

Impact of photon addition and subtraction on nonclassical and phase properties of a displaced Fock state

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Abstract

Various nonclassical and quantum phase properties of photon added then subtracted displaced Fock state have been examined systematically and rigorously. Higher-order moments of the relevant bosonic operators are computed to test the nonclassicality of the state of interest, which reduces to various quantum states (having applications in quantum optics, metrology and information processing) in different limits ranging from the coherent (classical) state to the Fock (most nonclassical) states. The nonclassical features are discussed using Klyshko's, Vogel's, and Agarwal-Tara's criteria as well as the criteria of lower- and higher-order antibunching, sub-Poissonian photon statistics and squeezing. In addition, phase distribution function and quantum phase fluctuation have been studied. These properties are examined for various combinations of number of photon addition and/or subtraction and Fock parameter. The examination has revealed that photon addition generally improves nonclassicality, and this advantage enhances for the large (small) values of displacement parameter using photon subtraction (Fock parameter). The higher-order sub-Poissonian photon statistics is only observed for the odd orders. In general, higher-order nonclassicality criteria are found to detect nonclassicality even in the cases when corresponding lower-order criteria failed to do so. Photon subtraction is observed to induce squeezing, but only large number of photon addition can be used to probe squeezing for large values of displacement parameter. Further, photon subtraction is found to alter the phase properties more than photon addition, while Fock parameter has an opposite effect of the photon addition/subtraction. Finally, nonclassicality and non-Gaussianity is also established using Q function.

1 Introduction

As nonclassical states do not have any classical counterpart they are essential for achieving quantum supremacy ([1] and references therein). The notion of nonclassicality used in the present work is defined by the negative values of Glauber-Sudarshan P function which implies that a nonclassical state cannot be expressed as a mixture of coherent states [2, 3]. Such states have been studied for long, but their importance has been enhanced with the recent developments in quantum computation, communication and sensing (see [4–6] and references therein). In fact, in the recent past, various exciting applications of nonclassicality ranging from satellite based quantum key distribution (QKD) [7, 8] to the detection of gravitational wave in LIGO [9, 10], have been reported, and those have helped to establish that quantum supremacy cannot be established without the use of nonclassical states [1]. Nonclassical states, in general, can be created using various types of physical resources (e.g., PT symmetric systems [11, 12], quantum walk [13–15], atom-optical interactions [16, 17], nonlinear optical couplers [18, 19], nonlinear optical processes [20, 21], Bose-Einstein condensate [22]). We are particularly interested in the realizations based on photonics, where nonclassical states are generated by using linear and nonlinear optical components, including mirrors, beam splitters, detectors, wave plates, nonlinear crystals, pentaprism, beam displacer and retroreflectors [23]. The art of generating the required quantum states is known as the quantum state engineering [24–27], which plays a very crucial role in generating various quantum states which are required for the quantum information processing.

There are some distinct theoretical tools for performing quantum state engineering, like quantum scissoring [28], hole-burning [29–31] or filtering out a particular Fock state from the photon number distribution [32], applying non-Gaussianity inducing operations [33]. However, these distinct mechanisms are realized primarily by appropriately using beam-splitter, mirror and single photon detectors or single photon counting module. Without going into finer details of optical realization of quantum state engineering tools, we may note that these tools can be used to generate various nonclassical states, e.g.,

displaced Fock state (DFS)¹ [34], photon added DFS [35], photon subtracted DFS [35], photon added squeezed coherent state [36], photon subtracted squeezed coherent state [36], number state filtered coherent state [32]. Some of these states, like photon added coherent state (PACS), have already been realized experimentally [37].

Many of the above mentioned engineered quantum states have already been studied in detail. Primarily, three types of investigations have been performed on the engineered quantum states- (i) study of various nonclassical features of these states (and their variation with the state parameters) as reflected through different witnesses of nonclassicality. Initially, such studies were restricted to the lower-order nonclassical features. In the recent past, various higher-order nonclassical features have been predicted theoretically [38–42] and confirmed experimentally ([43, 44] and references therein) in quantum states generated in nonlinear optical processes. (ii) Phase properties of the nonclassical states have been studied [45] by computing quantum phase fluctuations, phase dispersion, phase distribution functions, etc., under various formalisms, like Susskind and Glogower [46], Pegg-Barnett [47] and Barnett-Pegg [48] formalisms. (iii) Various applications of the engineered quantum states have been designed. For example, antibunching is used for the characterization of the single photon source [40], squeezing (specifically, squeezed vacuum) is used to achieve the required precision in measurement in the LIGO experiment [9, 10], and entanglement is used in quantum teleportation [49] and quantum cryptography [50]. More interestingly, many of the engineered quantum states have been used in continuous variable quantum cryptography (some of which will be mentioned in the next section) ([51, 52] and references therein).

