# New Physics in $b \rightarrow s \mu^{+} \mu^{-}$: CP-Violating Observables 

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#### Abstract

We perform a comprehensive study of the impact of new-physics operators with different Lorentz structures on CP-violating observables involving the $b \rightarrow s \mu^{+} \mu^{-}$transition. We examine the effects of new vector-axial vector (VA), scalar-pseudoscalar (SP) and tensor (T) interactions on the CP asymmetries in the branching ratios and forward-backward asymmetries of $\bar{B}_{s}^{0} \rightarrow \mu^{+} \mu^{-}, \bar{B}_{d}^{0} \rightarrow X_{s} \mu^{+} \mu^{-}$, $\bar{B}_{s}^{0} \rightarrow \mu^{+} \mu^{-} \gamma, \bar{B}_{d}^{0} \rightarrow \bar{K} \mu^{+} \mu^{-}$, and $\bar{B}_{d}^{0} \rightarrow \bar{K}^{*} \mu^{+} \mu^{-}$. In $\bar{B}_{d}^{0} \rightarrow \bar{K}^{*} \mu^{+} \mu^{-}$, we also explore the direct CP asymmetries in the longitudinal polarization fraction $f_{L}$ and the angular asymmetries $A_{T}^{(2)}$ and $A_{L T}$, as well as the triple-product CP asymmetries $A_{T}^{(i m)}$ and $A_{L T}^{(i m)}$. We find that, in almost all cases, the CP-violating observables are sensitive only to new physics which involves VA operators. The VA new physics may therefore be unambiguously identified by a combined analysis of future measurements of these CP-violating observables.


Keywords: B Physics, Beyond Standard Model.

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## 1. Introduction

The $B$ factories have taken us to the luminosity frontier with more than a billion $B^{+} / B_{d}$ mesons, and the Tevatron experiments have provided us with invaluable data on $B_{s}$ mesons. We have now entered the precision era of $B$ physics. The Standard Model (SM) has been successful in explaining most of the data to date. However, this is now the time to look forward to precision tests, with the ATLAS and CMS experiments already running, the LHCb expected to start recording data soon, and the Super- $B$ factories on their way. One can now be ambitious and not only look for new-physics (NP) effects, but also try to identify the kind of NP involved.

Though there is no unambiguous signal of NP so far in all of the $B$ decays we have observed, some possible hints of NP have recently surfaced in modes involving $b \rightarrow s$
transitions. These include measurements of CP-averaged quantities such as the large transverse polarization in $B \rightarrow \phi K^{*}$ [1, 2], and the anomalous forward-backward asymmetry in $B \rightarrow K^{*} \mu^{+} \mu^{-}[3,4,5]$. There are also measurements of CP-violating quantities such as the difference between the mixing-induced CP asymmetries seen in $b \rightarrow s$ penguin decays and in $B_{d} \rightarrow J / \psi K_{S}[6,7,8]$, the large CP asymmetry in $B_{s} \rightarrow J / \psi \phi[9]$, and the anomalous CP asymmetry in like-sign dimuon signals [10].

In the companion paper [11], we performed a general analysis with all possible Lorentz structures of NP in the transition $b \rightarrow s \mu^{+} \mu^{-}$. We included NP vector-axial vector (VA), scalar-pseudoscalar (SP), and tensor (T) b $\rightarrow s \mu^{+} \mu^{-}$operators, and explored their possible effects on the decays $\bar{B}_{s}^{0} \rightarrow \mu^{+} \mu^{-}, \bar{B}_{d}^{0} \rightarrow X_{s} \mu^{+} \mu^{-}, \bar{B}_{s}^{0} \rightarrow \mu^{+} \mu^{-} \gamma$, $\bar{B}_{d}^{0} \rightarrow \bar{K} \mu^{+} \mu^{-}$, and $\bar{B}_{d}^{0} \rightarrow \bar{K}^{*} \mu^{+} \mu^{-}$. We focused on CP-conserving observables such as differential branching ratios, forward-backward asymmetries, polarization fractions, and the asymmetries $A_{T}^{(2)}, A_{L T}$ in $\bar{B}_{d}^{0} \rightarrow \bar{K}^{*} \mu^{+} \mu^{-}$. Because we only considered CP-conserving observables, all the NP couplings were taken to be real. We computed the effects of all NP operators, individually and in all combinations, on these observables.

The CP-violating observables in various $b \rightarrow s \mu^{+} \mu^{-}$decays in the SM as well as in some NP models have been studied in Refs. [12, 13, 14, 15, 16, 17, 18, 19, $20,21,22,23,24,25,26,27]$ In this paper, we explore the CP-violating quantities that may be measured in the same decay modes by allowing the new couplings to be complex. The introduction of complex couplings has two effects. First, some quantities which were taken to be CP-conserving above now display CP-violation, i.e. the quantities take different values in the CP-conjugate decays. The difference between the value of a measurement in a decay and in its CP-conjugate counterpart is then a CP-violating observable. Second, new observables appear which vanish in the CP-conserving limit. (These were not considered in Ref. [11] for this reason.) These essentially correspond to the CP-violating triple-product asymmetries $A_{T}^{(i m)}$ and $A_{L T}^{(i m)}$ in $\bar{B}_{d}^{0} \rightarrow \bar{K}^{*} \mu^{+} \mu^{-}$, which may be obtained from the angular distribution in this decay. Our goal is to identify those quantities for which there may be large effects due to the presence of NP. In such cases, we try to find salient features of the effects of NP, which may help us identify the Lorentz structure of the NP involved.

Here we have taken the NP to be present only in the effective $b \rightarrow s \mu^{+} \mu^{-}$ operator. While this can, in principle, contribute to CP violation in $B_{d}-\bar{B}_{d}$ and $B_{s}-\bar{B}_{s}$ mixing, it is a higher-order effect, and hence negligible compared to the SM contribution. We therefore neglect mixing-induced (indirect) CP violation in this work, and focus only on CP violation in the decay. In the SM, such CP violation is expected to be close to zero in $b \rightarrow s$ transitions. A naive estimate indicates that this asymmetry will be $\sim 10^{-3}$ [20, 23], but even if next-to-leading order (NLO) QCD corrections and hadronic uncertainties are included, it is observed that the CP asymmetry will not exceed $1 \%[24,25,28]$. Thus, if a large CP-violating effect, more than a few percent, is observed in any of the $b \rightarrow s \mu^{+} \mu^{-}$channels, this will
therefore be a clear signature of NP. In this paper, we go further and explore the extent to which the Lorentz structure of NP can be ascertained from the CP-violating measurements.

The paper is organized as follows. We begin in Sec. 2 by describing the effective Hamiltonian with NP operators and new couplings. Although the formalism is the same as that used in Ref. [11], the constraints on the NP couplings are now more relaxed since the couplings are allowed to be complex. We also present an overview of the types of CP-violating observables which are examined. In Sec. 3 we note that there are essentially no measurable CP-violating quantities in the mode $\bar{B}_{s}^{0} \rightarrow \mu^{+} \mu^{-}$. We then consider the decays $\bar{B}_{d}^{0} \rightarrow X_{s} \mu^{+} \mu^{-}$(Sec. 4), $\bar{B}_{s}^{0} \rightarrow \mu^{+} \mu^{-} \gamma$ (Sec. 5), and $\bar{B}_{d}^{0} \rightarrow \bar{K} \mu^{+} \mu^{-}$(Sec. 6). In these sections we examine the same observables as in Ref. [11], this time looking at the asymmetries between these processes and their CP-conjugates. In Sec. 7, we study the CP asymmetries in $\bar{B}_{d}^{0} \rightarrow \bar{K}^{*} \mu^{+} \mu^{-}$for the observables considered in Ref. [11], and in addition we explore new observables that vanish in the CP-conserving limit (triple products). We summarize our findings in Sec. 8 and discuss their implications.

## 2. $b \rightarrow s \mu^{+} \mu^{-}$Operators

### 2.1 Effective Hamiltonian in the SM and with NP

Our formalism is identical to that used in Ref. [11]. We repeat it here briefly for the sake of completeness. Within the SM, the effective Hamiltonian for the quark-level transition $b \rightarrow s \mu^{+} \mu^{-}$is

$$
\begin{align*}
\mathcal{H}_{\mathrm{eff}}^{S M}= & -\frac{4 G_{F}}{\sqrt{2}} V_{t s}^{*} V_{t b}\left\{\sum_{i=1}^{6} C_{i}(\mu) \mathcal{O}_{i}(\mu)+C_{7} \frac{e}{16 \pi^{2}}\left[\bar{s} \sigma_{\mu \nu}\left(m_{s} P_{L}+m_{b} P_{R}\right) b\right] F^{\mu \nu}\right. \\
& \left.+C_{9} \frac{\alpha_{e m}}{4 \pi}\left(\bar{s} \gamma^{\mu} P_{L} b\right) \bar{\mu} \gamma_{\mu} \mu+C_{10} \frac{\alpha_{e m}}{4 \pi}\left(\bar{s} \gamma^{\mu} P_{L} b\right) \bar{\mu} \gamma_{\mu} \gamma_{5} \mu\right\}+h . c . \tag{2.1}
\end{align*}
$$

where $P_{L, R}=\left(1 \mp \gamma_{5}\right) / 2$. The operators $\mathcal{O}_{i}(i=1, . .6)$ correspond to the $P_{i}$ in Ref. [29], and $m_{b}=m_{b}(\mu)$ is the running $b$-quark mass in the $\overline{\mathrm{MS}}$ scheme. We use the SM Wilson coefficients $\left(C_{i}\right)$ as given in Ref. [25].

