

A note on critical flow section in collector channels

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Abstract. Generalized solution for the location of critical flow section in collector channels is presented. Based on the concept of the singularity, the dynamic equation of spatially varied flow (SVF) is solved using the flow resistance equations of von Karman (for rough regime) and Jain (for transitional and smooth regimes). The advantage of using Jain's equations is that they provide the explicit forms of the Colebrook–White and Nikuradse equations. Computational steps for the determination of critical flow section in a collector channel, being dependent on channel geometry, roughness, longitudinal bed slope and inflow discharge, are given for different channel shapes.

Keywords. Collector channel; critical flow section; hydraulics; open channel flow; spatially varied flow.

1. Introduction

In a spatially varied flow (SVF), the flow discharge varies towards the downstream direction. For example, a collector channel collects surplus water through side-weirs of a storage system. From the point of view of practical interest, a collector channel, designed for having zero discharge at the upstream starting point, receives the lateral discharge at a constant rate per unit length of channel. With discharge increasing downstream in a collector channel, a critical flow section prevails where the flow changes from subcritical to supercritical. When a control section is known, the dynamic equation of SVF can be solved. Location of a control section is rather important to design channel depth.

Bremen & Hager (1989) and Hager (1985) showed that the critical flow section could be determined with the aid of the singular-point method, if a critical flow section exists in collector channels. However, the critical flow section is dependent on channel characteristic parameters and lateral inflow discharge. An arbitrary selection of channel characteristic and flow parameters may result in supercritical flow even though a critical flow section does not exist. Guo (1999) and Dey (2000) put forward a set of design charts for the determination of critical flow section in trapezoidal and circular channels respectively, considering the Manning and Chezy equations. However, the use of flow resistance equations for rough,

A list of symbols is given at the end of the paper

transitional and smooth regimes, being the most realistic and appropriate approach, has received no attention so far.

The aim of the present study is to report a methodology for generalized solution of critical flow section in collector channels. The solution is obtained using the flow resistance equations of von Karman (for rough regime) and Jain (1976) (for transitional and smooth regimes).

2. The model

The discharge in a collector channel, having zero inflow at the starting point upstream, is the product of the constant lateral inflow discharge per unit length and the length of the channel from the starting point upstream to the section of interest. Thus, one can write

$$Q = qx, \quad (1)$$

where Q = discharge at distance x measured from the starting point upstream, and q = lateral inflow discharge per unit length. According to Chow (1959), in case of the lateral flow being normal to the collector channel (assuming hydrostatic pressure and uniform velocity distribution), the dynamic equation of an SVF is written in terms of the variation of free surface profile

$$\frac{dy}{dx} = \frac{S_o - S - 2(h/x)Fr^2}{1 - Fr^2}, \quad (2)$$

where y = flow depth, S_o = channel bed slope, S = friction slope, h = hydraulic depth ($= A/T$), A = flow area, T = top width of flow, and Fr = Froude number of flow. The Froude number is given by

$$Fr = (Q/A)/(gh)^{1/2} = Q(T)^{1/2}/(gA^3)^{1/2}, \quad (3)$$

where g = gravitational constant.

Solution of (2) can be obtained using a tail water condition or control section. The present analysis deals with the case where the discharge increases downstream resulting in a change of flow regime from subcritical to supercritical in an SVF. However, the critical flow section serves as a control section. Thus, (2) is applied to the critical flow section, where Froude number F is unity. Thus, one gets

$$S_o - S_c - 2(h_c/x_c) = 0. \quad (4)$$

In the above, subscript c refers to the parameters associated with the critical flow section. At the critical flow section, (3) becomes

$$Q_c = (gA_c^3/T_c)^{1/2}. \quad (5)$$

The location of the critical flow section x_c can be obtained using (1) and (5) as

$$x_c = (gA_c^3/q^2T_c)^{1/2}. \quad (6)$$

As no physical solution can be obtained from (4) [conditions for no solution have been discussed by Dey (1994, 1998)] for certain design parameters, designers find it difficult to confirm the existence of a critical section without any mathematical guidance. To

establish such guidance for obtaining feasible solution of (4), substituting (6) into (4) and normalizing, one gets

$$(S_c/S_o) + \left[2\hat{q} / \left(\hat{T}_c \hat{A}_c \right)^{1/2} \right] = 1, \quad (7)$$

where $\hat{q} = q/(S_o g^{0.5} l^{1.5})$, $\hat{T} = T/l$, $\hat{A} = A/l^2$, and $l =$ length parameter of channel. Also, (6) can be expressed as a normalized form as

$$\hat{x}_c = \left(\hat{A}_c^3 / \hat{T}_c \right)^{1/2} / \hat{q}, \quad (8)$$

where $\hat{x} = x S_o / l$. Henderson (1966) derived S_c using the Chezy equation. French (1985) obtained the solution for wide channels following the Henderson method. In the present study, the term S_c is determined using the flow resistance equations of von Karman (for rough regime) and Jain (1976) (for transitional and smooth regimes).