Motivated by the above observations, in what follows, we would like to perform an investigation on a particularly interesting engineered quantum state which would have the flavor of all the three facets of studies mentioned above. Specifically, in what follows, we aim to study the nonclassical (both lower- and higher-order) and phase properties of a photon added then subtracted DFS (PASDFS) which can be obtained by applying non-Gaussianity inducing operators on DFS. The reason behind selecting this particular state lies in the fact that this is a general state in the sense that in the limiting cases, this state reduces to different quantum states having known applications in continuous variable quantum cryptography (this point will be further elaborated in the next section).

As it appears from the above discussion, this investigation has two facets. Firstly, we wish to study nonclassical features of PASDFS, namely Klyshko’s [53], Agarwal-Tara’s [54], Vogel’s [55] criteria, lower- and higher-order antibunching [40], squeezing [56–58], and sub-Poissonian photon statistics (HOSPS) [59]. We subsequently study the phase properties of PASDFS by computing phase distribution function [60, 61], phase fluctuation parameters [48, 62], and phase dispersion [63]. A detailed analysis of the obtained results will also be performed to reveal the usefulness of the obtained results.

The rest of the paper is organized as follows. In the next section, we describe the quantum state of interest (i.e., PASDFS) in Fock basis and calculate the analytic expressions for the higher-order moments of the relevant field operators for this state. In Section 3, we investigate the possibilities of witnessing various nonclassical features in PASDFS and its limiting cases by using a set of moments-based criteria for nonclassicality. Variations of nonclassical features (witnessed through different criteria) with various physical parameters are also discussed here. In Section 4, phase properties of PASDFS are studied. Q function for PASDFS is obtained in Section 5. Finally, we conclude in Section 6.

2 Quantum states of our interest

As mentioned in the previous section, this paper is focused on PASDFS. Before, expressing PASDFS in Fock basis, we may note that a DFS can be prepared by applying displacement operator on Fock state $|n\rangle$, and the same can be expanded in Fock basis as

$$|\phi(n, \alpha)\rangle = \hat{D}(\alpha)|n\rangle = \sum_{m=0}^{\infty} C_m(\alpha, n)|m\rangle, \quad (1)$$

where $C_m(\alpha, n) = \langle m | \hat{D}(\alpha) | n \rangle$. A PASDFS can be obtained by sequentially applying appropriate number of annihilation (photon subtraction) and creation (photon addition) operators on a DFS. Analytical expression for PASDFS (specifically, a k photon added and then q photon subtracted DFS) in Fock basis can be shown to be

$$|\psi(k, q, n, \alpha)\rangle = N \hat{a}^q \hat{a}^{\dagger k} |\phi(n, \alpha)\rangle = N \sum_{m=0}^{\infty} C_m(\alpha, n, k, q) |m + k - q\rangle, \quad (2)$$

where $N = \left[\sum_{m=0}^{\infty} |C_m(\alpha, n, k, q)|^2 \right]^{-\frac{1}{2}}$ is the normalization factor. A bit of computation yields the expression for higher-order moment of annihilation and creation operator as

¹DFS is also referred to as generalized coherent state and displaced number state ([34, 35] and references therein).

$$\langle \hat{a}^{\dagger t} \hat{a}^j \rangle \equiv \langle \psi(k, q, n, \alpha) | \hat{a}^{\dagger t} \hat{a}^j | \psi(k, q, n, \alpha) \rangle = N^2 \sum_{m=0}^{\infty} C_m^*(\alpha, n, k, q) C_{m-j+t}(\alpha, n, k, q) \frac{\sqrt{(m+k-q)!(m+k-q-j+t)!}}{(m+k-q-j)!}. \quad (3)$$

For different values of t and j , moments of any order can be obtained, and the same may be used to investigate the nonclassical properties of PASDFS and its limiting cases by using various moments-based criteria of nonclassicality. The same will be performed in the following section, but before proceeding, it would be apt to briefly state our motivation behind the selection of this particular state for the present study (or why do we find this state as interesting?).

Due to the difficulty in realizing single photon on demand sources, the unconditional security promised by various QKD schemes, like BB84 [64] and B92 [65], does not remain unconditional in the practical situations. This is where continuous variable QKD (CVQKD) becomes relevant as they do not require single photon sources. Special cases of PASDFS has already been found useful in the realization of CVQKD. For example, protocols for CVQKD have been proposed using PACS ($k = 1, q = 0, n = 0$) [66, 67], photon added then subtracted coherent states ($k = 1, q = 1, n = 0$) [51, 52], and coherent state ($k = 0, q = 0, n = 0$) [68–71]. Further, boson sampling with displaced single photon Fock states and single PACS [72] has been reported, and an m PACS ($k = m, q = 0, n = 0$) has been used for quantum teleportation [66]. Apart from these schemes of CVQKD, which can be realized by using PASDFS or its limiting cases, the fact that the photon addition and/or subtraction operation from a classical or nonclassical state can be performed experimentally using the existing technology [37, 73] has enhanced the importance of PASDFS.