The effective Hamiltonian in the presence of NP is

$$
\begin{equation*}
\mathcal{H}_{\mathrm{eff}}\left(b \rightarrow s \mu^{+} \mu^{-}\right)=\mathcal{H}_{\mathrm{eff}}^{S M}+\mathcal{H}_{\mathrm{eff}}^{V A}+\mathcal{H}_{\mathrm{eff}}^{S P}+\mathcal{H}_{\mathrm{eff}}^{T}+h . c . \tag{2.2}
\end{equation*}
$$

where

$$
\begin{gather*}
\mathcal{H}_{\mathrm{eff}}^{V A}=-\frac{4 G_{F}}{\sqrt{2}} \frac{\alpha_{e m}}{4 \pi} V_{t s}^{*} V_{t b}\left\{R_{V}\left(\bar{s} \gamma^{\mu} P_{L} b\right) \bar{\mu} \gamma_{\mu} \mu+R_{A}\left(\bar{s} \gamma^{\mu} P_{L} b\right) \bar{\mu} \gamma_{\mu} \gamma_{5} \mu\right. \\
\left.+R_{V}^{\prime}\left(\bar{s} \gamma^{\mu} P_{R} b\right) \bar{\mu} \gamma_{\mu} \mu+R_{A}^{\prime}\left(\bar{s} \gamma^{\mu} P_{R} b\right) \bar{\mu} \gamma_{\mu} \gamma_{5} \mu\right\}  \tag{2.3}\\
\mathcal{H}_{\mathrm{eff}}^{S P}=-\frac{4 G_{F}}{\sqrt{2}} \frac{\alpha_{e m}}{4 \pi} V_{t s}^{*} V_{t b}\left\{R_{S}\left(\bar{s} P_{R} b\right) \bar{\mu} \mu+R_{P}\left(\bar{s} P_{R} b\right) \bar{\mu} \gamma_{5} \mu\right. \\
+  \tag{2.4}\\
\left.+R_{S}^{\prime}\left(\bar{s} P_{L} b\right) \bar{\mu} \mu+R_{P}^{\prime}\left(\bar{s} P_{L} b\right) \bar{\mu} \gamma_{5} \mu\right\} \tag{2.5}
\end{gather*}
$$

are the new contributions. Here, $R_{V}, R_{A}, R_{V}^{\prime}, R_{A}^{\prime}, R_{S}, R_{P}, R_{S}^{\prime}, R_{P}^{\prime}, C_{T}$ and $C_{T E}$ are the NP effective couplings. In our numerical analysis in this paper, we take all NP couplings to be complex. As in Ref. [11], we do not include NP through the $O_{7}=\bar{s} \sigma^{\alpha \beta} P_{R} b F_{\alpha \beta}$ operator or its chirally-flipped counterpart $O_{7}^{\prime}=\bar{s} \sigma^{\alpha \beta} P_{L} b F_{\alpha \beta}$.

### 2.2 Constraints on NP couplings

The constraints on the NP couplings in $b \rightarrow s \mu^{+} \mu^{-}$come mainly from the upper bound on the branching ratio $B\left(\bar{B}_{s}^{0} \rightarrow \mu^{+} \mu^{-}\right)$and the measurements of the total branching ratios $B\left(\bar{B}_{d}^{0} \rightarrow X_{s} \mu^{+} \mu^{-}\right)$and $B\left(\bar{B}_{d}^{0} \rightarrow \bar{K} \mu^{+} \mu^{-}\right)[30,31,32]$ :

$$
\begin{align*}
B\left(\bar{B}_{s}^{0} \rightarrow \mu^{+} \mu^{-}\right) & <4.70 \times 10^{-8} \quad(90 \% \text { C.L. }),  \tag{2.6}\\
B\left(\bar{B}_{d}^{0} \rightarrow X_{s} \mu^{+} \mu^{-}\right) & =\left\{\begin{array}{l}
(1.60 \pm 0.50) \times 10^{-6}\left(\text { low } q^{2}\right) \\
(0.44 \pm 0.12) \times 10^{-6}\left(\text { high } q^{2}\right)
\end{array},\right.  \tag{2.7}\\
B\left(\bar{B}_{d}^{0} \rightarrow \bar{K} \mu^{+} \mu^{-}\right) & =\left(4.5_{-1.0}^{+1.2}\right) \times 10^{-7}, \tag{2.8}
\end{align*}
$$

where the low- $q^{2}$ and high- $q^{2}$ regions correspond to $1 \mathrm{GeV}^{2} \leq q^{2} \leq 6 \mathrm{GeV}^{2}$ and $q^{2} \geq 14.4 \mathrm{GeV}^{2}$, respectively. Here $q^{2}$ is the invariant mass squared of the two muons.

We consider all the NP couplings $R_{i}$ to be complex and parametrize them as

$$
\begin{equation*}
R_{i}=\left|R_{i}\right| e^{i \phi_{R_{i}}} \tag{2.9}
\end{equation*}
$$

where $i=V, A, S, P, T, T E$ and $-\pi \leq \phi_{R_{i}} \leq \pi$. The bounds on these couplings will in general depend on which operators are present. While we take the correlations in these constraints into account in our numerical calculations, for the sake of simplicity we only give the bounds when the NP operators (VA, SP, T) are present individually.

If the only NP couplings present are $R_{V, A}$, we obtain

$$
\begin{equation*}
\frac{\left|\operatorname{Re}\left(R_{V}\right)+2.8\right|^{2}}{(6.3)^{2}}+\frac{\left|\operatorname{Im}\left(R_{V}\right)\right|^{2}}{(6.0)^{2}} \lesssim 1.0, \quad \frac{\left|\operatorname{Re}\left(R_{A}\right)-4.1\right|^{2}}{(6.1)^{2}}+\frac{\left|\operatorname{Im}\left(R_{A}\right)\right|^{2}}{(6.0)^{2}} \lesssim 1.0 \tag{2.10}
\end{equation*}
$$

If the only NP couplings present are $R_{V, A}^{\prime}$, the constraints are

$$
\begin{equation*}
\frac{\left|\operatorname{Re}\left(R_{V}^{\prime}\right)\right|^{2}}{(3.5)^{2}}+\frac{\left|\operatorname{Im}\left(R_{V}^{\prime}\right)\right|^{2}}{(4.0)^{2}} \lesssim 1.0, \quad \frac{\left|\operatorname{Re}\left(R_{A}^{\prime}\right)\right|^{2}}{(3.5)^{2}}+\frac{\left|\operatorname{Im}\left(R_{A}^{\prime}\right)\right|^{2}}{(4.0)^{2}} \lesssim 1.0 \tag{2.11}
\end{equation*}
$$

For the SP operators, the present upper bound on $B\left(\bar{B}_{s}^{0} \rightarrow \mu^{+} \mu^{-}\right)$provides the limit

$$
\begin{equation*}
\left|R_{S}-R_{S}^{\prime}\right|^{2}+\left|R_{P}-R_{P}^{\prime}\right|^{2} \lesssim 0.44 \tag{2.12}
\end{equation*}
$$

This constitutes a severe constraint on the NP couplings if only $R_{S, P}$ or $R_{S, P}^{\prime}$ are present. However, if both types of operators are present, these bounds can be evaded due to cancellations between the $R_{S, P}$ and $R_{S, P}^{\prime}$. In that case, $B\left(\bar{B}_{d}^{0} \rightarrow X_{s} \mu^{+} \mu^{-}\right)$and $B\left(\bar{B}_{d}^{0} \rightarrow \bar{K} \mu^{+} \mu^{-}\right)$can still bound these couplings. The stronger bound is obtained from the measurement of the latter quantity, which yields

$$
\begin{equation*}
\left|R_{S}\right|^{2}+\left|R_{P}\right|^{2} \lesssim 9, \quad R_{S} \approx R_{S}^{\prime}, \quad R_{P} \approx R_{P}^{\prime} \tag{2.13}
\end{equation*}
$$

Finally, the constraints on the NP tensor operators come entirely from $B\left(\bar{B}_{d}^{0} \rightarrow\right.$ $\left.X_{s} \mu^{+} \mu^{-}\right)$. When only the T operators are present,

$$
\begin{equation*}
\left|C_{T}\right|^{2}+4\left|C_{T E}\right|^{2} \lesssim 1.0 . \tag{2.14}
\end{equation*}
$$

The constraints are not affected significantly if more than one type (VA, SP or T) of NP operators is present simultabeously.

### 2.3 CP-violating effects

All CP-violating effects are due to the interference of (at least) two amplitudes with a relative weak phase. In principle, there can be three types of interference: SM-SM, SM-NP, NP-NP. In the SM, all contributions to the $b \rightarrow s \mu^{+} \mu^{-}$modes are proportional to the Cabibbo-Kobayashi-Maskawa (CKM) factors $V_{t b}^{*} V_{t s}, V_{c b}^{*} V_{c s}$, or $V_{u b}^{*} V_{u s}$. The term $V_{c b}^{*} V_{c s}$ can be eliminated in terms of the other two using the unitarity of the CKM matrix. Furthermore, although $V_{u b}^{*} V_{u s}$ has a large weak phase, its magnitude is greatly suppressed relative to that of $V_{t b}^{*} V_{t s}$. Thus, to a good approximation, all nonzero SM contributions have the same weak phase, and so all CP-violating effects are predicted to be tiny in the SM.

There are two types of CP violation. The first is direct CP-violating asymmetries. Suppose that a particular $\bar{B}$ decay has two contributing amplitudes: iM $(\bar{B}$ decay $)=$ $\mathcal{A}_{1}+\mathcal{A}_{2}$. Each amplitude has both a weak and a strong phase. The matrix element $i \overline{\mathcal{M}}$ for the CP-conjugate decay is the same as $i \mathcal{M}$, except that the weak phases change signs. CP violation is indicated by a nonzero value of $|\mathcal{M}|^{2}-|\overline{\mathcal{M}}|^{2}$. It is straightforward to show that this is proportional to $\sin \phi_{w} \sin \delta$, where $\phi_{w}$ and $\delta$ are, respectively, the relative weak and strong phases between $\mathcal{A}_{1}$ and $\mathcal{A}_{2}$. Direct CPviolating asymmetries therefore require that the interfering amplitudes have both a nonzero relative weak and strong phase.

The second type of CP violation is triple-product (TP) asymmetries. Suppose that the matrix element for the $\bar{B}$ decay takes the form $i \mathcal{M}(\bar{B}$ decay $)=\mathcal{A}_{1}+$ $i \mathcal{A}_{2} \epsilon_{\mu \nu \rho \sigma} p_{\bar{B}}^{\mu} v_{1}^{\nu} v_{2}^{\rho} v_{3}^{\sigma}$, where the $v_{i}$ are spins or momenta of the final-state particles. The difference $|\mathcal{M}|^{2}-|\overline{\mathcal{M}}|^{2}$ is proportional to $m_{\bar{B}} \vec{v}_{1} \cdot\left(\vec{v}_{2} \times \vec{v}_{3}\right) \sin \phi_{w} \cos \delta$. By measuring the TP $\vec{v}_{1} \cdot\left(\vec{v}_{2} \times \vec{v}_{3}\right)$ in both $\bar{B}$ and $B$ decays, the TP asymmetry can be obtained. Note that the measurement of a nonzero TP in the $\bar{B}$ decay alone is not sufficient to establish CP violation, i.e. it does not necessarily imply a nonzero weak phase. A fake, CP-conserving TP can be produced if $\mathcal{A}_{1}$ and $\mathcal{A}_{2}$ have a relative strong phase. It is only by measuring the difference of TPs in $\bar{B}$ and $B$ decays that the fake TP can be eliminated and a true, CP-violating signal produced [33].

