \varepsilon}{\partial r} \right) + \frac{1}{W^2} \left( \frac{\partial v}{\partial r} + v \frac{\partial v}{\partial \tau} \right) + 2 \frac{v}{r} = 0 \quad (16)$$

and

$$\frac{1}{\omega} \left( \frac{\partial P}{\partial r} + v \frac{\partial P}{\partial \tau} \right) + \frac{1}{W^2} \left( \frac{\partial v}{\partial \tau} + v \frac{\partial v}{\partial r} \right) = 0, \quad (17)$$

where,  $v = \frac{\partial r}{\partial \tau}$  is the front velocity in the NM rest frame and  $k = \frac{\partial P}{\partial \varepsilon}$  is taken as the square of the effective sound speed in the medium and  $W = 1/\gamma_i$  is the inverse of Lorentz factor.

The above equations are solved to give a single equation

$$\frac{dv}{dr} = \frac{2vkW^2(1+v^2)}{r[4v^2 - k(1+v^2)^2]}. \quad (18)$$

The eqn. (18) is integrated, with respect to  $r(t)$ , starting from the center towards the surface of the star. Using the above EoS we construct a star following the standard Tolman-Oppenheimer-Volkoff equations (Oppenheimer & Volkoff 1939). The velocity at the center of the star should be zero from symmetry considerations. On the other hand, the  $1/r$  dependence of the  $\frac{dv}{dr}$ , in eqn. (18) suggests a steep rise in velocity near the center of the star.

We first construct the density profile of the star for a fixed central density (here it is 6 times normal nuclear saturation density). Equations (14) and (15) then specify the respective flow velocities  $v_1$  and  $v_2$  of the nuclear and quark matter in the rest frame of the front, at a radius infinitesimally close to the center of the star. This would give the initial velocity of the front ( $-v_1$ ), at that radius, in the NM rest frame. We next start with eqn. (18) from a point infinitesimally close to the center of the star and integrate it outwards along the radius of the star. The solution gives the variation of the velocity with the distance from the center of the star. For a static star, being spherically symmetric, the problem is rather simple; for a rotating star, however, the asymmetry has to be taken care of.

As due to rotation the star is no more spherical, but is oblate spheroid, therefore to describe the star we introduce a new parameter  $\chi = \cos \theta$ , where  $\theta$  is the angle made with the vertical axis (axis of rotation) of the star. The detail general relativistic (GR) calculations can be obtained from Bhattacharyya et al. (Bhattacharyya et al. 2007). We here mention only the important results needed for our purpose. We start with the metric (Cook et al. 1994)

$$ds^2 = -e^{\gamma+\rho} dt^2 + e^{2\alpha} (dr^2 + r^2 d\theta^2) + e^{\gamma-\rho} r^2 \sin^2 \theta (d\phi - \omega dt)^2 \quad (19)$$

describing the structure of the star, with the four gravitational potentials  $\alpha, \gamma, \rho$  and  $\omega$ , which are functions of  $\theta$  and

$r$  only. The Einstein's equations for the potentials are solved through the 'rns' code, with the input of our EoS and a fixed central density. Similar hydrodynamic equation for the front propagation can be written down and solved to get the final equation for the velocity of the front. The equation is quite similar to that of (18), only there is an extra term due to GR effect (Bhattacharyya et al. 2007).

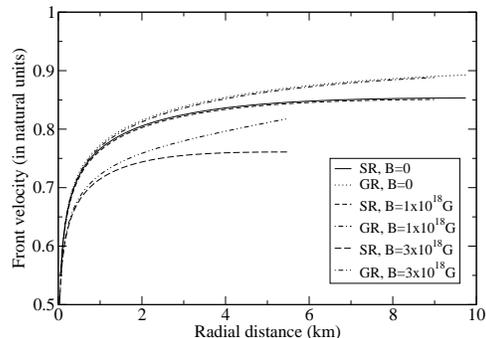
In fig. 6 we plot the velocity of the conversion front along the radius of the star in the presence of magnetic field for different  $\mathcal{B}_C$ , with field profile corresponding to  $\beta = 0.1$  and  $\gamma = 1$ . We show both results obtained from special and general relativistic calculations. The figure shows that the velocity of the front, for all cases, shoots up near the center and then saturates at a certain velocity for higher radius. Such a behaviour of velocity near the central point is apparent from the eqn. (18) above. The velocity of the front further increases with the inclusion of GR effect. The rise in the front velocity is due to the fact that now GR effect of the curvature of the front adds up with the  $P$  contribution which provides the thrust to the propagation front. We can further observe that as the magnetic field increases the velocity of the front decreases, this is due to the fact that the magnetic contribution ( $\mathcal{B}^2/8\pi$ ) acts in the opposite direction of the matter  $P$ , reducing the effective  $P$  term. This  $P$  is the fact which provides the thrust to the propagation front, and thus the velocity of the front decreases with the decrease of effective  $P$ . Fig. 7 shows the same nature only now we plot for the EoS with magnetic field profile corresponding to  $\beta = 0.2$  and  $\gamma = 2$ . The difference in the graphs is only due to the difference in the nature of the magnetic field (mainly the slope of the growth of the field which is governed by  $\beta$  and  $\gamma$ ).