3 Nonclassicality witnesses and the nonclassical features of PASDFS witnessed through those criteria

The negative value of the Glauber-Sudarshan P function characterizes nonclassicality of an arbitrary state [2, 3]. As P function is not directly measurable in experiments, many witnesses of nonclassicality have been proposed, such as, negative values of Wigner function [74, 75], zeroes of Q function [76, 77], several moments-based criteria [16, 78]. An infinite set of such moments-based criteria of nonclassicality is equivalent to P function in terms of necessary and sufficient conditions to detect nonclassicality [79]. Here, we discuss some of these moments-based criteria of nonclassicality and Q function (in Section 5) to study nonclassical properties of the state of our interest.

3.1 Lower- and higher-order antibunching

Nonclassicality witnessed through this criterion ensures that the state under consideration is a suitable choice to be used as single photon source [33, 41]. The criteria of lower- and higher-order antibunching is given in terms of moments of number operator [40]

$$d(l-1) = \langle \hat{a}^{\dagger l} \hat{a}^l \rangle - \langle \hat{a}^{\dagger} \hat{a} \rangle^l < 0. \quad (4)$$

Special case $d(1)$ corresponds to lower-order antibunching, while $l > 2$ represents $(l-1)$ th higher-order antibunching. The higher-order nonclassicality criteria are shown to be good witnesses of weak nonclassicality [18, 36, 80, 81].

The relevance of photon addition, photon subtraction, Fock, and displacement parameters in the nonclassical properties of the class of PASDFSs is studied here rigorously. Specifically, using Eq. (3) with the criterion of antibunching (4) we can study the possibilities of observing lower- and higher-order antibunching in the quantum states of PASDFS class, where the class of PASDFSs refers to all the states that can be reduced from state (2) in the limiting cases. The outcome of such a study is illustrated in Fig. 1. It is observed that the depth of lower- and higher-order nonclassicality witnesses can be increased by increasing the value of the displacement parameter, but large values of α deteriorate the observed nonclassicality (cf. Fig. 1 (a)-(b)). The nonclassicality for higher-values of displacement parameter α can be induced by subtracting photons at the cost of reduction in the depth of nonclassicality witnessed for smaller α , as shown in Fig. 1 (a). However, photon addition is always more advantageous than subtraction. Therefore, both addition and subtraction of photons illustrate these collective effects by showing nonclassicality for even higher values of α at the cost of that observed for the small values of displacement parameter. Fock parameter has completely opposite effect of photon subtraction as it shows the advantage (disadvantage) for small (large) values of displacement parameter. Figure 1 (b) shows benefit of studying higher-order nonclassicality as depth of corresponding witness of nonclassicality can be observed to increase with the order. The higher-order nonclassicality criterion is also able to detect nonclassicality for certain values of displacement parameter for which the corresponding lower-order criterion failed to do so.

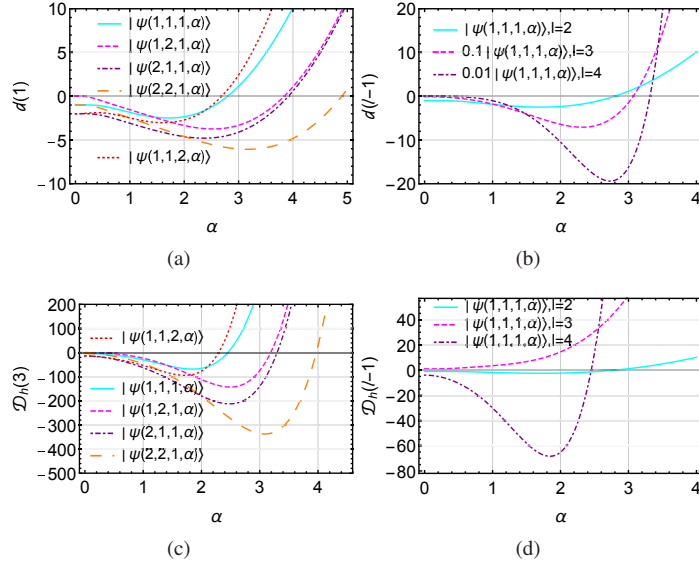


Figure 1: (Color online) For PASDFS the lower-and higher-order antibunching is given as a function of displacement parameter α . (a) Lower-order antibunching for different values of parameters of the state. (b) Higher-order antibunching for particular state. HOSPS for PASDFS for different values of (c) state parameters and (d) order of nonclassicality.