Let us first turn to direct CP violation, which requires both a relative weak and strong phase between two interfering amplitudes. Now, strong phases are generated through the rescattering of the operators in the effective Hamiltonian. The NP strong phases involve only the (constrained) NP operators, and are therefore small [34]. Thus, direct CP asymmetries can never arise from NP-NP interference.

On the other hand, the SM strong phase is not so small. It is generated because the Wilson coefficient $C_{9}^{\text {eff }}$, which gets a contribution from a $c \bar{c}$ quark loop, has an imaginary piece. ( $C_{9}^{\text {eff }}$ also gets a contribution from a $u \bar{u}$ quark loop. But this is proportional to $V_{u b}^{*} V_{u s}$, and hence negligible.) The quantity $C_{9}^{\text {eff }}$ can be written as [25]

$$
\begin{align*}
C_{9}^{\mathrm{eff}}= & C_{9}\left(m_{b}\right)+h\left(z, \hat{m}_{c}\right)\left(\frac{4}{3} C_{1}+C_{2}+6 C_{3}+60 C_{5}\right) \\
& -\frac{1}{2} h\left(z, \hat{m}_{b}\right)\left(7 C_{3}+\frac{4}{3} C_{4}+76 C_{5}+\frac{64}{3} C_{6}\right)  \tag{2.15}\\
& -\frac{1}{2} h(z, 0)\left(C_{3}+\frac{4}{3} C_{4}+16 C_{5}+\frac{64}{3} C_{6}\right)+\frac{4}{3} C_{3}+\frac{64}{9} C_{5}+\frac{64}{27} C_{6} .
\end{align*}
$$

Here $z \equiv q^{2} / m_{b}^{2}$, and $\hat{m}_{q} \equiv m_{q} / m_{b}$ for all quarks $q$. The function $h(z, \hat{m})$ represents the one-loop correction to the four-quark operators $O_{1}-O_{6}$ and is given by [35, 23, 25]

$$
\begin{align*}
h(z, \hat{m})= & -\frac{8}{9} \ln \frac{m_{b}}{\mu_{b}}-\frac{8}{9} \ln \hat{m}+\frac{8}{27}+\frac{4}{9} x  \tag{2.16}\\
& -\frac{2}{9}(2+x)|1-x|^{1 / 2} \begin{cases}\left(\ln \left|\frac{\sqrt{1-x}+1}{\sqrt{1-x}-1}\right|-i \pi\right), & \text { for } x \leq 1 \\
2 \arctan \frac{1}{\sqrt{x-1}}, & \text { for } x>1\end{cases}
\end{align*}
$$

where $x \equiv 4 \hat{m}^{2} / z$. Thus, a nontrivial strong phase is generated when $z \geq 4 \hat{m}^{2}$. This leads to the complex nature of $C_{9}^{\text {eff }}$ in the SM. For example, typical values of $C_{9}^{\mathrm{eff}}$ in the low- and high- $q^{2}$ regions are $C_{9}^{\text {eff }}\left(m_{b}\right)=4.75+0.09 i(z=0.1), C_{9}^{\text {eff }}\left(m_{b}\right)=4.76+$ $0.88 i(z=0.7) . C_{9}^{\text {eff }}$ therefore has a nontrivial imaginary component, which implies that direct CP asymmetries can arise due to SM-NP interference. Since the SM
operator $\left(C_{9}^{\text {eff }}\right)$ is of VA type, the NP operator must also be VA in order to generate a significant direct CP asymmetry. Other NP operators can also interfere with the SM, but the effect is suppressed by $m_{\mu} / m_{b}$, and hence very small. Note that, although this argument has used the total decay rate for illustration, we could have used (almost) any observable which is related to the square of the matrix element. This includes the differential branching ratio, forward-backward asymmetry, polarization asymmetries, etc.

The TP asymmetries, on the other hand, do not need a difference in strong phases between two amplitudes. Indeed, they are proportional to $\cos \delta$, though they do require a weak-phase difference. This means that a TP asymmetry can be produced by either SM-NP or NP-NP interference. Given that all SM operators are of VA type, the NP must also be VA if SM-NP interference is the reason for the TP. On the other hand, if NP-NP interference is involved, this can arise due to new SP and T operators (other NP-NP interference are possible, but the effects are suppressed by $m_{\mu} / m_{b}$ ).

In this paper, we explore both sources of CP asymmetries, direct CP violation and TPs. While we have checked the effects of SP and T NP operators on all the observables, we find them to be insignificant in most places (as expected from the arguments above), and we will mention them only during the discussion of TP asymmetries, where, in principle, they may play a significant role.

## 3. $\bar{B}_{s}^{0} \rightarrow \boldsymbol{\mu}^{+} \boldsymbol{\mu}^{-}$

We begin by considering the direct CP asymmetry in $\bar{B}_{s}^{0} \rightarrow \mu^{+} \mu^{-}$. Helicity conservation in the decay of $B_{s}$ or $\bar{B}_{s}$ implies that the only final states can be $\mu_{L}^{+} \mu_{L}^{-}$ or $\mu_{R}^{+} \mu_{R}^{-}$, which are $C P$ conjugates. The only CP-violating observables that can be constructed are then

$$
\begin{align*}
A_{C P}^{R L}(t) & \equiv \frac{B\left(\bar{B}_{s}^{0}(t) \rightarrow \mu_{R}^{+} \mu_{R}^{-}\right)-B\left(B_{s}^{0}(t) \rightarrow \mu_{L}^{+} \mu_{L}^{-}\right)}{B\left(\bar{B}_{s}^{0}(t) \rightarrow \mu_{R}^{+} \mu_{R}^{-}\right)+B\left(B_{s}^{0}(t) \rightarrow \mu_{L}^{+} \mu_{L}^{-}\right)} \\
A_{C P}^{L R}(t) & \equiv \frac{B\left(\bar{B}_{s}^{0}(t) \rightarrow \mu_{L}^{+} \mu_{L}^{-}\right)-B\left(B_{s}^{0}(t) \rightarrow \mu_{R}^{+} \mu_{R}^{-}\right)}{B\left(\bar{B}_{s}^{0}(t) \rightarrow \mu_{L}^{+} \mu_{L}^{-}\right)+B\left(B_{s}^{0}(t) \rightarrow \mu_{R}^{+} \mu_{R}^{-}\right)} \tag{3.1}
\end{align*}
$$

The CP asymmetry in the longitudinal polarization fraction $A_{L P}$ may be written in terms of these two CP asymmetries. The measurement of either of these CP asymmetries requires the measurement of muon polarization, which will be an impossible task for the upcoming experiments [11]. And even if this were doable, the lack of any sources for different strong phases in the two CP-conjugate final states implies that the direct CP asymmetry would vanish even with NP. We therefore do not study CP violation in $\bar{B}_{s}^{0} \rightarrow \mu^{+} \mu^{-}$.

## 4. $\bar{B}_{d}^{0} \rightarrow X_{s} \mu^{+} \mu^{-}$

A model-independent analysis of the CP asymmetry in the differential branching ratio (DBR) of $\bar{B}_{d}^{0} \rightarrow X_{s} \mu^{+} \mu^{-}$was previously carried out in Ref. [14]. There, the CP asymmetry in the DBR was predicted for some specific values of the NP couplings. However, no experimental constraints on the parameters were used. In this paper we study the CP asymmetry in the DBR, taking into account the constraints from the present measurements of other related observables. Moreover, in addition to the CP asymmetry in the DBR , we also study the CP asymmetry in the forward-backward asymmetry.

The CP asymmetry in DBR of $\bar{B}_{d}^{0} \rightarrow X_{s} \mu^{+} \mu^{-}$is defined as

$$
\begin{equation*}
A_{\mathrm{CP}}\left(q^{2}\right)=\frac{(d B / d z)-(d \bar{B} / d z)}{(d B / d z)+(d \bar{B} / d z)} \tag{4.1}
\end{equation*}
$$

where $z \equiv q^{2} / m_{b}^{2}$, and $d B / d z$ and $d \bar{B} / d z$ are the DBRs of $\bar{B}_{d}^{0} \rightarrow X_{s} \mu^{+} \mu^{-}$and its CP-conjugate $B_{d}^{0} \rightarrow X_{s} \mu^{+} \mu^{-}$, respectively. The expression for $(d B / d z)$ has been given in Ref. [11].

The CP asymmetry in the forward-backward asymmetry $A_{F B}$ is defined as

$$
\begin{equation*}
\Delta A_{F B}\left(q^{2}\right) \equiv A_{F B}\left(q^{2}\right)-\bar{A}_{F B}\left(q^{2}\right) \tag{4.2}
\end{equation*}
$$

where the definition of $A_{F B}$ is given in Ref. [11], and $\bar{A}_{F B}$ is the analogous quantity for the CP-conjugate decay. Note that while the relevant angle $\theta$ in $\bar{B}_{d}^{0} \rightarrow X_{s} \mu^{+} \mu^{-}$is defined relative to the direction of $\mu^{+}$, for the CP-conjugate decay one should define $\theta$ in relation to the direction of $\mu^{-}$, and similarly for $A_{F B}$ in other $b \rightarrow s \mu^{+} \mu^{-}$decay modes below.

Fig. 1 shows $A_{\mathrm{CP}}\left(q^{2}\right)$ and $\Delta A_{F B}\left(q^{2}\right)$ for $\bar{B}_{d}^{0} \rightarrow X_{s} \mu^{+} \mu^{-}$in the presence of new VA couplings. We make the following observations:

- When only $R_{V, A}$ couplings are present, $A_{\mathrm{CP}}\left(q^{2}\right)$ can be enhanced up to $6 \%$ at low $q^{2}$. On the other hand, its value at high $q^{2}$ can be as high as $12 \%$. $A_{\mathrm{CP}}\left(q^{2}\right)$ can have either sign at both low and high $q^{2}$. At high $q^{2}$, the magnitude of $A_{\mathrm{CP}}\left(q^{2}\right)$ is almost independent of $q^{2}$.
- When only $R_{V, A}^{\prime}$ couplings are present, $A_{\mathrm{CP}}\left(q^{2}\right)$ cannot be enhanced above the SM value. This is because $R_{V, A}^{\prime}$ couplings do not contribute to the numerator of $A_{\mathrm{CP}}\left(q^{2}\right)$ in Eq. (4.1). They can only affect the DBR, which may be enhanced by up to $50 \%$, thus decreasing $A_{\mathrm{CP}}\left(q^{2}\right)$.
- In the presence of $R_{V, A}$ couplings, $\Delta A_{F B}$ can be enhanced up to $3 \%$ at low $q^{2}$. At high $q^{2}$, the enhancement can be up to $12 \%$. The impact of $R_{V, A}^{\prime}$ couplings is negligible $(<1 \%)$.