From the Rankine-Hugoniot condition we can find that for the conservation condition to hold the number density of the SQM should be greater than that of the NM, but by a small amount. In fig. 8 we plot the magnetic field variation with the number density of both matter phases for magnetic field profile corresponding to  $\beta = 0.1$  and  $\gamma = 1$ . For the same baryon number density the SQM can support much lesser magnetic field than what the NM can support. Therefore, although there is a rise in number density due to conversion of NM to SQM, the magnetic field of the star reduces due to this conversion process. The conversion of NS to QS decreases the magnetic field of the star, resulting in a lesser magnetized QS. It is likely that the conversion process heats up the star, thereby gaining energy which is supplied through the conversion of magnetic energy to heat energy. Therefore the magnetic field of the resultant QS is lesser than the initial NS. In fig. 9 the same is plotted only for EoS with magnetic field profile corresponding to  $\beta = 0.2$  and  $\gamma = 2$ . The difference in the figures is due to same cause as discussed earlier.

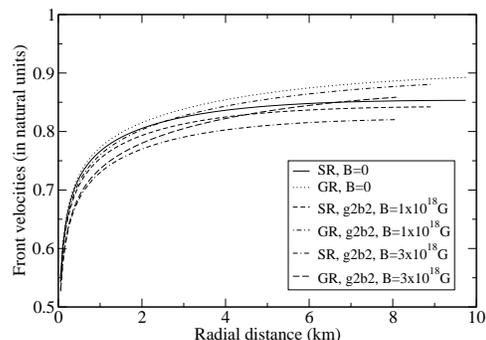
## 5 SUMMARY

We have studied the conversion of hyperon matter to SQM inside a NS. We have done both kinematic and dynamic study of the conversion process. We have seen the effect of magnetic field on the EoS and thereby the effect of it in the conversion process. We have found that beyond a certain density (on the lower side) the curve does not come down.

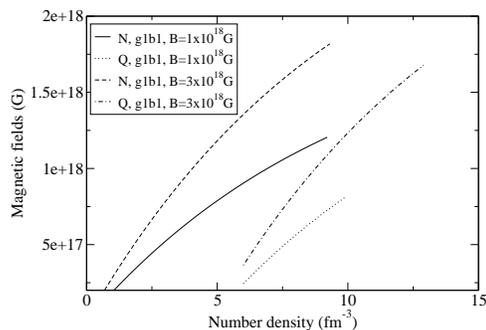
[t]



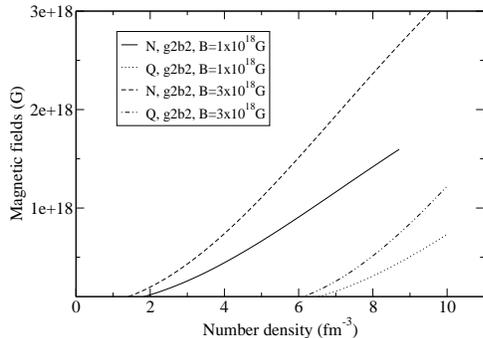
**Figure 6.** Variation of the velocity of the conversion front with radius of the star. The curves are plotted for the EoS with magnetic field profile corresponding to  $\beta = 0.1$  and  $\gamma = 1$ . The variation is plotted for three cases, one without magnetic field and the other two with  $\mathcal{B}_C = 1 \times 10^{18}$  G and  $3 \times 10^{18}$  G.