2.1 Rough regime

For a complete rough regime, the von Karman equation of flow resistance can be used for the determination of friction slope S_c , as was done by Dey (1998a,b). It is important to mention that the ASCE Task Force (1963) on the friction factor in open channels recommended that, for open channel roughness similar to that encountered in pipes, flow resistance equations similar to those of pipe flow are adequate for the estimation of friction factor. The von Karman equation is

$$1/\lambda^{1/2} = 1.14 - 0.86 \ln(k_s P_c / 4A_c), \quad (9)$$

where $\lambda =$ friction factor, $k_s =$ Nikuradse equivalent sand roughness, and $P =$ wetted perimeter. The friction factor can be extracted from the equation of bed shear as

$$\tau = (\lambda/8)\rho(Q_c/A_c)^2 = \gamma(A_c/P_c)S_c, \quad (10)$$

where $\tau =$ bed shear stress, $\rho =$ mass density of fluid, and $\gamma =$ specific weight of fluid. Using (9) and (10), one can write

$$S_c = P_c Q_c^2 / 8g A_c^3 [1.14 - 0.86 \ln(K_s P_c / 4A_c)]^2. \quad (11)$$

Replacing Q_c from (5) into (11) and normalizing, yields

$$S_c = \hat{P}_c / 8\hat{T}_c \left[1.14 - 0.86 \ln(\hat{k}_s \hat{P}_c / 4\hat{A}_c) \right]^2, \quad (12)$$

where $\hat{P} = P/l$, and $\hat{k}_s = k_s/l$.

2.2 Transitional regime

In transitional regime, the Colebrook–White equation is applicable. The advantage of using the Colebrook–White equation is that it covers the regime of flow from hydraulic smooth regime to rough regime. The rough or smooth regime can be verified from the values of relative roughness $\varepsilon (= 0.25k_s P A^{-1})$ and flow Reynolds number $Re (= 4Q P^{-1} n^{-1})$; where $\nu =$ kinematic viscosity being assumed as $10^{-6} \text{ m}^2/\text{s}$ using the well-known Moody

Table 1. Parameters of different channel sections.

l	\hat{A}	\hat{P}	\hat{T}
<i>Trapezoidal channel</i>			
b	$\hat{y}[1 + (\hat{y}/2)(m_l + m_r)]$	$1 + \hat{y}(1 + m_l^2)^{1/2} + (1 + m_r^2)^{1/2}$	$1 + \hat{y}(m_l + m_r)$
<i>Circular channel</i>			
d	$0.25[\cos^{-1}(1 - 2\hat{y}) - 2(1 - 2\hat{y}) / (\hat{y}(1 - \hat{y}))^{1/2}]$	$\cos^{-1}(1 - 2\hat{y})$	$2(\hat{y}(1 - \hat{y}))^{1/2}$

Note: In the above, b = channel width, m_l = left side slope, m_r = right side slope, d = channel diameter, and $\hat{y} = y/l$.

diagram. However, as the Colebrook–White equation is an implicit equation of λ , the use of flow resistance equation proposed by Jain (1976), which is the explicit form of the Colebrook–White equation, makes the solution simpler. The explicit form of the Colebrook–White equation is also reported elsewhere (Churchill 1977; Chue 1984; Chen 1985). The equation of Jain (1976) is

$$\frac{1}{(\lambda)^{1/2}} = 1.14 - 0.86 \ln \left((k_s P_c / 4A_c) + (6.1 v^{0.9} P_c^{0.9} / Q_c^{0.9}) \right). \tag{13}$$

Using (5) and (10), (13) is expressed in normalized form as

$$S_c = \frac{\hat{P}_c}{8\hat{T}_c \left[1.14 - 0.86 \ln \left((\hat{k}_s \hat{P}_c / 4\hat{A}_c) + (6.1 \beta^{0.9} \hat{P}_c^{0.9} \hat{T}_c^{0.45} / A_c^{1.35}) \right) \right]^2} \tag{14}$$

where $\beta = \nu / (g^{0.5} l^{1.5})$.