Figure 1: The left (right) panels of the figure show $A_{\mathrm{CP}}\left(q^{2}\right)$ and $\Delta A_{F B}$ for $\bar{B}_{d}^{0} \rightarrow X_{s} \mu^{+} \mu^{-}$ in the low- $q^{2}$ (high- $q^{2}$ ) region, in the scenario where only $\left(R_{V}, R_{A}\right)$ couplings are present. The green line corresponds to the SM prediction. The other lines show predictions for some representative values of the NP parameters $\left(R_{V}, R_{A}\right)$. For example, the blue curve in the low- $q^{2}$ and high- $q^{2}$ regions for the $A_{\mathrm{CP}}$ plot corresponds to $\left(5.68 e^{i 2.13}, 2.64 e^{-i 0.04}\right)$ and $\left(4.29 e^{i 1.68}, 4.15 e^{-i 0.26}\right)$, respectively, whereas the blue curve in the low- $q^{2}$ and high- $q^{2}$ regions for the $\Delta A_{\mathrm{FB}}$ plot corresponds to $\left(1.80 e^{i 2.91}, 5.45 e^{i 0.90}\right)$ and $\left(1.69 e^{-i 3.08}, 6.83 e^{-i 0.91}\right)$, respectively.

## 5. $\bar{B}_{s}^{0} \rightarrow \boldsymbol{\mu}^{+} \boldsymbol{\mu}^{-} \gamma$

Although $\bar{B}_{s}^{0} \rightarrow \mu^{+} \mu^{-} \gamma$ requires the emission of an additional photon as compared to $\bar{B}_{s}^{0} \rightarrow \mu^{+} \mu^{-}$, which suppresses the branching ratio ( BR ) by a factor of $\alpha_{e m}$, the photon emission also frees it from helicity suppression, making its BR much larger than $\bar{B}_{s}^{0} \rightarrow \mu^{+} \mu^{-}$. The SM prediction for the BR in the range $q^{2} \leq 9.5 \mathrm{GeV}^{2}$ and $q^{2} \geq 15.9 \mathrm{GeV}^{2}$ is $\approx 18.9 \times 10^{-9}$ [36]. As argued in Ref. [11], if we choose 2 $\mathrm{GeV}^{2} \leq q^{2} \leq 6 \mathrm{GeV}^{2}$ and $14.4 \mathrm{GeV}^{2} \leq q^{2} \leq 25 \mathrm{GeV}^{2}$ as the low- $q^{2}$ and high- $q^{2}$ regions, respectively, then the dominating contribution comes from the diagrams in which the final-state photon is emitted either from the $b$ or the $s$ quark, and the $\bar{B}_{s}^{0} \rightarrow \mu^{+} \mu^{-} \gamma$ decay is governed by the same $b \rightarrow s \mu^{+} \mu^{-}$effective Hamiltonian as the other decays considered in this paper.

The CP asymmetry in $\bar{B}_{s}^{0} \rightarrow \mu^{+} \mu^{-} \gamma$ is given in Eq. (4.1), where $d B / d x_{\gamma}$ and $d \bar{B} / d x_{\gamma}$ are the DBRs of $\bar{B}_{s}^{0} \rightarrow \mu^{+} \mu^{-} \gamma$ and its CP-conjugate $B_{s}^{0} \rightarrow \mu^{+} \mu^{-} \gamma$, respectively. The expression for $\left(d B / d x_{\gamma}\right)$ has been given in Ref. [11]. The CP asymmetry in $A_{F B}$ is given in Eq. (4.2), where the definition of $A_{F B}$ is given in Ref. [11], and


Figure 2: The left (right) panels of the figure show $A_{\mathrm{CP}}\left(q^{2}\right)$ and $\Delta A_{F B}$ for $\bar{B}_{s}^{0} \rightarrow \mu^{+} \mu^{-} \gamma$ in the low- $q^{2}$ (high- $q^{2}$ ) region, in the scenario where only $\left(R_{V}, R_{A}\right)$ couplings are present. For example, the blue curve in the low- $q^{2}$ and high- $q^{2}$ regions for the $A_{\mathrm{CP}}$ plot corresponds to $\left(2.95 e^{-i 0.38}, 4.56 e^{-i 0.04}\right)$, whereas the blue curve in the low- $q^{2}$ and high- $q^{2}$ regions for the $\Delta A_{\text {FB }}$ plot corresponds to $\left(1.60 e^{-i 0.08}, 4.14 e^{-i 0.12}\right)$.
$\bar{A}_{F B}$ is the analogous quantity for the CP-conjugate decay.
The CP asymmetry in the DBR of $B_{s} \rightarrow \mu \mu \gamma$ was studied in Refs. [17, 18], albeit only for the new-physics cases where $C_{7}=-C_{7}^{\mathrm{SM}}, C_{9}=-C_{9}^{\mathrm{SM}}$ and $C_{10}=-C_{10}^{\mathrm{SM}}$, and naturally only for VA operators. Here, we include a complete discussion of the possible enhancement of the asymmetry for all allowed values of $C_{9}$ and $C_{10}$, and in the presence of SP and T operators. In addition, we study the CP-violating asymmetry in $A_{F B}$, which also turns out to give possibly significant NP signals.

Fig. 2 shows $A_{\mathrm{CP}}\left(q^{2}\right)$ and $\Delta A_{F B}\left(q^{2}\right)$ for $\bar{B}_{s}^{0} \rightarrow \mu^{+} \mu^{-} \gamma$ in the presence of new VA couplings. We make the following observations:

- When only $R_{V, A}$ couplings are present, at low $q^{2}$ the magnitude of $A_{\mathrm{CP}}\left(q^{2}\right)$ can be enhanced up to $30 \%$ at certain $q^{2}$ values. At high $q^{2}$, the magnitude of $A_{\mathrm{CP}}\left(q^{2}\right)$ is almost independent of $q^{2}$, and can be enhanced to about $13 \%$. The asymmetry can have either sign at both low and high $q^{2}$.
- When only $R_{V, A}^{\prime}$ couplings are present, $A_{\mathrm{CP}}\left(q^{2}\right)$ cannot be enhanced in magnitude to more than $1.5 \%$ at low $q^{2}$, or more than $3 \%$ at high $q^{2}$. The detection of NP of this kind is therefore expected to be very difficult in this channel. When both primed and unprimed VA couplings are present, the results are the same as those obtained with only $R_{V, A}$ couplings.
- The behaviour of $\Delta A_{F B}\left(q^{2}\right)$ is similar to that of $A_{C P}\left(q^{2}\right)$. This quantity can be enhanced up to $40 \%$ for some values in the low- $q^{2}$ region. It can be as high as $18 \%$ throughout the high $-q^{2}$ region. The impact of $R_{V, A}^{\prime}$ couplings is negligible $(<1 \%)$.

The new VA operators can therefore enhance the asymmetries $A_{\mathrm{CP}}\left(q^{2}\right)$ and $\Delta A_{F B}\left(q^{2}\right)$ in $\bar{B}_{s}^{0} \rightarrow \mu^{+} \mu^{-} \gamma$ to $\sim 10 \%$ throughout the $q^{2}$ region. For a branching ratio of $O\left(2 \times 10^{-8}\right)$, a measurement of a CP asymmetry of $10 \%$ at the $3 \sigma$ level would require $\sim 10^{10} B$ mesons. It should therefore be possible to measure a CP asymmetry at the level of a few per cent at future colliders such as the Super- $B$ factories [37, 38, 39].

## 6. $\bar{B}_{d}^{0} \rightarrow \overline{\boldsymbol{K}} \boldsymbol{\mu}^{+} \boldsymbol{\mu}^{-}$

The CP asymmetry in $\bar{B}_{d}^{0} \rightarrow \bar{K} \mu^{+} \mu^{-}$is defined in a manner similar to that in Eq. (4.1), where $d B / d z$ and $d \bar{B} / d z$ are the DBRs of $\bar{B}_{d}^{0} \rightarrow \bar{K} \mu^{+} \mu^{-}$and its CPconjugate $B_{d}^{0} \rightarrow K \mu^{+} \mu^{-}$, respectively. The expression for $(d B / d z)$ has been given in Ref. [11]. A model-independent analysis of the CP asymmetry in the DBR, with specific chosen values of VA operators, was carried out in Ref. [19]. However, the constraints on the NP operators, coming from the measured branching ratio of $\bar{B}_{d}^{0} \rightarrow$ $X_{s} \mu^{+} \mu^{-}$, were not taken into account. Here, in addition to taking these constraints into account, we also consider new SP and T operators, and extend the analysis to study the CP asymmetry in $A_{F B}$.

The CP asymmetry in $A_{F B}$ is given in Eq. (4.2), where the definition of $A_{F B}$ is as given in Ref. [11], while $\bar{A}_{F B}$ is the analogous quantity for the CP-conjugate decay. Now, the decay mode $\bar{B}_{d}^{0} \rightarrow \bar{K} \mu^{+} \mu^{-}$is unique because the forward-backward asymmetry of muons is predicted to vanish exactly in the SM. This is due to the fact that the $\bar{B}_{d}^{0} \rightarrow \bar{K}$ hadronic matrix element does not have any axial-vector contribution. $A_{F B}$ can therefore have a nonzero value only if it receives a contribution from new physics. However, even in the presence of NP, the expressions in Ref. [11] indicate that the only term contributing to $\Delta A_{F B}\left(q^{2}\right)$ is that with VA+SP NP operators, and this is suppressed by the factor $m_{\mu} / m_{b}$. As a result, one does not expect a significant enhancement in $\Delta A_{F B}$ from any Lorentz structure of NP.