3.2 Higher-order sub-Poissonian photon statistics

Lower-order sub-Poissonian photon statistics is closely associated with lower-order antibunching (4) criteria ([36] and references therein). However, the presence of corresponding higher-order nonclassical feature, i.e., HOSPS, is independent of higher-order antibunching. HOSPS characterizes to the reduction of higher-order variance of the number operator calculated for certain quantum state with respect to the coherent state. The criterion can be expressed as [35, 36]

$$\mathcal{D}_h(l-1) = \sum_{e=0}^l \sum_{f=1}^e S_2(e, f) {}^l C_e (-1)^e d(f-1) \langle \hat{N} \rangle^{l-e} < 0, \quad (5)$$

where $S_2(e, f)$ is Stirling number of second kind, and ${}^l C_e$ is the usual binomial coefficient. HOSPS in PASDFS can be studied using Eq. (3) in Eq. (5). Variation of HOSPS nonclassicality witness for class of PASDFSs obtained by different nonclassicality inducing operations show the same effect as that of antibunching witness for all the odd orders of HOSPS, and as depicted in Fig. 1 (c). However, this nonclassical feature disappears for even orders of HOSPS (cf. Fig. 1 (d)). In case of the odd orders of HOSPS, though the depth of nonclassicality witness increases with the order, higher-order criterion is found to fail to detect nonclassicality for certain values of α when corresponding HOSPS criterion for smaller values of orders shown the nonclassicality.

3.3 Lower- and higher-order squeezing

The variance in quadrature below the corresponding value for displaced vacuum state (coherent state) can be defined as the squeezing in that quadrature [82]. Higher-order counterparts of this effect are studied in two ways Hong-Mandel-type [57, 58] and Hillery's amplitude-powered [56] squeezing. Here, we focus on the Hong-Mandel-type squeezing, which focuses on the reduction of higher-order variances of quadrature, that can be defined as [35, 36]

$$\mathcal{S}(l) = \frac{\langle (\Delta X)^l \rangle - \left(\frac{1}{2}\right)_{\frac{l}{2}}}{\left(\frac{1}{2}\right)_{\frac{l}{2}}} < 0, \quad (6)$$

where $(l)_x$ is Pochhammer symbol, and l th-order variance $\langle (\Delta X)^l \rangle$ can be written in terms of moments of annihilation and creation operators as [42]

$$\langle (\Delta X)^l \rangle = \sum_{r=0}^l \sum_{i=0}^{\frac{r}{2}} \sum_{k=0}^{r-2i} (-1)^r \frac{1}{2^{\frac{l}{2}}} (2i-1)! {}^{2i} C_k {}^l C_r {}^r C_{2i} \langle \hat{a}^\dagger + \hat{a} \rangle^{l-r} \langle \hat{a}^{\dagger k} \hat{a}^{r-2i-k} \rangle. \quad (7)$$

Only even values of l are allowed in the criterion of Hong-Mandel-type higher-order squeezing.

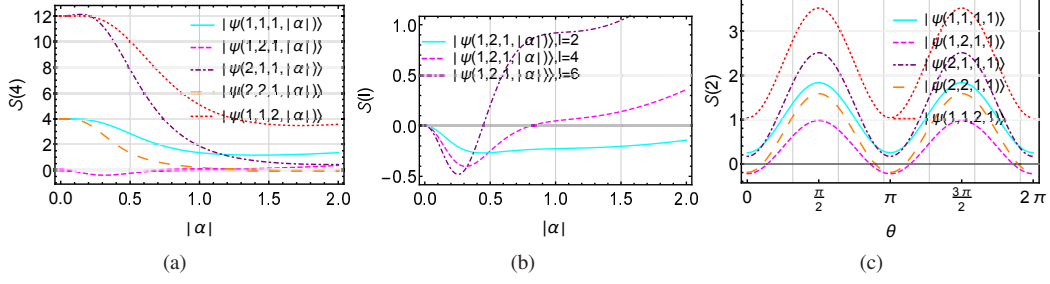


Figure 2: (Color online) Dependence of the Hong-Mandel-type higher-order squeezing witness on displacement parameter for (a) different state parameters and (b) order of squeezing. (c) Lower-order squeezing as a function of phase parameter of the state, i.e., phase θ of displacement parameter $\alpha = 1 \cdot \exp[i\theta]$.

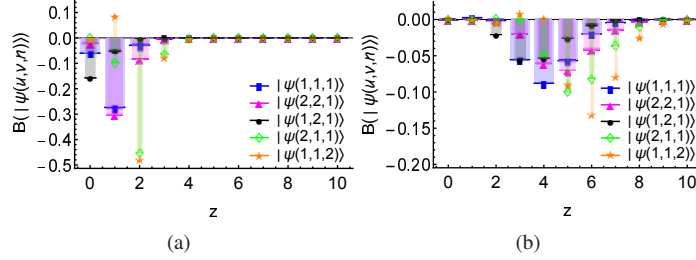


Figure 3: (Color online) Illustration of Klyshko's parameter $B(z)$ with respect to the photon number z for different values of state parameters with (a) $\alpha = 0.5$ and (b) $\alpha = 1$.