**Figure 7.** Variation of the velocity of the conversion front with radius of the star. The curves are plotted for the EoS with magnetic field profile corresponding to  $\beta = 0.2$  and  $\gamma = 2$ . The variation is plotted for three cases, one without magnetic field and the other two with  $\mathcal{B}_C = 1 \times 10^{18}$  G and  $3 \times 10^{18}$  G.



**Figure 8.** Variation of magnetic field with number density is shown for both nuclear and quark matter EoS. The curves are plotted for the EoS with magnetic field profile corresponding to  $\beta = 0.1$  and  $\gamma = 1$ . The variation is plotted for two different  $\mathcal{B}_C$ ,  $1 \times 10^{18}$  G and  $3 \times 10^{18}$  G.



**Figure 9.** Variation of magnetic field with number density is shown for both nuclear and quark matter EoS. The curves are plotted for the EoS with magnetic field profile corresponding to  $\beta = 0.2$  and  $\gamma = 2$ . The variation is plotted for two different  $B_C$ ,  $1 \times 10^{18}$  G and  $3 \times 10^{18}$  G.

This is due to the fact that beyond that density the SQM is not stable. To make the conversion from NM to SQM possible, the central density of a NS must not be smaller than the minimum density beyond which SQM is not stable. At that density there is a high chance of detonation wave formation. On the other extreme, at very high density a star may be formed but the conversion process would not start.

Due to introduction of the magnetic field the range of values of baryon density, for which the flow velocities are physical, increases. The magnetic field reduces the  $P$  for the same value of baryon number density, rendering the matter to be more compressible. That is for the same density, now the matter is less rigid or more compressible. The nature of the EoS depends on the magnetic field we choose, which depends on the central value and on the factor  $\beta$  and  $\gamma$ . This  $\beta$  and  $\gamma$  controls the way the magnetic field would vary along the star for a given  $B_C$ . Higher value of  $\beta$  and  $\gamma$  means the slope of the field variation is larger. Therefore regulating these parameters we can regulate the shock front propagation.

Coming to the dynamic picture, we have found that the velocity of the front shoots up near the center and then saturates at a certain velocity for higher radius. The GR effect increases the velocity of the front, and the rise is due to the fact that GR effect of the curvature of the front adds up with the  $P$  providing a much larger thrust to the front propagation. As the magnetic field increases the velocity of the front decreases due to the negative field pressure contribution which reduces the effective  $P$ . Thus the thrust to the propagation front decreases, reducing the velocity. The most interesting fact of the study is that for such EoS if we construct a hyperon star and by the conversion mechanism it converts to a QS the magnetic field of the star decreases. Thereby signalling that the phase transition of NS to QS is accompanied by decrease in magnetic field, which goes on to heat up the star.

Although it is an interesting result but such conclusion can only be made if such result are consistent with direct or indirect observational evidences from the NS. We have performed the calculation for the zero temperature EoS matter, as such EoS does not have the provision of finite tempera-

ture inclusion. We would like to perform similar calculation with other EoS of matter having provision for inclusion of both finite temperature and magnetic fields. In this work the magnetic field is included through the EoS but we would like to perform similar calculation where the magnetic field is also included through the Einsteins field equations. Such an analysis would give a better picture of the magnetic effect on phase transition mechanism and which is our immediate agenda.