Fig. 3 shows $A_{\mathrm{CP}}\left(q^{2}\right)$ for $\bar{B}_{d}^{0} \rightarrow \bar{K} \mu^{+} \mu^{-}$in the presence of new VA couplings. We make the following observations:

- When only $R_{V, A}$ couplings are present, $A_{\mathrm{CP}}\left(q^{2}\right)$ can be enhanced up to $7 \%$ at low $q^{2}$. On the other hand, its value at high $q^{2}$ can be as high as $12 \%$. $A_{\mathrm{CP}}\left(q^{2}\right)$ can have either sign at both low and high $q^{2}$, and its magnitude is almost independent of $q^{2}$ in these regions.


Figure 3: The left (right) panel of the figure shows $A_{\mathrm{CP}}\left(q^{2}\right)$ for $\bar{B}_{d}^{0} \rightarrow \bar{K} \mu^{+} \mu^{-}$in the low$q^{2}$ (high- $q^{2}$ ) region, in the scenario where only $\left(R_{V}, R_{A}\right)$ terms are present. The green line corresponds to the SM prediction. The other lines show predictions for some representative values of the NP parameters $\left(R_{V}, R_{A}\right)$. For example, the blue curve in the low- $q^{2}$ and high$q^{2}$ regions corresponds to $\left(5.97 e^{i 2.23}, 3.08 e^{-i 0.05}\right)$ and $\left(6.47 e^{i 2.30}, 3.11 e^{i 0.48}\right)$, respectively.

- When only $R_{V, A}^{\prime}$ couplings are present, $A_{\mathrm{CP}}\left(q^{2}\right)$ can be enhanced up to $4 \%$ at low $q^{2}$. On the other hand, its value at high $q^{2}$ can be as high as $12 \% . A_{\mathrm{CP}}\left(q^{2}\right)$ can have either sign at both low and high $q^{2}$, and its magnitude is almost independent of $q^{2}$ in these regions.
- When both primed and unprimed VA couplings are present, the results are the same as those obtained with only $R_{V, A}$ couplings.

For a $\bar{B}_{d}^{0} \rightarrow \bar{K} \mu^{+} \mu^{-}$branching ratio of $O\left(0.5 \times 10^{-6}\right)$, a measurement of a CP asymmetry of $1 \%$ at the $3 \sigma$ level would require $\sim 10^{11} \mathrm{~B}$ mesons. It should therefore be possible to measure a CP asymmetry at the level of a few per cent at future colliders such as the Super- $B$ factories [37, 38, 39].

## 7. $\bar{B}_{d}^{0} \rightarrow \overline{\boldsymbol{K}}^{*} \boldsymbol{\mu}^{+} \boldsymbol{\mu}^{-}$

The complete three-angle distribution for the decay $\bar{B}^{0} \rightarrow \bar{K}^{* 0}\left(\rightarrow K^{-} \pi^{+}\right) \mu^{+} \mu^{-}$in the presence of NP can be expressed in terms of $q^{2}$, two polar angles $\theta_{\mu}$ and $\theta_{K}$, and the azimuthal angle $\phi$ between the planes of the dimuon and $K \pi$ decays:

$$
\begin{align*}
& \quad \frac{d^{4} \Gamma^{\bar{B}}}{d q^{2} d \cos \theta_{\mu} d \cos \theta_{K} d \phi}=N_{F} \times \\
& \left\{\cos ^{2} \theta_{K}\left(I_{1}^{0}+I_{2}^{0} \cos 2 \theta_{\mu}+I_{3}^{0} \cos \theta_{\mu}\right)+\sin ^{2} \theta_{K}\left(I_{1}^{T}+I_{2}^{T} \cos 2 \theta_{\mu}+I_{3}^{T} \cos \theta_{\mu}\right.\right. \\
& \left.+I_{4}^{T} \sin ^{2} \theta_{\mu} \cos 2 \phi+I_{5}^{T} \sin ^{2} \theta_{\mu} \sin 2 \phi\right)+\sin 2 \theta_{K}\left(I_{1}^{L T} \sin 2 \theta_{\mu} \cos \phi\right. \\
& \left.\left.+I_{2}^{L T} \sin 2 \theta_{\mu} \sin \phi+I_{3}^{L T} \sin \theta_{\mu} \cos \phi+I_{4}^{L T} \sin \theta_{\mu} \sin \phi\right)\right\} \tag{7.1}
\end{align*}
$$

The expressions for the normalization $N_{F}$ and the I's are given in Ref. [11]. The I's are functions of the couplings, kinematic variables and form factors. The definitions of the angles in $\bar{B}_{d}^{0} \rightarrow \bar{K}^{*} \mu^{+} \mu^{-}$involve the directions of the $\mu^{+}$and $\bar{K}^{*}$. For the CP-conjugate decay $B^{0} \rightarrow K^{* 0}\left(\rightarrow K^{+} \pi^{-}\right) \mu^{+} \mu^{-}$, one defines these angles relative to the directions of the $\mu^{-}$and $K^{*}$. The $\bar{I}$ 's can be obtained from the $I$ 's by replacing $\theta_{\mu} \rightarrow \theta_{\mu}-\pi$ and $\phi \rightarrow-\phi$, and changing the signs of the weak phases.

The CP asymmetries in the branching ratio and forward-backward asymmetry were analyzed in Ref. [23] with the measurement of $B \rightarrow X_{s} \gamma$ and the limit on the $\bar{B}_{d}^{0} \rightarrow \bar{K}^{*} \mu^{+} \mu^{-}$branching ratio available then. An analysis of CP asymmetries in $\bar{B}_{d}^{0} \rightarrow \bar{K}^{*} \mu^{+} \mu^{-}$in the low- $q^{2}$ region was also performed earlier in Ref. [25]. We extend this analysis by including T operators, and present our results for all asymmetries, in both the low- $q^{2}$ and high- $q^{2}$ regions.

A detailed discussion of the CP-conserving observables in this decay distribution can be found in Ref. [11]. In this section we consider the direct CP asymmetries in the differential branching ratio ( DBR ), the forward-backward asymmetry $A_{F B}$, the longitudinal polarization fraction $f_{L}$, and the angular asymmetries $A_{T}^{(2)}$ and $A_{L T}$. We also examine the triple-product CP asymmetries $A_{T}^{(i m)}$ and $A_{L T}^{(i m)}$, which were not considered in Ref. [11] since they identically vanish in the CP-conserving limit (no strong or weak phases), regardless of the presence of NP.

### 7.1 Direct CP asymmetries in the DBR and $\boldsymbol{A}_{\text {FB }}$

The direct CP asymmetry in the differential branching ratio is defined as

$$
\begin{equation*}
A_{C P}\left(q^{2}\right)=\frac{\left(d \Gamma^{\bar{B}} / d q^{2}\right)-\left(d \Gamma^{B} / d q^{2}\right)}{\left(d \Gamma^{\bar{B}} / d q^{2}\right)+\left(d \Gamma^{B} / d q^{2}\right)}, \tag{7.2}
\end{equation*}
$$

where

$$
\begin{equation*}
\frac{d \Gamma^{\bar{B}}}{d q^{2}}=\frac{8 \pi N_{F}}{3}\left(A_{L}^{\bar{B}}+A_{T}^{\bar{B}}\right) \tag{7.3}
\end{equation*}
$$

Here the longitudinal and transverse polarization amplitudes $A_{L}^{\bar{B}}$ and $A_{T}^{\bar{B}}$ are obtained from Eq. (7.1):

$$
\begin{equation*}
A_{L}^{\bar{B}}=\left(I_{1}^{0}-\frac{1}{3} I_{2}^{0}\right), \quad A_{T}^{\bar{B}}=2\left(I_{1}^{T}-\frac{1}{3} I_{2}^{T}\right) . \tag{7.4}
\end{equation*}
$$

The expressions for $A_{L}^{B}$ and $A_{T}^{B}$ of the CP-conjugate mode can be obtained by replacing the $I$ 's with $\bar{I}$ 's.

The forward-backward asymmetry in $\bar{B}_{d}^{0} \rightarrow \bar{K}^{*} \mu^{+} \mu^{-}$has recently been measured, and shows features that may indicate a deviation from the SM. This measured quantity is actually the CP-averaged forward-backward asymmetry $A_{F B}$. However, the difference between the measurement of this quantity in $\bar{B}_{d}^{0} \rightarrow \bar{K}^{*} \mu^{+} \mu^{-}$and
its CP-conjugate mode may also reveal the presence of NP. This CP asymmetry is quantified as

$$
\begin{equation*}
\Delta A_{F B}\left(q^{2}\right)=A_{F B}^{\bar{B}}\left(q^{2}\right)+A_{F B}^{B}\left(q^{2}\right), \tag{7.5}
\end{equation*}
$$

where

$$
\begin{equation*}
A_{F B}^{\bar{B}(B)}\left(q^{2}\right)=\frac{\int_{0}^{1} d \cos \theta_{\mu} \frac{d^{2} \Gamma^{\bar{B}}(B)}{d q^{2} d \cos \theta_{\mu}}-\int_{-1}^{0} d \cos \theta_{\mu} \frac{d^{2} \Gamma^{\bar{B}(B)}}{d q^{2} d \cos \theta_{\mu}}}{\int_{0}^{1} d \cos \theta_{\mu} \frac{d^{2} \Gamma^{\bar{B}(B)}}{d q^{2} d \cos \theta_{\mu}}+\int_{-1}^{0} d \cos \theta_{\mu} \frac{d^{2} \Gamma^{\bar{B}(B)}}{d q^{2} d \cos \theta_{\mu}}} . \tag{7.6}
\end{equation*}
$$

It can be obtained by integrating over the two angles $\theta_{K}$ and $\phi$ in Eq. (7.1).
Fig. 4 shows $A_{C P}\left(q^{2}\right)$ and $\Delta A_{F B}\left(q^{2}\right)$ for $\bar{B}_{d}^{0} \rightarrow \bar{K}^{*} \mu^{+} \mu^{-}$in the presence of new VA couplings. We make the following observations:

- If only $R_{V, A}$ couplings are present, $A_{C P}\left(q^{2}\right)$ can be enhanced up to $5 \%$ at low $q^{2}$, and up to $14 \%$ at high $q^{2} . \Delta A_{F B}\left(q^{2}\right)$ can be enhanced up to $3 \%$ at low $q^{2}$, and up to $11 \%$ at high $q^{2}$. Both $A_{C P}\left(q^{2}\right)$, and $\Delta A_{F B}\left(q^{2}\right)$ can have either sign at both low and high $q^{2}$.
- If only $R_{V, A}^{\prime}$ couplings are present, $A_{C P}\left(q^{2}\right)$ can be enhanced up to $3 \%$ at low $q^{2}$, and up to $7 \%$ at high $q^{2}$. $\Delta A_{F B}\left(q^{2}\right)$ can be enhanced up to $1 \%$ at low $q^{2}$, and up to $4 \%$ at high $q^{2}$. Both $A_{C P}\left(q^{2}\right)$, and $\Delta A_{F B}\left(q^{2}\right)$ can have either sign at both low and high $q^{2}$.
- When both primed and unprimed VA couplings are present, $A_{C P}\left(q^{2}\right)$ can be enhanced up to $9 \%$ at low $q^{2}$, and up to $14 \%$ at high $q^{2} . \Delta A_{F B}\left(q^{2}\right)$ can be enhanced up to $6 \%$ at low $q^{2}$, and up to $19 \%$ at high $q^{2}$. Both $A_{C P}\left(q^{2}\right)$, and $\Delta A_{F B}\left(q^{2}\right)$ can have either sign at both low and high $q^{2}$ (see Fig. 4).