Out of all the nonclassicality inducing operations used in PASDFS only photon subtraction is squeezing inducing operation as shown in Fig. 2, which is consistent with some of our recent observations [35]. With photon addition higher-order squeezing can be induced for large values of modulus of displacement parameter at the cost of squeezing observed for small $|\alpha|$ as long as the number of photon subtracted is more than the value of Fock parameter. As far as higher-order squeezing is concerned, the observed nonclassicality disappears for large values of real displacement parameter with increase in the depth of the nonclassicality witness. Squeezing being a phase dependent nonclassical feature depends on the phase θ of the displacement parameter $\alpha = |\alpha| \exp[i\theta]$ (shown in Fig. 2 (c) for lower-order squeezing).

3.4 Klyshko's Criterion

Another interesting criterion of nonclassicality is based on the probability of three successive photon numbers, i.e., $z, z + 1, z + 2$, and is known as Klyshko's criterion [53]. It can be defined in terms of probability p_z of detecting z number of photons as

$$B(z) = (z + 2)p_z p_{z+2} - (z + 1)(p_{z+1})^2 < 0. \quad (8)$$

For PASDFS p_z can be obtained from Eq. (3). Nonclassicality reflected through Klyshko's criterion can be controlled by all the state engineering operations used here as shown in Fig. 3. The depth of this nonclassicality witness increases at higher values of photon numbers z due to increase in photon addition and/or Fock parameter. In contrast, depth of witness increases at smaller photon numbers z due to photon subtraction. The Klyshko's nonclassicality witness is positive for some photon numbers only if $k + n > q$. Additionally, with increase in displacement parameter the depth of nonclassicality witness decreases, and the weight of the distribution of witness shift to higher values of z .

3.5 Agarwal-Tara's criterion

A moments-based criterion of nonclassicality was introduced in terms of higher-order moments of number operator by Agarwal and Tara [54]. This criterion can be written as

$$A_3 = \frac{\det m^{(3)}}{\det \mu^{(3)} - \det m^{(3)}} < 0, \quad (9)$$

where the matrices are given as

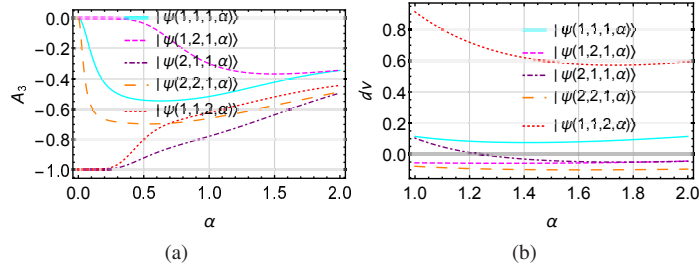


Figure 4: (Color online) Nonclassicality reflected through the negative values of (a) Agarwal-Tara's and (b) Vogel's criteria as a function of α for different state parameters.

$$s^{(3)} = \begin{bmatrix} 1 & s_1 & s_2 \\ s_1 & s_2 & s_3 \\ s_2 & s_3 & s_4 \end{bmatrix} \quad (10)$$

with $s_i \in \{m_i, \mu_i\}$ and $m_n = \langle \hat{a}^\dagger^n \hat{a}^n \rangle$ and $\mu_n = \langle (\hat{a}^\dagger \hat{a})^n \rangle$. This nonclassicality witness is able to detect nonclassicality in all the quantum states in the class of PASDFSs (cf. Fig. 4 (a)). Note that for $|\psi(1, 2, 1, \alpha)\rangle$ with small α , A_3 parameter is close to zero, which is due to very high probability for zero photon states.

3.6 Vogel's criterion

The moments-based nonclassicality criterion of the previous subsection was later extended to Vogel's nonclassicality criterion [55] in terms of matrix of moments as

$$v = \begin{bmatrix} 1 & \langle \hat{a} \rangle & \langle \hat{a}^\dagger \rangle \\ \langle \hat{a}^\dagger \rangle & \langle \hat{a}^\dagger \hat{a} \rangle & \langle \hat{a}^{\dagger 2} \rangle \\ \langle \hat{a} \rangle & \langle \hat{a}^2 \rangle & \langle \hat{a}^\dagger \hat{a} \rangle \end{bmatrix}. \quad (11)$$

The negative value of the determinant dv of matrix v in Eq. (11) is signature of nonclassicality. Fock parameter has adverse effect on the nonclassicality in PASDFS detected by this criterion. This adverse effect can be compensated by photon subtraction and can be further controlled by photon addition (as shown in Fig. 4 (b)). Notice that the nonclassical behavior illustrated by Agarwal-Tara's (Vogel's) criterion is related to higher-order antibunching (squeezing) criterion. However, nonclassicality witness of Vogel's criterion is a phase independent property unlike squeezing.