2.3 Smooth regime

The complete smooth regime is seldom obtained in practice. The channel is considered to be smooth when the roughness elements are submerged by the laminar-sublayer. The Nikuradse equation is applicable in smooth regime. As his equation is an implicit equation of λ , Jain (1976) also proposed an explicit form of the Nikuradse equation. The equation is

$$\frac{1}{(\lambda)^{1/2}} = 0.78 \ln (4Q_c / \nu P_c) - 1.5146. \tag{15}$$

Using (5) and (10), (15) is expressed in normalized form as

$$S_c = \hat{P}_c / 8\hat{T}_c \left[0.78 \ln \left(4A_c^{1.5} / \beta \hat{P}_c \hat{T}_c^{0.5} \right) - 1.5146 \right]^2. \tag{16}$$

3. Common channel sections

The channel parameters of some commonly available channels in practice, required for the direct use in the developed equations of the preceding section, are furnished in table 1. Hence, the two extreme cases of trapezoidal channel section are rectangular ($m_l = m_r = 0$) and triangular ($b = 0$) sections.

4. Computational steps

The equations developed in the preceding section are used to design a collector channel. As input data, the values of q , S_o , k_s and other channel characteristic parameters, such as for the rectangular channel b , the trapezoidal channel b , m_l and m_r , and the circular channel d . The steps involved for the computation are given below.

- (1) Compute \hat{q} , \hat{k}_s and β .
- (2) Consider complete rough regime. Substitute S_c from (12) in (7).
- (3) Compute \hat{y}_c numerically, using the Muller method (Conte & de Boor 1987).
- (4) Compute \hat{A}_c , \hat{P}_c and \hat{T}_c .
- (5) Using (8), compute \hat{x}_c .
- (6) Compute $x_c (= \hat{x}_c l / S_o)$.
- (7) Compute ε and Re.
- (8) Using the Moody diagram, check the hydraulic regime considered in step 2.
- (9) If agreeable to the consideration in step 2, stop computation. Otherwise, in step 2, use (14) or (16) in place of (12) and repeat steps 2 to 8.

5. Numerical illustration

To illustrate the application of the computational steps for designing a collector channel, the following numerical example is considered. The input data for a trapezoidal channel are: $q = 3.72 \text{ m}^2/\text{s}$, $S_o = 0.15$, $k_s = 0.01 \text{ m}$, $b = 3 \text{ m}$, and $m_l = m_r = 0.5$. The value of x_c is to be determined.

Using the given data, $\hat{q} = 1.524$, $\hat{k}_s = 3.333 \times 10^{-3}$, and $\beta = 6.144 \times 10^{-8}$, and considering the complete rough regime, S_c is substituted from (12) in (7). The value of \hat{y}_c , computed numerically using the Muller method (Conte & de Boor 1987), is found to be 1.828. Then, \hat{A}_c , \hat{P}_c , and \hat{T}_c are computed as 3.5, 5.088 and 2.828 respectively. Using (8), \hat{x}_c is obtained as 2.554 and $x_c (= \hat{x}_c l / S_o)$ is 51.083 m. The hydraulic regime, checked for $\varepsilon = 0.00121$ and $\text{Re} = 4.01 \times 10^6$ using the Moody diagram, confirms that the initial assumption of complete rough regime is valid.

6. Conclusions

Based on the concept of the singular-point, the dynamic equation of an SVF has been solved using the flow resistance equations of von Karman (for rough regime) and Jain (for transitional and smooth regimes) to derive a generalized solution for critical flow section in collector channels. The computational steps for the determination of critical flow section in SVF have been given for different channel shapes.

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List of symbols

A	flow area;
\hat{A}	A/l_2 ;
b	channel width;
Fr	Froude number of flow;
g	gravitational constant;
h	hydraulic depth;
k_s	Nikuradse equivalent sand roughness;
\hat{k}_s	k_s/l ;
l	length parameter of channel;
m_l, m_r	left and right side slopes respectively;
P	wetted perimeter;
\hat{P}	P/l ;
Q	discharge;
q	lateral inflow discharge per unit length;
\hat{q}	$q/(S_o g^{0.5} l^{1.5})$;
Re	Reynolds number;
S	friction slope;
S_o	channel bed slope;
T	top width of flow;
\hat{T}	T/l ;
x	distance along channel;
\hat{x}	$x S_o/l$;
y	depth of flow;
\hat{y}	y/l ;
β	$v/(g^{0.5} l^{1.5})$;
ε	relative roughness;
γ	specific weight of fluid;
λ	friction factor;
ν	kinematic viscosity of fluid;
ρ	mass density of fluid;
τ	bed shear stress.
<i>Subscript</i>	
c	critical flow.

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