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## REFERENCES

- Alcock, C., Farhi, E., & Olinto, A., *Astrophys. J.*, 310, 261 (1986)
- Anile, A. M., *Relativistic fluids and Magneto-fluids : with application in Astrophysics and Plasma Physics*, Cambridge University Press, U.K. (1989)
- Baade, W., & Zwicky, F., *Phys. Rep.*, 45, 138 (1934)
- Bandyopadhyay, D., Chakrabarty, S., & Pal, S., *Phys. Rev. Lett.*, 79, 2176 (1998)
- Bhattacharyya, A., Ghosh, S. K., Joarder, P., Mallick, R., & Raha, S., *Phys. Rev. C*, 74, 065804 (2006)
- Bhattacharyya, A., Ghosh, S. K., Mallick, R., & Raha, S., *Phys. Rev. C*, 76, 052801 (2007)
- Bocquet, M., Bonazzola, S., Gourgoulhon, E., & Novak, J., *Astron. and Astrophys.*, 301, 757 (1995)
- Boguta, J., & Bodmer, A. R., *Nucl. Phys. A*, 292, 413 (1977)
- Broderick, A., Prakash, M., & Lattimer, J. M., *Astrophys. J.*, 37, 351 (2001)
- Broderick, A., Prakash, M., & Lattimer, J. M., *Phys. Lett. B*, 531, 167 (2002)
- Cardall, C. Y., Prakash, M., & Lattimer, J. M., *Astrophys. J.*, 554, 322 (2001)
- Chakrabarty, S., *Phys. Rev. D*, 43, 627 (1991)
- Chakrabarty, S., *Phys. Rev. D*, 54, 1306 (1996)
- Chakrabarty, S., Bandyopadhyay, D., & Pal, S., *Phys. Rev. Lett.*, 78, 2898 (1997)
- Chen, W., Zhang, P. Q., & Liu, L. G., *Mod. Phys. Lett. A*, 22, 623 (2005)
- Chin, S. A., *Ann. Phys.*, 108, 301 (1977)
- Cook, G. B., Shapiro, S. L., & Teukolsky, S. A., *Astrophys. J.*, 398, 203 (1992); 424, 823 (1994)
- M. Dey, I. Bombaci, J. Dey, S. Ray, B. Samanta, *Phys. Lett. B*, 438, 123 (1998)
- Ellis, J., Kapusta, J. L., Olive, K. A., *Nucl. Phys. B*, 348, 345 (1991)
- Felipe, R. G., Martinez, A. P., Rojas, H. P., & Orsaria, M., *Phys. Rev. C*, 77, 015807 (2008)
- Folomeev, V., Gurovich, V., Nusser, A., & Tokareva, I., *Int. J. Mod. Phys. D*, 14, 33 (2005)
- Fowler, R. G., Raha, S., & Weiner, R. M., *Z Phys. C*, 9, 271 (1981)
- Ghosh, T., & Chakrabarty, S., *Int. J. Mod. Phys. D*, 10, 89 (2001); *Phys. Rev. D*, 63, 043006 (2001)

- Glendenning, N. K., Phys. Lett. B, 114, 392 (1982); Astrophys. J., 293, 470 (1985); Z. Phys. A, 326, 57 (1987); Z. Phys. A, 327, 295 (1987)
- Glendenning, N. K., 1991, Nucl. Phys. (Proc. Suppl.) B, 24, 110 (1992), Phys. Rev. D, 46, 1274 (1992)
- Hewish, A., Bell, S. J., Pilkington, J. D. H., Scott, P. F., & Collins, R. A., Nature, 217, 709 (1968)
- Kluzniak, W., & Ruderman, M., Astrophys. J., 505, L113 (1998)
- Landau, L. D., & Lifshitz, E. M., Fluid Mechanics, Pergamon Press (1987)
- Li, A., Xu, R. X., & Lu, J. F., Mon. Not. Roy. Astron. Soc., 402, 2715L (2010)
- Mereghetti, S., & Stella, L., Astrophys. J., 442, L17 (1995)
- Olinto, A., Phys. Lett. B, 192, 71 (1987); Nucl. Phys. Proc. Suppl. B, 24, 103 (1991)
- Oppenheimer, J. R., & Volkoff, G. M., Phys. Rev., 55, 374 (1939)
- Richardson J. L., 1979, PLB, 82, 272
- Serot, B. D., Phys. Lett. B, 86, 146 (1979)
- Serot, B. D., Walecka, J. D., Adv. Nucl. Phys., 16, 1 (1986)
- Sinha, M., Mukhopadhyay, B., arXiv:1005.4995 (2010)
- Sotani, H., Kokkotas, K. D., & Stergioulas, N., MNRAS, 319, 261 (2007)
- Stohmayer, T. E., & Watts, A. L., Astrophys. J., 653, 593 (2006)
- Thompson, C., & Duncan, R. C., Astrophys. J., 408, 194 (1993)
- Usov, V. V., Nature, 357, 472 (1992)
- Walecka, J. D., Ann. Phys., 83, 491 (1974)
- Wei, F. X., Mao, G. J., Ko, C. M., Kissinger, L. S., Stoecker, H., & Greiner, W., J. Phys. G, 32, 47 (2006)
- Witten, E., Phys. Rev. D, 30, 272 (1984)
- Yuan, Y. F., & Zhang, J. L., Astrophys. J., 25, 950 (1999)