4 Phase properties of PASDFS

The nonclassicality inducing operations are also expected to impact the phase properties of a quantum state [83, 84]. Recently, we have reported an extensive study on the role that such quantum state engineering tools can play in application oriented studies on quantum phase [45]. Specifically, relevance in quantum phase estimation, phase fluctuation, and phase distribution were discussed which can play an important role in quantum metrology [85]. Here, we briefly discuss some of the phase properties of the class of PASDFSs.

4.1 Phase distribution function

Phase distribution function for an arbitrary density operator ϱ is defined as [86]

$$P(\theta) = \frac{1}{2\pi} \langle \theta | \varrho | \theta \rangle, \quad (12)$$

where phase state $|\theta\rangle$ is

$$|\theta\rangle = \sum_{n=0}^{\infty} \exp^{in\theta} |n\rangle. \quad (13)$$

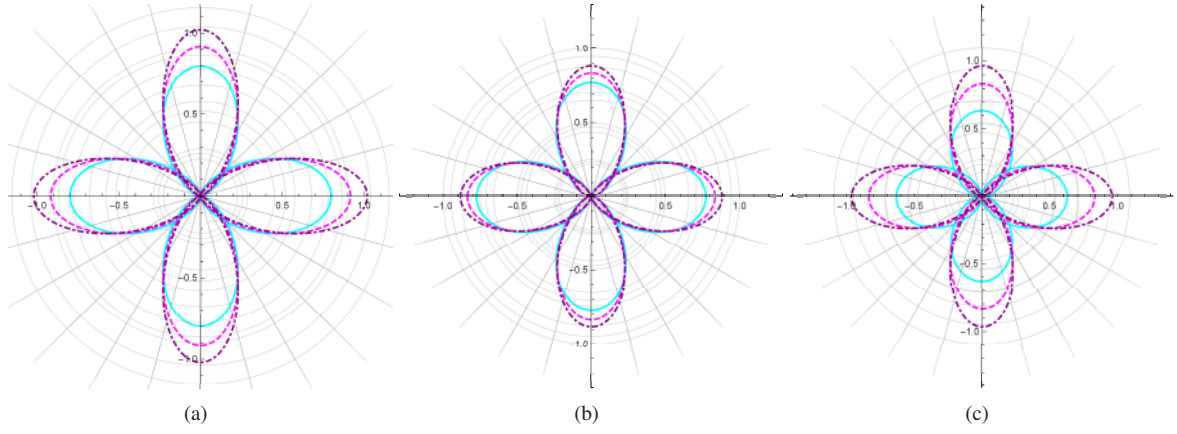


Figure 5: (Color online) Polar plot of phase distribution function for PASDFS $|\psi(k, q, n, \alpha)\rangle$ with respect to variation in displacement parameter for (a) $n = 1$, $k = 2$ and $q = 1, 2$, and 3 represented by the smooth (cyan), dashed (magenta), and dot-dashed (purple) lines, respectively; (b) $n = 2$, $q = 2$ and $k = 1, 2$, and 3 illustrated by the smooth (cyan), dashed (magenta), and dot-dashed (purple) lines, respectively; and (c) $n = 1$ with $k = q = 1, 2$, and 3 shown by the smooth (cyan), dashed (magenta), and dot-dashed (purple) lines, respectively.

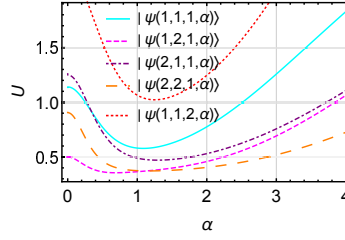


Figure 6: (Color online) Variation of phase fluctuation parameter with displacement parameter for various state parameters in PASDFS.

Fock states, used here for preparation of PASDFS, have uniformly distributed phase $P_\theta = \frac{1}{2\pi}$. Further, photon addition and subtraction are also used in PASDFS after applying displacement operator on Fock state. The analytical expression for phase distribution function for PASDFS can be computed as

$$P(\theta) = \frac{N^2}{2\pi} \left| \sum_{m=0}^{\infty} C_m(\alpha, n, k, q) \exp[l(m+k-q)\theta] \right|^2. \quad (14)$$

Photon subtraction can be observed to be a more effective tool to alter phase properties of PASDFS than photon addition, as shown in Fig. 5. Interestingly, photon addition shows similar behavior, though less prominent, as photon subtraction, Fock parameter has opposite effect.