These observations are consistent with the rough expectations in Ref. [23] about the effect of VA operators.

### 7.2 Direct CP asymmetry in the polarization fraction $f_{L}$

The CP asymmetry in the longitudinal polarization fraction $f_{L}$ is defined as

$$
\begin{equation*}
\Delta f_{L}=f_{L}^{\bar{B}}-f_{L}^{B} \tag{7.7}
\end{equation*}
$$

where

$$
\begin{equation*}
f_{L}^{\bar{B}(B)}=\frac{A_{L}^{\bar{B}(B)}}{A_{L}^{\bar{B}(B)}+A_{T}^{\bar{B}(B)}} . \tag{7.8}
\end{equation*}
$$

Fig. 5 shows $\Delta f_{L}\left(q^{2}\right)$ for $\bar{B}_{d}^{0} \rightarrow \bar{K}^{*} \mu^{+} \mu^{-}$in the presence of new VA couplings. We make the following observations:


Figure 4: The left (right) panels of the figure show $A_{C P}\left(q^{2}\right)$ and $\Delta A_{F B}\left(q^{2}\right)$ for $\bar{B}_{d}^{0} \rightarrow \bar{K}^{*} \mu^{+} \mu^{-}$in the low- $q^{2}$ (high- $q^{2}$ ) region, in the scenario where ( $R_{V}, R_{A}, R_{V}^{\prime}, R_{A}^{\prime}$ ) terms are all present. The green line corresponds to the SM prediction. The other lines show predictions for some representative values of the NP parameters. For example, the blue curve for $A_{C P}\left(q^{2}\right)$ in the low- $q^{2}$ and high $-q^{2}$ regions corresponds to $\left(2.77 e^{i 1.83}, 2.08 e^{i 0.5}, 3.8 e^{i 0.08}, 1.23 e^{-i 2.74}\right)$ and ( $5.88 e^{i 2.29}, 1.66 e^{i 0.82}, 3.49 e^{i 0.36}, 1.02 e^{i 0.98}$ ), respectively. The blue curve for $\Delta A_{F B}\left(q^{2}\right)$ in the low- $q^{2}$ and high $-q^{2}$ regions corresponds to $\left(1.56 e^{-i 2.59}, 1.80 e^{-i 0.35}, 4.23 e^{i 0.67}, 1.29 e^{i 1.43}\right)$ and $\left(3.21 e^{i 2.61}, 1.38 e^{i 2.26}, 5.55 e^{i 0.69}, 3.03 e^{i 1.92}\right)$, respectively.

- If only $R_{V, A}$ couplings are present, $\Delta f_{L}\left(q^{2}\right)$ can be enhanced up to $2 \%$ at very low $q^{2}$. On the other hand, $\Delta f_{L}\left(q^{2}\right)$ is almost the same as the SM at high $q^{2}$. It can have either sign at both low and high $q^{2}$.
- If only $R_{V, A}^{\prime}$ couplings are present, $\Delta f_{L}\left(q^{2}\right)$ can be enhanced up to $2 \%$ at both low and high $q^{2}$. It can have either sign at both low and high $q^{2}$.
- When both primed and unprimed VA couplings are present, $\Delta f_{L}\left(q^{2}\right)$ can be enhanced up to $9 \%$ at low $q^{2}$, and up to $6 \%$ at high $q^{2}$. It can have either sign at both low and high $q^{2}$ (see Fig. 5).


### 7.3 Direct CP asymmetries in the angular asymmetries $A_{T}^{(2)}$ and $A_{L T}$

The transverse asymmetry $A_{T}^{(2) \bar{B}(B)}$ is defined [40] through the double differential


Figure 5: The left (right) panel of the figure shows $\Delta f_{L}\left(q^{2}\right)$ for $\bar{B}_{d}^{0} \rightarrow \bar{K}^{*} \mu^{+} \mu^{-}$ in the low- $q^{2}$ (high- $q^{2}$ ) region, in the scenario where ( $R_{V}, R_{A}, R_{V}^{\prime}, R_{A}^{\prime}$ ) terms are all present. For example, the blue curve in the low- $q^{2}$ and high $-q^{2}$ regions corresponds to $\left(2.78 e^{i 2.98}, 2.19 e^{-i 0.77}, 6.91 e^{-i 0.29}, 3.34 e^{-i 0.56}\right)$.
decay rate as

$$
\begin{equation*}
\frac{d^{2} \Gamma^{\bar{B}(B)}}{d q^{2} d \phi}=\frac{1}{2 \pi} \frac{d \Gamma^{\bar{B}(B)}}{d q^{2}}\left[1+f_{T}^{\bar{B}(B)}\left(A_{T}^{(2) \bar{B}(B)} \cos 2 \phi+A_{T}^{(i m) \bar{B}(B)} \sin 2 \phi\right)\right] \tag{7.9}
\end{equation*}
$$

It can be obtained by integrating Eq. (7.1) over the two polar angles $\theta_{\mu}$ and $\theta_{K}$. Here $A_{T}^{(i m) \bar{B}(B)}$ is a triple product, and is discussed separately below. In terms of the coupling constants and matrix elements defined in Ref. [11], $A_{T}^{(2) \bar{B}(B)}$ can be expressed as

$$
\begin{equation*}
A_{T}^{(2) \bar{B}}=\frac{4 I_{4}^{T}}{3 A_{T}^{\bar{B}}}, \quad A_{T}^{(2) B}=\frac{4 \bar{I}_{4}^{T}}{3 A_{T}^{B}} . \tag{7.10}
\end{equation*}
$$

While $A_{T}^{(2) \bar{B}}\left(A_{T}^{(2) B}\right)$ is finite even in the CP-conserving limit (and was discussed in Ref. [11]), a CP asymmetry may be defined through the difference

$$
\begin{equation*}
\Delta A_{T}^{(2)} \equiv A_{T}^{(2) \bar{B}}-A_{T}^{(2) B} \tag{7.11}
\end{equation*}
$$

Fig. 6 shows $\Delta A_{T}^{(2)}$ for $\bar{B}_{d}^{0} \rightarrow \bar{K}^{*} \mu^{+} \mu^{-}$in the presence of new VA couplings. We make the following observations:

- If only $R_{V, A}$ couplings are present, $\Delta A_{T}^{(2)}$ cannot be enhanced more than $1 \%$ at both low and high $q^{2}$. It can have either sign at both low and high $q^{2}$.
- If only $R_{V, A}^{\prime}$ couplings are present, $\Delta A_{T}^{(2)}$ can be enhanced up to $4 \%$ at low $q^{2}$, and up to $6 \%$ high $q^{2}$. It can have either sign at both low and high $q^{2}$.
- When both primed and unprimed VA couplings are present, $\Delta A_{T}^{(2)}$ can be enhanced up to $11 \%$ at low $q^{2}$, and up to $12 \%$ at high $q^{2}$. It can have either sign at both low and high $q^{2}$ (see Fig. 6).


Figure 6: The left (right) panel of the figure shows $\Delta A_{T}^{(2)}\left(q^{2}\right)$ for $\bar{B}_{d}^{0} \rightarrow \bar{K}^{*} \mu^{+} \mu^{-}$in the low- $q^{2}$ (high- $q^{2}$ ) region, in the scenario where $\left(R_{V}, R_{A}, R_{V}^{\prime}, R_{A}^{\prime}\right)$ terms are all present. The green line corresponds to the SM prediction. The other lines show predictions for some representative values of the NP parameters. For example, the blue curve in the low $-q^{2}$ and high $-q^{2}$ regions corresponds to ( $0.11 e^{i 2.18}, 2.66 e^{-i 1.31}, 4.3 e^{i 0.03}, 0.23 e^{-i 2.27}$ ) and $\left(2.32 e^{i 2.51}, 4.89 e^{i 1.27}, 3.12 e^{i 0.42}, 0.14 e^{-i 1.55}\right)$, respectively.

The longitudinal-transverse asymmetry $A_{L T}^{\bar{B}(B)}$ is defined through

$$
\begin{equation*}
\frac{d^{2} \Gamma_{L T}^{\bar{B}(B)}}{d q^{2} d \phi}=\frac{d \Gamma^{\bar{B}(B)}}{d q^{2}}\left(A_{L T}^{(r e) \bar{B}(B)} \cos \phi+A_{L T}^{(i m) \bar{B}(B)} \sin \phi\right), \tag{7.12}
\end{equation*}
$$

where

$$
\begin{equation*}
\frac{d^{2} \Gamma_{L T}^{\bar{B}(B)}}{d q^{2} d \phi}=\int_{0}^{1} d \cos \theta_{K} \frac{d^{3} \Gamma^{\bar{B}(B)}}{d q^{2} d \cos \theta_{K} d \phi}-\int_{-1}^{0} d \cos \theta_{K} \frac{d^{3} \Gamma^{\bar{B}(B)}}{d q^{2} d \cos \theta_{K} d \phi} \tag{7.13}
\end{equation*}
$$

Here $A_{L T}^{(i m) \bar{B}(B)}$ is a triple product, and is discussed separately below. In terms of the coupling constants and matrix elements defined in Ref. [11], $A_{L T}^{(r e) \bar{B}(B)}$ can be expressed as

$$
\begin{equation*}
A_{L T}^{(r e) \bar{B}}=\frac{I_{3}^{L T}}{4\left(A_{L}^{\bar{B}}+A_{T}^{\bar{B}}\right)}, \quad A_{L T}^{(r e) B}=-\frac{\bar{I}_{3}^{L T}}{4\left(A_{L}^{B}+A_{T}^{B}\right)} . \tag{7.14}
\end{equation*}
$$

Note that $A_{L T}^{(r e) B}=-A_{L T}^{(r e) \bar{B}}$ in the CP-conserving limit. Thus, a CP asymmetry may be defined through the sum

$$
\begin{equation*}
\Delta A_{L T}\left(q^{2}\right) \equiv A_{L T}^{(r e) \bar{B}}\left(q^{2}\right)+A_{L T}^{(r e) B}\left(q^{2}\right) . \tag{7.15}
\end{equation*}
$$

We now assume the presence of new VA couplings. However, we find that these couplings cannot enhance $\Delta A_{L T}\left(q^{2}\right)$ to more than $3 \%$ at both low and high $q^{2}$.