4.2 Phase Fluctuation

The idea of phase operator was first given by Dirac [87] with the assumption that \hat{a} can be written as multiplication of a unitary operator and a Hermitian function of \hat{N} but it led to an uncertainty relation that lacked physical meaning. Later, Louisell came up with an idea of periodic phase [88], following which Susskind and Glogower developed the Sine and Cosine operators [46], which was further modified by Barnett and Pegg [48] as

$$\hat{S} = \frac{1}{2l} \left[\frac{1}{(\bar{N} + 0.5)^{\frac{1}{2}}} \hat{a} - \hat{a}^\dagger \frac{1}{(\bar{N} + 0.5)^{\frac{1}{2}}} \right] \quad (15)$$

and

$$\hat{C} = \frac{1}{2} \left[\frac{1}{(\bar{N} + 0.5)^{\frac{1}{2}}} \hat{a} + \hat{a}^\dagger \frac{1}{(\bar{N} + 0.5)^{\frac{1}{2}}} \right]. \quad (16)$$

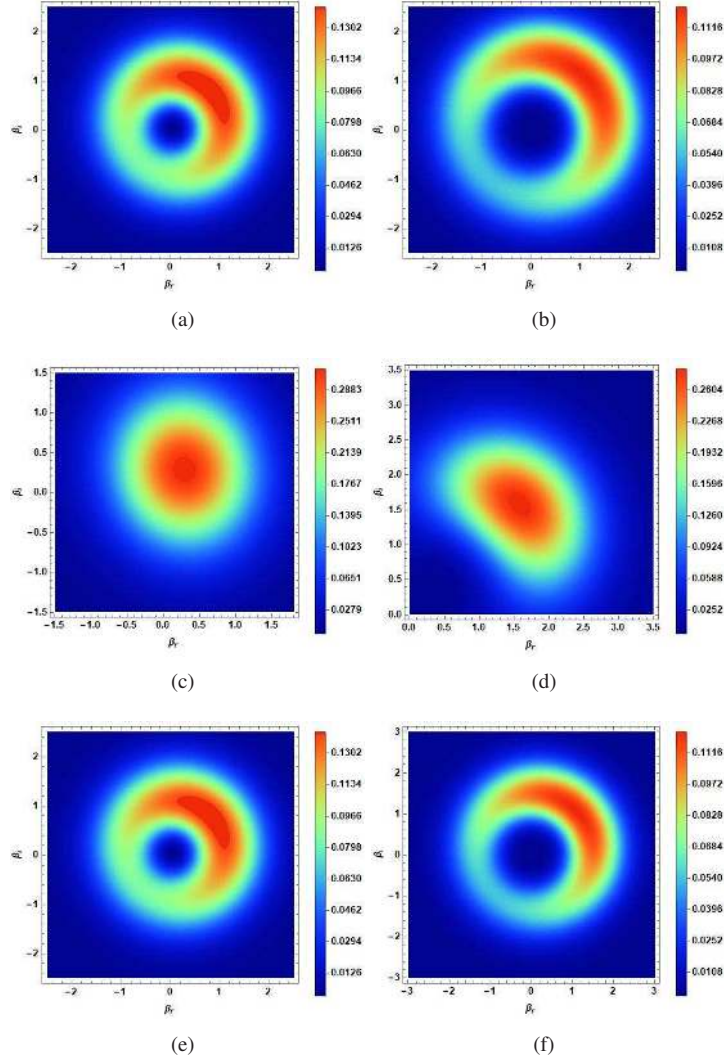


Figure 7: (Color online) Q function for PASDFS $|\psi(k, q, n, \alpha)\rangle$ with (a) $k = q = n = 1$, (b) $k = 2, q = n = 1$, and (c) $q = 2, k = n = 1$ with $\alpha = \frac{1}{5\sqrt{2}} \exp(i\pi/4)$. (d) Similarly, Q function of PASDFS with $q = 2, k = n = 1$ and $\alpha = \sqrt{2} \exp(i\pi/4)$. Q function for $|\psi(k, q, n, \alpha)\rangle$ with (e) $k = q = 1, n = 2$ and (f) $q = 1, k = n = 2$ for $\alpha = \frac{1}{5\sqrt{2}} \exp(i\pi/4)$.

Here, \bar{N} is the mean number of photons. Carruthers and Nieto [62] provided phase fluctuation parameters in terms of these operators

$$U = (\Delta N)^2 [(\Delta S)^2 + (\Delta C)^2] / [\langle \hat{S} \rangle^2 + \langle \hat{C} \rangle^2], \quad (17)$$

and $S_s = (\Delta N)^2 (\Delta S)^2$, $Q = S_s / \langle \hat{C} \rangle^2$. Here, we focus only on the first phase fluctuation parameter U , which is related to antibunching if U is below its value for coherent state (i.e., 0.5), remaining consistent with Barnett-Pegg formalism [89, 90]. One can observe that the phase fluctuation parameter is able to detect nonclassicality (specifically antibunching) only in three cases where the role of the photon subtraction is relevant (cf. Fig. 6). The observation can be seen analogous to that observed for Vogel's nonclassicality criterion.