Note that $\Delta A_{L T}\left(q^{2}\right)$ is related to the observable $A_{5}^{D}$ in Ref. [24]: $\Delta A_{L T}\left(q^{2}\right) \approx$ $A_{5}^{D} / 4$. Our limit of $3 \%$ on the maximum value of $\Delta A_{L T}\left(q^{2}\right)$ is then consistent with the limit of 0.07 on the average value $\left\langle A_{5}^{D}\right\rangle$ over the low- $q^{2}$ region, as calculated in Ref. [24].

### 7.4 CP-violating triple-product asymmetries

In this subsection, we consider the triple products (TPs) in the decays $\bar{B}^{0} \rightarrow \bar{K}^{* 0}(\rightarrow$ $\left.K^{-} \pi^{+}\right) \mu^{+} \mu^{-}$and $B^{0} \rightarrow K^{* 0}\left(\rightarrow K^{+} \pi^{-}\right) \mu^{+} \mu^{-}$. For the decaying $\bar{B}$ meson, the TP is proportional to $\left(\hat{n}_{K} \times \hat{n}_{\mu}\right) \cdot \hat{n}_{z}$ in its rest frame, where the unit vectors are given in terms of the momenta of the final-state particles as

$$
\begin{equation*}
\hat{n}_{K}=\frac{\hat{p}_{K^{-}} \times \hat{p}_{\pi^{+}}}{\left|\hat{p}_{K^{-}} \times \hat{p}_{\pi^{+}}\right|}, \quad \hat{n}_{z}=\frac{\hat{p}_{K^{-}}+\hat{p}_{\pi^{+}}}{\left|\hat{p}_{K^{-}}+\hat{p}_{\pi^{+}}\right|}, \quad \hat{n}_{\mu}=\frac{\hat{p}_{\mu^{-}} \times \hat{p}_{\mu^{+}}}{\left|\hat{p}_{\mu^{-}} \times \hat{p}_{\mu^{+}}\right|} . \tag{7.16}
\end{equation*}
$$

In terms of the azimuthal angle $\phi$, one gets

$$
\begin{equation*}
\cos \phi=\hat{n}_{K} \cdot \hat{n}_{\mu}, \quad \sin \phi=\left(\hat{n}_{K} \times \hat{n}_{\mu}\right) \cdot \hat{n}_{z} \tag{7.17}
\end{equation*}
$$

and hence the quantities that are coefficients of $\sin \phi$ (or of $\sin 2 \phi=2 \sin \phi \cos \phi$ ) are the TPs.

As noted above, while the angular distribution for the $\bar{B}$ decay involves $\phi$, for $B$ it involves $-\phi$. Thus, the CP-violating triple-product asymmetry is proportional to the sum of $\bar{B}$ and $B$ TPs.

The first TP is $A_{T}^{(i m) \bar{B}(B)}$, introduced above in Eq. (7.9). In terms of the coupling constants and matrix elements defined in Ref. [11], $A_{T}^{(i m) \bar{B}(B)}$ can be written as

$$
\begin{equation*}
A_{T}^{(i m) \bar{B}}=\frac{4 I_{5}^{T}}{3 A_{T}^{\bar{B}}}, \quad A_{T}^{(i m) B}=-\frac{4 \bar{I}_{5}^{T}}{3 A_{T}^{B}} . \tag{7.18}
\end{equation*}
$$

We observe that $A_{T}^{(i m)}$ depends only on the VA couplings. The CP-violating tripleproduct asymmetry is

$$
\begin{equation*}
A_{T}^{(i m)}=\frac{1}{2}\left(A_{T}^{(i m) \bar{B}}+A_{T}^{(i m) B}\right) \tag{7.19}
\end{equation*}
$$

Fig. 7 shows $A_{T}^{(i m)}\left(q^{2}\right)$ for $\bar{B}_{d}^{0} \rightarrow \bar{K}^{*} \mu^{+} \mu^{-}$in the presence of new VA couplings. We make the following observations:

- If only $R_{V, A}$ couplings are present, $A_{T}^{(i m)}\left(q^{2}\right)$ can be enhanced up to $5 \%$ at low $q^{2}$ and can have either sign. On the other hand, $A_{T}^{(i m)}\left(q^{2}\right)$ is almost same as the SM prediction $(\simeq 0)$ at high $q^{2}$.
- If only $R_{V, A}^{\prime}$ couplings are present, $A_{T}^{(i m)}\left(q^{2}\right)$ can be enhanced up to $49 \%$ at low $q^{2}$, and up to $46 \%$ at high $q^{2}$. It can have either sign at both low and high $q^{2}$.
- When both primed and unprimed VA couplings are present, the results for $A_{T}^{(i m)}\left(q^{2}\right)$ are almost the same as those obtained with only $R_{V, A}^{\prime}$ couplings (see Fig. 7).


Figure 7: The left (right) panel of the figure shows $A_{T}^{(i m)}\left(q^{2}\right)$ for $\bar{B}_{d}^{0} \rightarrow \bar{K}^{*} \mu^{+} \mu^{-}$in the low- $q^{2}$ (high- $q^{2}$ ) region, in the scenario where $\left(R_{V}, R_{A}, R_{V}^{\prime}, R_{A}^{\prime}\right)$ terms are all present. The green line corresponds to the SM prediction. The other lines show predictions for some representative values of the NP parameters. For example, the blue curve in the low- $q^{2}$ and high- $q^{2}$ regions corresponds to $\left(1.33 e^{-i 2.96}, 0.78 e^{i 2.47}, 0.83 e^{-i 0.27}, 3.15 e^{i 1.75}\right)$ and $\left(2.15 e^{-i 2.77}, 0.7 e^{-i 2.43}, 8.20 e^{-i 0.16}, 4.8 e^{-i 1.62}\right)$, respectively.

The second TP is $A_{L T}^{(i m) \bar{B}(B)}$, introduced above in Eq. (7.12). In terms of the coupling constants and matrix elements defined in Ref. [11], $A_{L T}^{(i m) \bar{B}(B)}$ can be written as

$$
\begin{equation*}
A_{L T}^{(i m) \bar{B}}=\frac{I_{4}^{L T}}{4\left(A_{L}^{B}+A_{T}^{\bar{B}}\right)}, \quad A_{L T}^{(i m) B}=\frac{\bar{I}_{4}^{L T}}{4\left(A_{L}^{B}+A_{T}^{B}\right)} . \tag{7.20}
\end{equation*}
$$

We observe that $A_{L T}$ depends on the VA couplings, as well as on V-S and SP-T interference terms. The CP-violating triple-product asymmetry is

$$
\begin{equation*}
A_{L T}^{(i m)}=\frac{1}{2}\left(A_{L T}^{(i m) \bar{B}}-A_{L T}^{(i m) B}\right) . \tag{7.21}
\end{equation*}
$$

Fig. 8 shows $A_{L T}^{(i m)}\left(q^{2}\right)$ for $\bar{B}_{d}^{0} \rightarrow \bar{K}^{*} \mu^{+} \mu^{-}$in the presence of new VA couplings. We make the following observations:

- If only $R_{V, A}$ couplings are present, $A_{L T}^{(i m)}\left(q^{2}\right)$ can be enhanced up to $6 \%$ at very low $q^{2}$, and is almost same as the SM prediction $(\approx 0)$ at high $q^{2}$. It can have either sign at both low and high $q^{2}$.
- If only $R_{V, A}^{\prime}$ couplings are present, $A_{L T}^{(i m)}\left(q^{2}\right)$ can be enhanced up to $8 \%$ at low $q^{2}$ and is almost same as the SM prediction $(\approx 0)$ at high $q^{2}$. It can have either sign at both low and high $q^{2}$.
- When both primed and unprimed VA couplings are present, $A_{L T}^{(i m)}\left(q^{2}\right)$ can be enhanced up to $10 \%$ at low $q^{2}$ and up to $0.5 \%$ at high $q^{2}$. It can have either sign at both low and high $q^{2}$ (see Fig. 8).


Figure 8: The figure shows $A_{L T}^{(i m)}\left(q^{2}\right)$ for $\bar{B}_{d}^{0} \rightarrow \bar{K}^{*} \mu^{+} \mu^{-}$in the low- $q^{2}$ region, in the scenario where $\left(R_{V}, R_{A}, R_{V}^{\prime}, R_{A}^{\prime}\right)$ terms are all present. The green line corresponds to the SM prediction. The other lines show predictions for some representative values of the NP parameters. For example, the blue curve corresponds to $\left(1.68 e^{i 1.92}, 2.27 e^{i 0.53}, 4.22 e^{i 0.28}, 0.14 e^{-i 1.91}\right)$.

Note that $A_{L T}^{(i m)}\left(q^{2}\right)$ is related to the observable $A_{7}^{D}$ in Ref. [24]: $A_{L T}^{(i m)}\left(q^{2}\right) \approx$ $A_{7}^{D} / 8$. Our limit of $10 \%$ on the maximum value of $A_{L T}^{(i m)}\left(q^{2}\right)$ is then consistent with the limit of 0.76 on the average value $\left\langle A_{7}^{D}\right\rangle$ over the low- $q^{2}$ region, as calculated in Ref. [24]. However, in addition we present the full $q^{2}$-dependence of this quantity.

In principle, $A_{L T}^{(i m) \bar{B}(B)}$ can be generated due to NP SP-T interference. However, we find that the effect is tiny: $A_{L T}^{(i m)}\left(q^{2}\right)$ can be enhanced up to $0.4 \%$ at low $q^{2}$ and can have either sign; $A_{L T}^{(i m)}\left(q^{2}\right)$ is same as the SM $(\simeq 0)$ at high $q^{2}$.