5 Quasidistribution function: Q function

Quasidistribution functions allow us to calculate the average values of an operator analogous to classical phase space and are also witnesses of nonclassicality. For instance, negative values of Wigner and P functions and zeros of Q function are

S. No.	Nonclassical Properties	Observed in PASDFS
1	Lower-order and higher-order Antibunching	yes
2	Higher-order sub Poissonian photon statistics	yes
3	Lower-order and higher-order squeezing	yes
4	Klyshko's criterion	yes
5	Agarwal-Tara's criterion	yes
6	Vogel's criterion	yes
7	Phase distribution function	-
8	Phase fluctuation	yes
9	Q function	yes

Table 1: Summary of the nonclassical properties of PASDFS.

signatures of nonclassicality present in the quantum state [33, 91]. All these quasidistribution functions can be defined in terms of each other and are also related to the corresponding characteristic function. Therefore, one can also use the quasidistribution functions to study non-Gaussianity both qualitatively and quantitatively [92]. Non-Gaussian states are also found useful in a set of secure quantum communication schemes [51, 52, 93].

Here, we discuss only Q function of an arbitrary state ρ which can be defined as [76]

$$Q = \frac{1}{\pi} \langle \beta | \rho | \beta \rangle, \quad (18)$$

where $|\beta\rangle$ is a coherent state. The analytic form of Q function for PASDFS can be expressed as

$$Q = \frac{N^2}{\pi} \exp[-|\beta|^2] \left| \sum_{m=0}^{\infty} C_m(\alpha, n, k, q) \frac{\beta^{*(m+k-q)}}{\sqrt{(m+k-q)!}} \right|^2. \quad (19)$$

Here, We will establish non-Gaussianity inducing behavior of photon addition and Fock parameter (cf. Fig. 7), which are so far illustrated as nonclassicality inducing and phase altering operations. Clearly, with photon addition tendency of quasidistribution away from Gaussian behavior is visible, while with photon subtraction squeezing along particular phase angle chosen by displacement parameter can be observed. This squeezing can be noticed to be more appreciable for higher values of displacement parameter (cf. Fig. 7 (c)-(d)). From Fig. 7 (e)-(f), it can be observed that Fock parameter and photon addition have a similar effect in the phase space. As zeros of Q function are signature of nonclassicality, PASDFS shows nonclassicality in Fig. 7 (b), (e), and (f).

6 Conclusions

Finally, we would like to throw some light on the nonclassical behavior of PASDFS using different witnesses of lower- and higher-order nonclassicality. The significance of this choice of state is its uniqueness that a class of engineered quantum states can be achieved as the reduced case of PASDFS $|\psi(k, q, n, \alpha)\rangle$, like photon added DFS ($q = 0$), photon subtracted DFS ($k = 0$), DFS ($q = k = 0$), PACS ($n = q = 0$), photon subtracted coherent state ($n = k = 0$), coherent state ($n = k = q = 0$), and Fock state ($n = k = q = \alpha = 0$). Some of the reduced states have been experimentally realized and in some cases optical schemes for generation have been proposed, so this family of states is apt for various challenging tasks to establish quantum dominance. The state under consideration requires various non-Gaussianity inducing quantum engineering operations and thus our focus here was to analyze the relevance of each operation independently in the nonclassical features (listed in Table 1) observed in PASDFS. To study the nonclassical properties of PASDFS, a set of moments-based criteria for Klyshko's, Agrwal-Tara's, and Vogel's criteria, as well as lower- and higher-order antibunching, HOSPS, and squeezing. Further, phase properties for the same state are also studied using phase distribution function and phase fluctuation. Finally, non-Gaussianity and nonclassicality of PASDFS is also studied using Q function.

The present study reveals that with an increase in the order of nonclassicality the depth of nonclassicality witnesses increase. Additionally, higher-order nonclassicality criteria were able to detect nonclassicality in the cases when corresponding lower-order criteria failed to do so. Different nonclassical features are observed for smaller values of displacement parameter, which can be sustained for higher values by increasing the number of subtracted photon. Photon addition generally improves nonclassicality, and this advantage can be further enhanced for the higher (smaller) values of displacement parameter using photon subtraction (Fock parameter). The HOSPS nonclassical feature is only observed for the odd orders. As far as squeezing is concerned, only photon subtraction could induce this nonclassicality. Large number of photon addition can be used to

observe squeezing at higher values of displacement parameter at the cost of that present for smaller α . Photon subtraction alters the phase properties more than photon addition, while Fock parameter has an opposite effect of the photon addition/subtraction. The nonclassicality revealed through phase fluctuation parameter shows similar behavior as Vogel's criterion. Finally, we have shown the nonclassicality and non-Gaussianity of PASDFS with the help of a quasidistribution function, namely Q function.

We hope the present work focused on the characterization of nonclassicality in the class of states obtained from PASDFS will be useful in the applications like non-Gaussian quantum information processing. The present work can be further extended to quantify the amount of nonclassicality and non-Gaussianity using different measures.

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