## 8. Discussion and summary

Even after the successful start of the LHC that will search for new physics (NP) at the TeV scale and beyond, $B$ decays still remain one of the best avenues of detecting indirect NP signals. The copious amount of data on $B$ decays, expected from future experiments like the LHC and super- $B$ factories, will allow us to explore in detail many decay modes that are currently considered to be rare. The combined analysis of many such decay modes will allow us to look for NP in a model-independent manner.

We consider all possible Lorentz structures of new physics (NP) in the $b \rightarrow$ $s \mu^{+} \mu^{-}$transition, and analyze their effects on the CP-violating observables in (i) $\bar{B}_{s}^{0} \rightarrow \mu^{+} \mu^{-}$, (ii) $\bar{B}_{d}^{0} \rightarrow X_{s} \mu^{+} \mu^{-}$, (iii) $\bar{B}_{s}^{0} \rightarrow \mu^{+} \mu^{-} \gamma$, (iv) $\bar{B}_{d}^{0} \rightarrow \bar{K} \mu^{+} \mu^{-}$, (v) $\bar{B}_{d}^{0} \rightarrow$ $\bar{K}^{*} \mu^{+} \mu^{-}$, and their CP-conjugate modes. These are the same modes we explored in the companion paper [11], where we considered only CP-conserving quantities. We find that for $\bar{B}_{s}^{0} \rightarrow \mu^{+} \mu^{-}$, the only CP-violating quantities that can be constructed even in principle require the measurement of muon polarization, a task not possible
in foreseeable detectors. Therefore, we do not dwell on this mode further. For the rest of the modes, we focus on

- CP violation in the differential branching ratio $\left(A_{C P}\right)$, and
- CP violation in the forward-backward asymmetry $\left(\Delta A_{F B}\right)$.

In addition, for $\bar{B}_{d}^{0} \rightarrow \bar{K}^{*} \mu^{+} \mu^{-}$, we analyze

- the CP asymmetry in the longitudinal polarization fraction $\left(\Delta f_{L}\right)$,
- the CP asymmetries $\Delta A_{T}^{(2)}$ and $\Delta A_{L T}$ arising in the angular distributions, and
- the triple-product (TP) CP asymmetries $\Delta A_{T}^{(i m)}$ and $\Delta A_{L T}^{(i m)}$.

We determine the constraints on the coupling constants in the effective NP operators by using the currently available data. On the basis of these limits and general arguments, we expect that the CP-violating quantities in most of the modes can only be sensitive to the vector-axial vector (VA) couplings, while the scalar-pseudoscalar (SP) and the tensor (T) NP operators can only contribute, if at all, to certain TP asymmetries. Our later detailed exploration of the allowed parameter space for all the NP couplings vindicates this argument. The effects of SP and T NP operators are therefore discussed only briefly in this paper.

On the other hand, the VA operators can have a significant impact on the CPviolating observables. (See Table 1). The SM predicts $A_{C P}\left(q^{2}\right) \lesssim 10^{-3}$ for all the modes, while VA NP operators allow this quantity to be as large as $\sim 10 \%$ (for $\bar{B}_{d}^{0} \rightarrow X_{s} \mu^{+} \mu^{-}, \bar{B}_{d}^{0} \rightarrow \bar{K} \mu^{+} \mu^{-}$and $\left.\bar{B}_{d}^{0} \rightarrow \bar{K}^{*} \mu^{+} \mu^{-}\right)$and even up to $\sim 30 \%$ for $\bar{B}_{s}^{0} \rightarrow \mu^{+} \mu^{-} \gamma$. Even $\Delta A_{F B}$, expected to be $\lesssim 10^{-4}$ in the SM, can be enhanced up to $\sim 10 \%$ (for $\bar{B}_{d}^{0} \rightarrow X_{s} \mu^{+} \mu^{-}$) and up to $\sim 40 \%$ (for $\bar{B}_{s}^{0} \rightarrow \mu^{+} \mu^{-} \gamma$ ). While $\Delta A_{F B}$ in $\bar{B}_{d}^{0} \rightarrow \bar{K} \mu^{+} \mu^{-}$stays zero even with VA NP, its value in $\bar{B}_{d}^{0} \rightarrow \bar{K}^{*} \mu^{+} \mu^{-}$may be enhanced to $\sim 10 \%$ from its SM expectation of $\lesssim 10^{-4}$.

In $\bar{B}_{d}^{0} \rightarrow \bar{K}^{*} \mu^{+} \mu^{-}$the SM predicts $\Delta f_{L} \lesssim 10^{-4}$, while VA NP operators allow this quantity to be enhanced up to $\sim 10 \% . \Delta A_{T}^{(2)}, \Delta A_{L T}, A_{T}^{(i m)}$ and $A_{L T}^{(i m)}$ are all zero in the SM. VA NP operators can enhance $\Delta A_{T}^{(2)}$ up to $\sim 12 \%, A_{T}^{(i m)}$ even up to $\sim 50 \%$, and $A_{L T}^{(i m)}$ up to $\sim 10 \%$. $\Delta A_{L T}$ can not be enhanced more than $\sim 3 \%$ even in the presence of VA NP operators. Note that while in almost all the cases the impact of the left-handed VA NP couplings $R_{V, A}$ is dominant, for the TP asymmetry $\Delta A_{T}^{(i m)}$, the $R_{V, A}^{\prime}$ couplings play a dominating role.

TP's can also be generated by NP-NP interference. However, we do not find large effects. The interference of SP-T operators can increase $A_{L T}^{(i m)}\left(q^{2}\right)$ up to only $0.4 \%$ at low $q^{2}$.

It is quite possible that if the NP is of the VA type, its presence would first be indicated through the CP-conserving/CP-averaged quantities considered in Ref. [11]. However, the CP-violating signals considered in this paper are so robust (orders of

| Observable | SM | Only new VA | Only new SP | Only new T |
| :---: | :---: | :---: | :---: | :---: |
| $\begin{array}{r} \bar{B}_{d}^{0} \rightarrow X_{s} \mu^{+} \mu^{-} \\ A_{\mathrm{CP}} \\ \Delta A_{F B} \end{array}$ | $\begin{aligned} & \text { - } 10^{-3} \rightarrow 10^{-4} \\ & \left(\text { low } \rightarrow \text { high } q^{2}\right) \\ & 10^{-4} \rightarrow 10^{-5} \\ & \left(\text { low } \rightarrow \text { high } q^{2}\right) \end{aligned}$ | - $(6 \rightarrow 12) \%$ <br> (low $\rightarrow$ high $q^{2}$ ) <br> - $(3 \rightarrow 12) \%$ <br> (low $\rightarrow$ high $q^{2}$ ) | - Marginal S $\bullet<1 \%$ | - Marginal S/E <br> No effect |
| $\begin{array}{r} \overline{\bar{B}}_{s}^{0} \rightarrow \mu^{+} \mu^{-} \gamma \\ A_{\mathrm{CP}} \\ \Delta A_{F B} \end{array}$ | $\begin{aligned} & \text { - } 10^{-3} \rightarrow 10^{-4} \\ & \left(\text { low } \rightarrow \text { high } q^{2}\right) \\ & 10^{-4} \rightarrow 10^{-5} \\ & \left(\text { low } \rightarrow \text { high } q^{2}\right) \end{aligned}$ | - $(30 \rightarrow 13) \%$ <br> (low $\rightarrow$ high $q^{2}$ ) <br> - $(40 \rightarrow 18) \%$ <br> (low $\rightarrow$ high $q^{2}$ ) | No effect <br> No effect | $\begin{aligned} & \bullet<1 \% \\ & \bullet<1 \% \end{aligned}$ |
| $\begin{array}{r} \bar{B}_{d}^{0} \rightarrow \bar{K} \mu^{+} \mu^{-} \\ A_{\mathrm{CP}} \\ \Delta A_{F B} \end{array}$ | $\begin{aligned} & \text { - } 10^{-3} \rightarrow 10^{-4} \\ & \left(\text { low } \rightarrow \text { high } q^{2}\right) \\ & \text { Zero } \end{aligned}$ | $\begin{aligned} & \text { - }(7 \rightarrow 12) \% \\ & \left(\text { low } \rightarrow \text { high } q^{2}\right) \\ & \text { No effect } \end{aligned}$ | - Marginal S $\bullet<1 \%$ | - Marginal S/E <br> No effect |
| $\begin{array}{r} \bar{B}_{d}^{0} \rightarrow \bar{K}^{*} \mu^{+} \mu^{-} \\ A_{\mathrm{CP}} \end{array}$ | $\begin{aligned} & -10^{-3} \rightarrow 10^{-4} \\ & \left(\text { low } \rightarrow \text { high } q^{2}\right) \end{aligned}$ | $\begin{aligned} & \bullet(9 \rightarrow 14) \% \\ & \left(\operatorname{low} \rightarrow \text { high } q^{2}\right) \end{aligned}$ | No effect | - < 1\% |
| $\Delta A_{F B}$ | $\begin{aligned} & \text { - } 10^{-4} \rightarrow 10^{-6} \\ & \left(\text { low } \rightarrow \text { high } q^{2}\right) \end{aligned}$ | $\begin{aligned} & \bullet(6 \rightarrow 19) \% \\ & \left(\text { low } \rightarrow \text { high } q^{2}\right) \end{aligned}$ | No effect | - < $1 \%$ |
| $\Delta f_{L}$ | $\begin{aligned} & \bullet 10^{-4} \rightarrow 10^{-7} \\ & \left(\text { low } \rightarrow \text { high } q^{2}\right) \end{aligned}$ | $\begin{aligned} & \bullet(9 \rightarrow 16) \% \\ & \left(\operatorname{low} \rightarrow \text { high } q^{2}\right) \end{aligned}$ | No effect | - < $1 \%$ |
| $\Delta A_{T}^{(2)}$ | Zero | - ~ $12 \%$ | No effect | No effect |
| $\Delta A_{L T}$ | Zero | - $<3 \%$ | No effect | No effect |
| $A_{T}^{(i m)}$ | Zero | - ~ $50 \%$ | No effect | No effect |
| $A_{L T}^{(i m)}$ | Zero | - $\sim 10 \%$ | No effect | No effect |

Table 1: The effect of NP couplings on observables. E: enhancement, S: suppression. The numbers given are optimistic estimates.
magnitude more than the SM predictions) that these may be the ones that will unambiguously establish the presence of NP of the VA kind. Moreover, hadronic
uncertainties play a very minor role in the CP-violating asymmetries considered in this paper. A combined analysis of CP-violating and CP-conserving signals may allow even the determination of the magnitudes and phases of the NP coupling constants, in addition to confirming the NP Lorentz structure.

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